APPLICATION OF HOMOTOPY PERTURBATION METHOD ON OSCILLATOR OF COMPLEX VARIABLE

Hamed Goodarzian
Islamic Azad University, Mahdishahr Branch, Mahdishahr, Iran

ABSTRACT
In this work a nearly new established analytical approach, He's homotopy-perturbation method (HPM) has been applied to solve the differential equation of oscillator with a high degree of The objective of this paper is to present an analytical investigation to analyze the vibration cubic positive nonlinearity of complex variable in nonlinear dynamic systems. The comparison of the solution obtained by HPM has are done against the numerical solution in confirmation of the applicability of the method and its effectiveness in solving problems with high degree of nonlinearity.

Keywords: *Homotopy -Perturbation method, Runge-Kutta method, Parametrically excited oscillator.*

1. INTRODUCTION
Nonlinear phenomena are of fundamental importance in various fields of science and engineering. Most nonlinear models of real-life problems are still very difficult to solve, either numerically or theoretically. Many assumptions have to be made unnecessarily to make nonlinear models solvable. One particularly interesting aspect of the physical behavior of microelectromechanical and nanoelectromechanical systems (MEMS) and (NEMS) is their nonlinear mechanical response at relatively small deviations from equilibrium. Since there are some limitations in using the common perturbation method together with the fact that this method is based on the existence of small parameter, developing the method for different applications is very difficult. Therefore, a number of semi analytical methods, including the homotopy-perturbation method (HPM), proposed by He [1-18], and the variational iteration method [19-20] have recently been developed by using new techniques to eliminate the small parameter.
It is important to investigate the dynamic behavior of these systems. Vibrations of such systems occur in wide range of mechanics, due to time-varying loads, especially periodic ones. The motivation of this paper is to extend the homotopy perturbation method (HPM) proposed oscillator of Complex Variable
In the present work, the HPM method is applied on the governing equation of Mathieu-Duffing system and the results are compared with that of the numerical method to demonstrate the effectiveness of the approach. The governing equation of Mathieu-Duffing system which is considered in this study is described by the following high-order nonlinear differential equation:

$$\ddot{\varphi}+\gamma\dot{\varphi}+[\xi+2\varepsilon\cos(2t)]\varphi+\beta\varphi^3=F_0\sin 2t \quad (1)$$

where dots indicate differentiation with respect to the time (t), $\varepsilon<<1$ is a small parameter, β is the parameter of nonlinearity, and ξ is the transient curve .
The initial condition considered in this study is defined by $\varphi=1, \dot{\varphi}=0.$

2. THE BASIC CONCEPT OF HPM
consider the following equation:

$$A(x)-f(r)=0 \quad r\in\Omega \quad (2)$$

with the boundary condition of:

$$B\left(x,\frac{\partial x}{\partial t}\right)=0 \quad r\in\Gamma \quad 3$$

where A is a general differential operator, B a boundary operator, $f(r)$ a known analytical function and Γ is the boundary of the domain Ω. A can be divided into two parts of L and N, where L is linear and N is nonlinear. Eq. (2.1) can therefore be rewritten as follows:

$$L(x)+N(x)-f(r)=0 \quad r\in\Omega \quad (4)$$

Homotopy perturbation structure is shown as follows:

$$H(\nu,p)=(1-p)\left[L(\nu)-L(x_0)\right]+p\left[A(\nu)-f(r)\right]=0 \quad (5)$$

where,

$$\nu(r,p):\ \Omega\times[0,1]\to R \qquad 6$$

In Eq. (2.4), $p\in[0\,,1]$ is an embedding parameter and x_0 is the first approximation that satisfies the boundary condition. We can assume that the solution of Eq. (2) can be written as a power series in p , as following:

$$\nu=\nu_0+p\nu_1+p^2\nu_2+\cdots=\sum_{i=0}^{n}\nu_i p^i \qquad (7)$$

and the best approximation for the solution is:

$$x=\lim_{p\to 1}\nu=\nu_0+\nu_1+\nu_2+\cdots \qquad (8)$$

3. APPLICATION OF HPM

applying the HPM on the nonlinear equation of (1),

$$(1-p)(\ddot{\varphi}+\gamma\dot{\varphi}+\xi\varphi)+p(\ddot{\varphi}+\gamma\dot{\varphi}+[\xi+2\varepsilon\cos(2t)]\varphi+\beta\varphi^3-F_0\sin 2t)=0 \qquad (9)$$

expanding the equation and collecting based on the coefficients of p -terms:

$$\begin{cases} p^0:\ddot{\varphi}_0+c\dot{\varphi}_0+k_1\varphi_0=0\\ p^1:\ddot{\varphi}_1+\gamma\dot{\varphi}_1+\xi_1\varphi_1-2\varepsilon\varphi_0+\beta\varphi_0^{\,3}+4\varepsilon\cos(t)^2\varphi_0-F_0\sin(2t)=0\\ p^2:\ddot{\varphi}_2+\gamma\dot{\varphi}_2+\xi\varphi_2-2\varepsilon\varphi_1+4\varepsilon\cos(t)^2\varphi_1+3\beta\varphi_0\varphi_1^{\,2}=0\\ p^3:.... \end{cases} \qquad (10)$$

One can now try to obtain the solution of different iterations (3.2), in the form of:

$$\varphi_0(t)=\frac{1}{2}\frac{(\gamma+\sqrt{\gamma^2-4\xi})e^{(-\frac{1}{2}\gamma+\frac{1}{2}\sqrt{\gamma^2-4\xi})t}}{\sqrt{\gamma^2-4\xi}}-\frac{1}{2}\frac{(\gamma-\sqrt{\gamma^2-4\xi})e^{(-\frac{1}{2}\gamma-\frac{1}{2}\sqrt{\gamma^2-4\xi})t}}{\sqrt{\gamma^2-4\xi}} \qquad (11)$$

The obtained iteration is used to generate the equation for the next iteration, and therefore the second and third iterations are obtained.

4. RESULTS AND DISCUSSIONS

The results of the HPM solution are tabulated in Table 1. Also to demonstrate the effectiveness of the proposed method, HPM, the results are also pictured in figures. From the figures, Figure1 through 3, it's obvious that HPM is powerful in finding analytical solutions for a wide class of nonlinear problems. These figures show obviously the excellent agreement between HPM and numerical solution.

Table 1. A comparative table for error detection of the analytic method, for $\gamma=3,\xi=2.01,\varepsilon=0.01,\beta=2,F_0=1$.

	x		$\dot{x}$	
t	HPM	RK	HPM	RKf45
0	1	1	0.00	0.00
1	0.4544060072	0.505374334790630874	-0.3695403461	-0.303432300570351254
2	0.2039961695	0.257540517444629858	-0.3068046675	-0.340792164974021261
3	-0.1254177687	-0.100146714434942219	-0.1981001184	-0.218168996091851976
4	-0.02917428662	-0.0173018074459907502	0.3111043130	0.301619978381819809
5	0.1519734217	0.156520938641227126	-0.0788630729	-0.0846998347044815026
6	-0.1004193148	-0.0993175983679624314	-0.2447736236	-0.245921904022073950
7	-0.07004090632	-0.0680631878741948804	0.2840196936	0.284616101932424870
8	0.1580626315	0.159265518808038852	0.0089960259	0.00719991014298755182
9	-0.06177042964	-0.0619039606809274820	-0.2912643749	-0.291523322750080804
10	-0.1067465441	-0.105412445157567314	0.2335204042	0.235326410676588772

The results shown in table1 indicates that the HPM experiences a high accuracy. In addition, in comparison with RK method and other analytical methods, a considerable reduction of the volume of the calculation can be seen in HPM. It can be approved that HPM is powerful in finding analytical solutions for a wide class of nonlinear problems. These figures show obviously the excellent agreement between HPM and RK method.

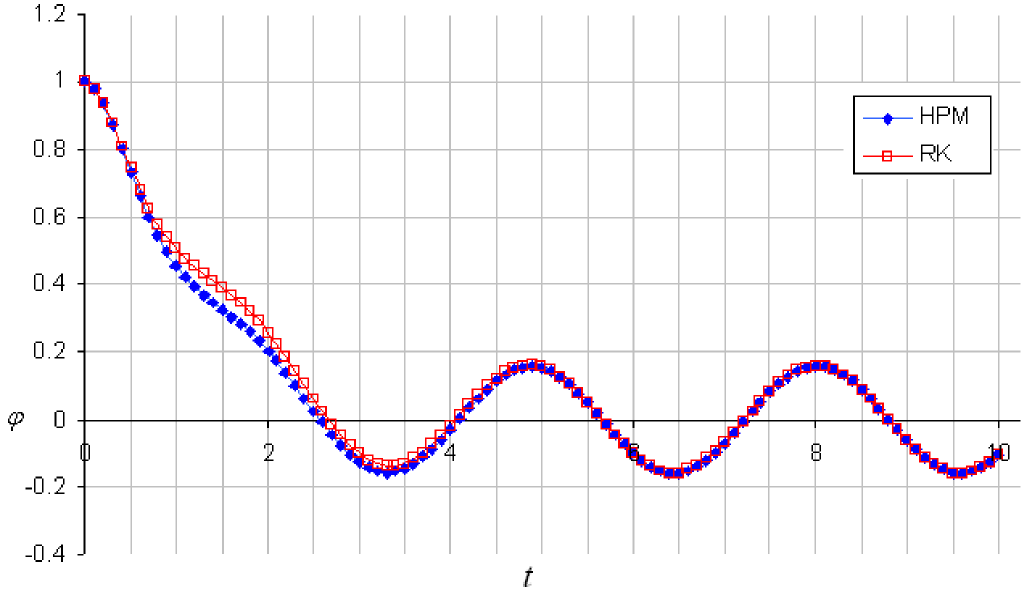

Figure (1): comparison of HPM and RK.Displacement φ based on time t

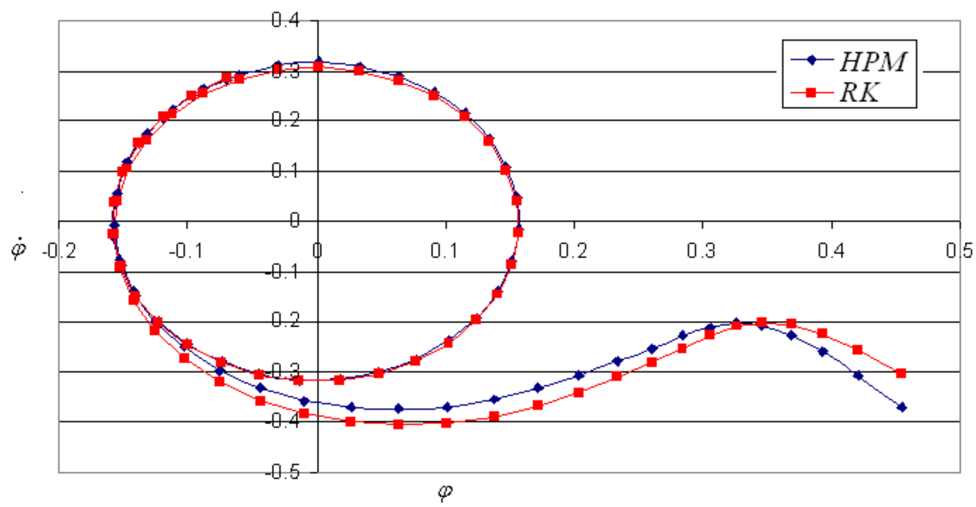

Figure (2): comparison of HPM and RK for Velocity $\dot{\varphi}$ based on displacement φ .

5. CONCLUSIONS

In this work, the HPM has been employed to analyze of parametrically excited oscillator with strong cubic positive nonlinearity of Complex Variable in Nonlinear Dynamic Systems with Forcing. The results obtained from this method have been compared with those obtained from numerical method using RK algorithm. This comparison shows excellent agreement between the two methods. Also, HPM does not require small parameters, so the limitation of the conventional perturbation method could be eliminated.

REFERENCES

[1]. D.W. Jordan, P. Smith, Nonlinear Ordinary Differential Equations, third ed., Oxford University Press, 1999

[2]. J. Biazar, Solution of the epidemic model by Adomian decomposition method, Applied Mathematics and Computation 173 (2) (2006)

[3]. 1101–1106

[4]. J.H. He, The homotopy perturbation method for nonlinear oscillators with discontinuities, Applied Mathematics and Computation151 (2004) 287–292

[5]. J.H. He, Comparison of homotopy perturbation method and homotopy analysis method, Applied Mathematics and Computation156 (2004) 527–539

[6]. J.H. He, Asymptotology by homotopy perturbation method, Applied Mathematics and Computation 156 (3) (2004) 591–596

[7]. J.H. He, Homotopy perturbation method for bifurcation of nonlinear problems, International Journal of Non-linear Science Numerical Simulation 6 (2) (2005) 207–208.

[8]. J.H. He, Application of homotopy perturbation method to nonlinear wave equations, Chaos, Solitons and Fractals 26 (2005) 695–70

[9]. J.H. He, Limit cycle and bifurcation of nonlinear problems, Chaos, Solitons and Fractals 26 (3) (2005) 827–83

[10]. J.H. He, Homotopy perturbation method for solving boundary value problems, Physics Letters A 350 (1–2) (2006) 87–88.

[11]. J.H. He, Some asymptotic methods for strongly nonlinear equations, International Journal of Modern Physics B 20 (10) (2006) 1141–1199.

[12]. J.H. He, Homotopy perturbation technique, Computer methods in applied mechanics and engineering 178 (1999) 257–26.

[13]. M. Rafei, D.D. Ganji, Explicit solutions of Helmholtz equation and fifth-order KdV equation using homotopy perturbation method, International Journal of Nonlinear Science and Numerical Simulation 7 (3) (2006) 321–329.

[14]. D.D. Ganji, M. Rafei, Solitary wave solutions for a generalized Hirota–Satsuma coupled KdV equation by homotopy perturbation method, Physics Letters A 356 (2006) 131–137.

[15]. A.M. Siddiqui, R. Mahmood, Q.K. Ghori, Homotopy perturbation method for thin film flow of a fourth grade fluid down a vertical cylinder, Physics Letters A 352 (2006) 404–410.

[16]. S. Abbasbandy, Homotopy perturbation method for quadratic Riccati differential equation and comparison with Adomian's decomposition method, Applied Mathematics and Computation 172 (2006) 485–490.

[17]. S. Abbasbandy, Iterated He's homotopy perturbation method for quadratic Riccati differential equation, Applied Mathematics and Computation 175 (2006) 581–589.

[18]. S. Abbasbandy, Numerical solutions of the integral equations: Homotopy perturbation method and Adomian's decomposition method, Applied Mathematics and Computation 173 (2006) 493–500.

[19]. J.H. He, Variational iteration method — a kind of non-linear analytical technique: Some examples, International Journal of Non-Linear Mechanics 34 (4) (1999) 699–708

[20]. J.H. He, Variational iteration method for autonomous ordinary differential systems, Applied Mathematics and Computation 118 (2–3) (2000)115–123