CONSTRUCTION OF α - CUT FUZZY $\tilde{\tilde{X}} - \tilde{R}$ AND $\tilde{\tilde{X}} - \tilde{S}$ CONTROL CHARTS USING FUZZY TRAPEZOIDAL NUMBER

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ABSTRACT
Statistical Process Control (SPC) is used to monitor the process stability which ensures the predictability of the process. In 1920's Shewhart introduced the control chart techniques that are one of the most important techniques of quality control to detect if assignable causes exist. The widely used control chart techniques are $\overline{X} - R$ and $\overline{X} - S$. These are called traditional variable control charts. A traditional variable control chart, consists of three lines namely Center Line, Upper Control Limit and Lower Control Limit. These limits are represented by the numerical values. The process is either "in-control" or "out-of-control" depending on numerical observations. For many problems, control limits could not be so precise. Uncertainty comes from the measurement system including operators and gauges and environmental conditions. In this situation, fuzzy set theory is a useful tool to handle this uncertainty. Numeric control limits can be transformed to fuzzy control limits by using membership functions. If a sample mean is too close to the control limits and the used measurement system is not so sensitive, the decision may be faulty. Fuzzy control limits provide a more accurate and flexible evaluation. In this paper, the fuzzy α cut $\tilde{\tilde{X}} - \tilde{R}$ and $\tilde{\tilde{X}} - \tilde{S}$ control charts are constructed by using fuzzy trapezoidal numbers. An application is presented for the proposed fuzzy $\tilde{\tilde{X}} - \tilde{R}$ control charts.

Keywords: *Fuzzy numbers, Statistical Process Control, α cut and α -level fuzzy midrange.*

1. INTRODUCTION
Statistical Process Control (SPC) is a technique applied towards improving the quality of characteristics by monitoring the process under study continuously, in order to detect assignable causes and take required actions as quickly as possible. Control charts are viewed as the most commonly applied SPC tools. A control chart consists of three horizontal lines called; Upper Control Limit (UCL), Center Line (CL) and Lower Control Limit (LCL). The center line in a control chart denotes the average value of the quality characteristic under study. If a point lies within UCL and LCL, then the process is deemed to be under control. Otherwise, a point plotted outside the control limits can be regarded as evidence representing that the process is out of control and, hence preventive or corrective actions are necessary in order to find and eliminate the assignable cause or causes, which subsequently result in improving quality characteristics [7].
The control chart may be classified into two types namely variable and attribute control charts. The fuzzy set theory was first introduced by Zadeh [11] and studied by many authors [2], [3], [4], [5] and [9]. It is mostly used when the data is attribute in nature and these types of data may be expressed in linguistic terms such as "very good", "good", "medium", "bad" and "very bad".
Till now a few authors have discussed on fuzzy variable control charts and their applications. For example, Rowlands and Wang [8], El-Shal et al.[2] and Zarandi et al. [12]. In the consideration of real production process, it is assumed that there are no doubts about observations and their values. But, such type of values can be obtained by human judgments, evaluations and decisions. Suppose the collected information is a continuous random variable of a production process should include the variability caused by human subjectivity or measurement devices, or environmental conditions. These variability causes create vagueness in the measurement system. Thus, linguistic terms like ''a range between 5.5 and 6.1" or ''a range approximately equal to 5.8" can be used instead of an exact value of continuous random variable. This type of situation can't reflect the real system like a deterministic model a probabilistic model. Real situations are very often uncertain or vague in a number of ways. The fuzzy set theory provides a useful methodology for modeling such uncertain data. So, representing X_i values by fuzzy numbers is a

reasonable way to analyze and evaluate the process. Flexibility of control limits can be provided by fuzzy X_i ‘s. Thus, one can get rid of strict control limits.

The measures of central tendency in descriptive Statistics are used in variable control charts. These measures can be used to convert fuzzy sets into scalars which are fuzzy mode, α -level fuzzy midrange, and fuzzy median and fuzzy average. There is no theoretical basis to select the appropriate fuzzy measures among these four.

The objective of this study is first to construct the fuzzy $\tilde{\bar{\bar{X}}} - \tilde{R}$ and $\tilde{\bar{\bar{X}}} - \tilde{S}$ control charts with α cuts by using α -level fuzzy midrange. The following procedures are used to construct the fuzzy $\tilde{\bar{\bar{X}}} - \tilde{R}$ and $\tilde{\bar{\bar{X}}} - \tilde{S}$ control charts.

1. First transform the traditional $\bar{X} - R$ and $\bar{X} - S$ control charts to fuzzy control charts. To obtain fuzzy $\tilde{\bar{\bar{X}}} - \tilde{R}$ and $\tilde{\bar{\bar{X}}} - \tilde{S}$ control charts, the trapezoidal fuzzy number (a, b, c, d) are used.
2. The α cut fuzzy $\tilde{\bar{\bar{X}}} - \tilde{R}$ control charts and α cut fuzzy $\tilde{\bar{\bar{X}}} - \tilde{S}$ control charts are developed by using α cut approach.
3. The α -level fuzzy midrange for fuzzy $\tilde{\bar{\bar{X}}} - \tilde{R}$ and $\tilde{\bar{\bar{X}}} - \tilde{S}$ control charts are calculated by using α -level fuzzy midrange transformation techniques.
4. Finally, the application of $\tilde{\bar{\bar{X}}} - \tilde{R}$ control charts is highlighted by using the numerical example.

2. FUZZY TRANSFORMATION TECHNIQUES

Mainly four fuzzy transformation techniques, which are similar to the measures of central tendency used in descriptive statistics: α - level fuzzy midrange, fuzzy median, fuzzy average, and fuzzy mode [10] are used. In this paper, among the above four transformation techniques, the α - level fuzzy midrange transformation technique is used for the construction of fuzzy $\tilde{\bar{\bar{X}}} - \tilde{R}$ and $\tilde{\bar{\bar{X}}} - \tilde{S}$ control charts based on fuzzy trapezoidal number.

3. α - LEVEL FUZZY MIDRANGE

This is defined as the mid point of the ends of the α - level cuts, denoted by A^α , is a non fuzzy set that comprises all elements whose membership is greater than or equal to α . If a^α and d^α are the end points of A^α , then

$$f_{mr}^{\alpha} = \frac{1}{2}(a^\alpha + d^\alpha)$$

In fact, the fuzzy mode is a special case of α - level fuzzy midrange when α =1.

α - level fuzzy midrange of sample j, $S_{mr,j}^{\alpha}$ is used to transform the fuzzy control limits into scalar and is determined as follows

$$S_{mr,j}^{\alpha} = \frac{(a_j + d_j) + \alpha\{(b_j - a_j) - (d_j - c_j)\}}{2}$$

4. FUZZY $\tilde{\bar{\bar{X}}}$ CONTROL CHART BASED ON RANGES

In monitoring the production process, the control of process averages or quality level is usually done by $\bar{X}$ charts. The process variability or dispersion can controlled by either a control chart for the range, called R chart, or a control chart for the standard deviation, called S chart. In this section, fuzzy $\tilde{\bar{\bar{X}}} - \tilde{R}$ control charts are introduced based on fuzzy trapezoidal number. The fuzzy $\tilde{\bar{\bar{X}}} - \tilde{S}$ control charts are presented in the next section.

Montgomery [7] has proposed the control limits for $\bar{X}$ control chart based on sample range is given below

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R} \text{ , } CL_{\bar{X}} = \bar{\bar{X}} \text{ and } LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$$

where A_2 is a control chart co-efficient and $\bar{R}$ is the average of R_i ’s that are the ranges of samples.

In the case of fuzzy control chart, each sample or subgroup is represented by a trapezoidal fuzzy number (a, b, c, d) as shown in Fig. 1.

In this study, trapezoidal fuzzy numbers are represented as (X_a, X_b, X_c, X_d) for each observation. Note that a trapezoidal fuzzy number becomes triangular when b=c. For the case of representation and calculation, a triangular fuzzy number is also represented as a trapezoidal fuzzy number by (a, b, b, d) or (a, c, c, d).The center line, $\tilde{CL}$ is the arithmetic mean of the fuzzy sample means, which are represented by $(\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d)$. Here $\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c$ and $\overline{\overline{X}}_d$ are called the overall means and is calculated as follows

$$\overline{X}_{rj} = \frac{\sum_{i=1}^{n} \overline{X}_{rij}}{n} \text{ ; r = a, b, c, d ; i=1, 2, 3, …n; j=1, 2, 3, …m}$$

$$\overline{\overline{X}}_r = \frac{\sum_{j=1}^{m} \overline{X}_{rj}}{m} \text{ ; r = a, b, c, d ; j=1, 2, 3, …m}$$

$$\tilde{CL} = (\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d) = \left\{ \frac{\sum_{j=1}^{m} \overline{X}_{aj}}{m}, \frac{\sum_{j=1}^{m} \overline{X}_{bj}}{m}, \frac{\sum_{j=1}^{m} \overline{X}_{cj}}{m}, \frac{\sum_{j=1}^{m} \overline{X}_{dj}}{m} \right\}$$

Where n is the fuzzy sample size, m is the number of fuzzy samples and $\tilde{CL}$ is the center line for fuzzy $\tilde{\overline{\overline{X}}}$ control chart.

4.1 Control Limits for Fuzzy $\tilde{\overline{\overline{X}}}$ Control Chart

By using the traditional $\overline{X}$ control chart procedure, the control limits of fuzzy $\tilde{\overline{\overline{X}}}$ control charts with ranges based on fuzzy trapezoidal number are calculated as follows

$$U\tilde{C}L_{\overline{X}} = \tilde{CL} + A_2\overline{R} = (\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d) + A_2(\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d)$$

$$= (\overline{\overline{X}}_a + A_2\overline{R}_a, \overline{\overline{X}}_b + A_2\overline{R}_b, \overline{\overline{X}}_c + A_2\overline{R}_c, \overline{\overline{X}}_d + A_2\overline{R}_d)$$

$$= (U\tilde{C}L_1, U\tilde{C}L_2, U\tilde{C}L_3, U\tilde{C}L_4)$$

$$\tilde{CL} = (\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d) = (\tilde{CL}_1, \tilde{CL}_2, \tilde{CL}_3, \tilde{CL}_4)$$

$$L\tilde{C}L_{\overline{X}} = \tilde{CL} - A_2\overline{R} = (\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d) - A_2(\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d)$$

$$= (\overline{\overline{X}}_a - A_2\overline{R}_a, \overline{\overline{X}}_b - A_2\overline{R}_b, \overline{\overline{X}}_c - A_2\overline{R}_c, \overline{\overline{X}}_d - A_2\overline{R}_d)$$

$$= (L\tilde{C}L_1, L\tilde{C}L_2, L\tilde{C}L_3, L\tilde{C}L_4)$$

Where $\overline{R}_r = \frac{\sum R_{rj}}{m}$; r = a, b, c, d ; j = 1, 2, 3, ….m, the procedure for calculating R_{rj} is as follows

$$R_{rj} = X_{\max.aj} - X_{\min.cj} ; R_{rj} = X_{\max.bj} - X_{\min.bj} ; R_{rj} = X_{\max.cj} - X_{\min.cj} ; \; R_{rj} = X_{\max.dj} - X_{\min.aj} ;$$

j = 1, 2, 3, ….m.

Where $(X_{\max.aj}, X_{\max.bj}, X_{\max.cj}, X_{\max.dj})$ is the maximum fuzzy number in the sample and $(X_{\min.aj}, X_{\min.bj}, X_{\min.cj}, X_{\min.dj})$ is the minimum fuzzy number in the sample. Then

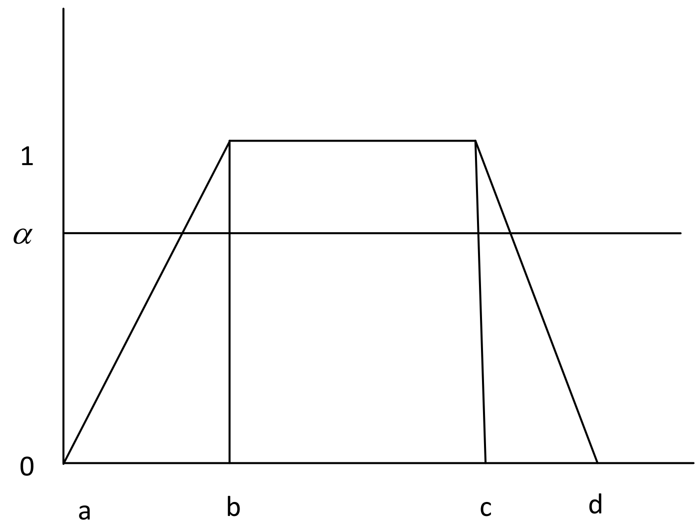

Fig.1. Representation of a sample by trapezoidal fuzzy numbers

4.2 Control Limits for α – Cut Fuzzy $\tilde{\overline{\overline{X}}}$ Control Chart

Introducing the α - cut procedure to the above fuzzy $\tilde{\overline{\overline{X}}}$ control limits, it can be rewritten as follows (the value of α can be selected according to the nature of the given problem and the selected α value must should lies between 0 and 1)

$$U\tilde{C}L_{\overline{X}}^{\alpha} = (\overline{\overline{X}}_a^{\alpha}, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d^{\alpha}) + A_2(\overline{R}_a^{\alpha}, \overline{R}_b, \overline{R}_c, \overline{R}_d^{\alpha})$$

$$= (\overline{\overline{X}}_a^{\alpha} + A_2\overline{R}_a^{\alpha}, \overline{\overline{X}}_b + A_2\overline{R}_b, \overline{\overline{X}}_c + A_2\overline{R}_c, \overline{\overline{X}}_d^{\alpha} + A_2\overline{R}_d^{\alpha})$$

$$= (U\tilde{C}L_1^{\alpha}, U\tilde{C}L_2, U\tilde{C}L_3, U\tilde{C}L_4^{\alpha})$$

$$C\tilde{L}_{\overline{x}}^{\alpha} = (\overline{\overline{X}}_a^{\alpha}, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d^{\alpha}) = (\tilde{C}L_1^{\alpha}, \tilde{C}L_2, \tilde{C}L_3, \tilde{C}L_4^{\alpha})$$

and $$L\tilde{C}L_{\overline{X}}^{\alpha} = (\overline{\overline{X}}_a^{\alpha}, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d^{\alpha}) - A_2(\overline{R}_a^{\alpha}, \overline{R}_b, \overline{R}_c, \overline{R}_d^{\alpha})$$

$$= (\overline{\overline{X}}_a^{\alpha} - A_2\overline{R}_a^{\alpha}, \overline{\overline{X}}_b - A_2\overline{R}_b, \overline{\overline{X}}_c - A_2\overline{R}_c, \overline{\overline{X}}_d^{\alpha} - A_2\overline{R}_d^{\alpha})$$

$$= (L\tilde{C}L_1^{\alpha}, L\tilde{C}L_2, L\tilde{C}L_3, L\tilde{C}L_4^{\alpha})$$

Where, $\overline{\overline{X}}_a^{\alpha} = \overline{\overline{X}}_a + \alpha(\overline{\overline{X}}_b - \overline{\overline{X}}_a)$, $\overline{\overline{X}}_d^{\alpha} = \overline{\overline{X}}_d - \alpha(\overline{\overline{X}}_d - \overline{\overline{X}}_c)$

$$\overline{R}_a^{\alpha} = \overline{R}_a + \alpha(\overline{R}_b - \overline{R}_a),\ \overline{R}_d^{a} = \overline{R} - \alpha(\overline{R}_d - \overline{R}_c)$$

The α - cut fuzzy $\tilde{\overline{\overline{X}}}$ control limits based on ranges are shown in fig.2

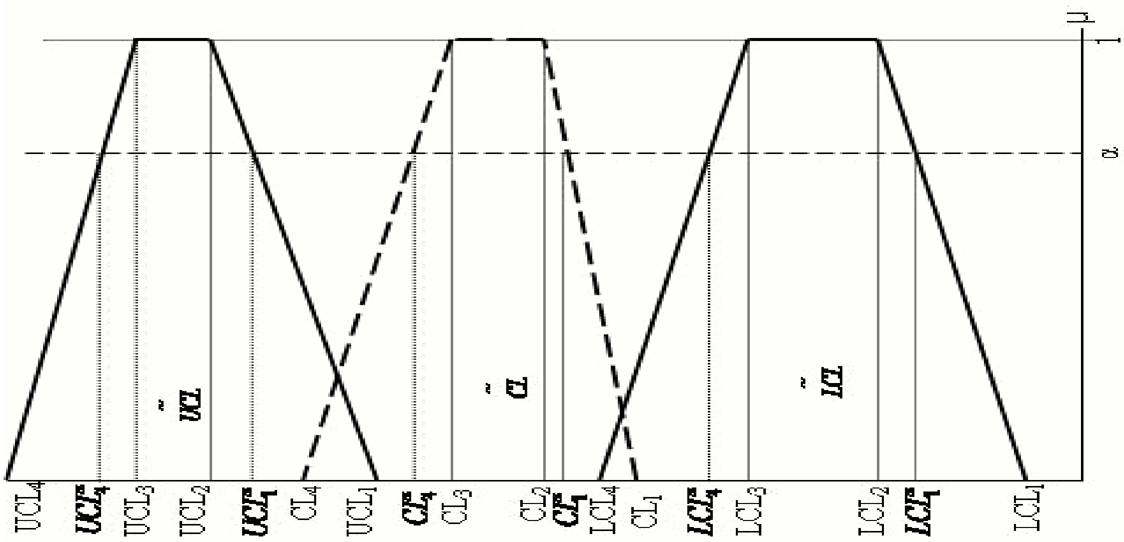

Fig.2. α **- Cut Fuzzy** $\tilde{\bar{\bar{X}}}$ **control chart based on ranges using fuzzy trapezoidal number**

4.3 α - LEVEL FUZZY MIDRANGE FOR α - CUT FUZZY $\tilde{\bar{\bar{X}}}$ CONTROL CHART BASED ON RANGES

The α - level fuzzy midrange is one of the transformation techniques (among the four) used to transform the fuzzy set into scalar. It is used to check the production process, whether the process is "in-control" or "out-of-control". The control limits for α - level fuzzy midrange for α - Cut Fuzzy $\tilde{\bar{\bar{X}}}$ control chart based on ranges can be obtained as follows

$$UCL^{\alpha}_{mr-\bar{X}} = CL^{\alpha}_{mr-\bar{x}} + A_2\left(\frac{\bar{R}^{\alpha}_a + R^{\alpha}_d}{2}\right)$$

$$CL^{\alpha}_{mr-\bar{X}} = f^{\alpha}_{mr-\bar{x}}(\tilde{C}L) = \frac{\bar{\bar{X}}^{\alpha}_a + \bar{\bar{X}}^{\alpha}_d}{2}$$

$$LCL^{\alpha}_{mr-\bar{X}} = CL^{\alpha}_{mr-\bar{x}} - A_2\left(\frac{\bar{R}^{\alpha}_a + R^{\alpha}_d}{2}\right)$$

The definition of α - level fuzzy midrange of sample j for fuzzy $\tilde{\bar{\bar{X}}}$ control chart is

$$S^{\alpha}_{mr\ \bar{X}.j} = \frac{(\bar{X}_{aj} + \bar{X}_{dj}) + \alpha\left[(\bar{X}_{bj}\quad \bar{X}_{aj})\quad (\bar{X}_{dj}\quad \bar{X}_{cj})\right]}{2}$$

Then, the condition of process control for each sample can be defined as:

$$\text{Process control} = \begin{cases} incontrol;\ for LCL^{\alpha}_{mr-\bar{X}} \le S^{\alpha}_{mr-\bar{X}j} \le UCL^{\alpha}_{mr-\bar{X}} \\ out-of-control; otherwise \end{cases}$$

5. FUZZY $\tilde{R}$ CONTROL CHART

The control limits for Shewhart R control chart is given by

$$UCL_R = D_4\bar{R},\ CL_R = \bar{R} \text{ and } LCL_R = D_3\bar{R}$$

Where D_4 and D_3 are control chart co-efficient [6].

By using the traditional R control chart procedure, the control limits for fuzzy $\tilde{R}$ control chart with trapezoidal fuzzy number is obtained as follows

$$U\tilde{C}L_R = D_4(\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d)$$

$$C\tilde{L}_R = (\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d)$$

and $L\tilde{C}L_R = D_3(\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d)$

5.1 Control Limits for α – Cut Fuzzy $\overline{\mathrm{R}}$ Control Chart

The control limits of α - cut fuzzy $\tilde{R}$ control chart based on trapezoidal fuzzy numbers are obtained as follows

$$U\tilde{C}L_R^{\alpha} = D_4(\overline{R}_a^{\alpha}, \overline{R}_b, \overline{R}_c, \overline{R}_d^{\alpha})$$

$$C\tilde{L}_R = (\overline{R}_a^{\alpha}, \overline{R}_b, \overline{R}_c, \overline{R}_d^{\alpha})$$

and $$L\tilde{C}L_R = D_3(\overline{R}_a^{\alpha}, \overline{R}_b, \overline{R}_c, \overline{R}_d^{\alpha})$$

5.2 α - LEVEL FUZZY MIDRANGE FOR α - CUT FUZZY $\tilde{R}$ CONTROL CHART

The control limits of α - Level fuzzy midrange for α - Cut Fuzzy $\tilde{R}$ control chart based on fuzzy Trapezoidal number can be calculated as follows

$$UCL^{\alpha}_{mr-R} = D_4 f^{\alpha}_{mr-R}(\tilde{C}L)$$

$$CL^{\alpha}_{mr-R} = f^{\alpha}_{mr-R}(\tilde{C}L) = \frac{\overline{R}_a^{\alpha} + \overline{R}_d^{\alpha}}{2}$$

$$LCL^{\alpha}_{mr-R} = D_3 f^{\alpha}_{mr-R}(\tilde{C}L)$$

The definition of α - level fuzzy midrange of sample j for fuzzy $\tilde{R}$ control chart can be calculated as follows

$$S^{\alpha}_{mr\ R.j} = \frac{(R_{aj} + R_{dj}) + \alpha\left[(R_{bj}\quad R_{aj})\quad (R_{dj}\quad R_{cj})\right]}{2}$$

Then, the condition of process control for each sample can be defined as:

$$\text{Decision} = \begin{cases} incontrol;\ for LCL^{\alpha}_{mr-R} \le S^{\alpha}_{mr-R.j} \le UCL^{\alpha}_{mr-R} \\ out-of-control; otherwise \end{cases}$$

6. FUZZY $\tilde{\tilde{\mathrm{X}}}$ CONTROL CHART BASED ON STANDARD DEVIATION

The R chart is used to monitor the dispersion associated with a quality characteristic. Its simplicity of construction and maintenance make the R chart very commonly used and the range is a good measure of variation for small subgroup sizes. When the sample size increases (n>10), the utility of the range as a measure of dispersion falls off and the standard deviation measure is preferred (Montgomery 2002)

The Shewhart $\overline{X}$ chart based on standard deviation is given below

$$UCL_{\overline{X}} = \overline{\overline{X}} + A_3\overline{S},\ CL_{\overline{X}} = \overline{\overline{X}} \text{ and } LCL_{\overline{X}} = \overline{\overline{X}} - A_3\overline{S}$$

Where A_3 is a control chart co-efficient (Kolarik 1995)

The value of $\overline{S}$ is

$$S_j = \sqrt{\frac{\sum_{i=1}^{n}(X_{ij} - \overline{X}_j)^2}{n-1}};\ \overline{S} = \frac{\sum_{j=1}^{m} S_j}{m}$$

Where S_j is the standard deviation of sample j and $\overline{S}$ is the average of S_j 's.

6.1 FUZZY $\tilde{\tilde{X}}$ CONTROL CHART BASED ON STANDARD DEVIATION

The theoretical structure of fuzzy $\tilde{\tilde{X}}$ control chart and fuzzy $\overline{S}$ control chart has been developed by Senturk and Erginel (2009). The fuzzy $\tilde{S}_j$ is the standard deviation of sample j and it is calculated as follows

$$S_j = \sqrt{\frac{\sum_{i=1}^{n}[(X_a, X_b, X_c, X_d)_{ij} - (\overline{X}_a, \overline{X}_b, \overline{X}_c, \overline{X}_d)_j]^2}{n-1}}$$

and the fuzzy average $\tilde{\bar{S}}$ is calculated by using standard deviation represented by the following Trapezoidal fuzzy number

$$\tilde{\bar{S}} = \left\{ \frac{\sum_{j=1}^{m} S_{aj}}{m}, \frac{\sum_{j=1}^{m} S_{bj}}{m}, \frac{\sum_{j=1}^{m} S_{cj}}{m}, \frac{\sum_{j=1}^{m} S_{dj}}{m} \right\} = (\overline{S}_a, \overline{S}_b, \overline{S}_c, \overline{S}_d)$$

And the control limits of fuzzy $\tilde{\bar{X}}$ control chart based on standard deviation are defined as follows

$$U\tilde{C}L_{\overline{X}} = \tilde{C}L + A_3\overline{S} = (\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d) + A_3(\overline{S}_a, \overline{S}_b, \overline{S}_c, \overline{S}_d)$$
$$= (\overline{\overline{X}}_a + A_3\overline{S}_a, \overline{\overline{X}}_b + A_3\overline{S}_b, \overline{\overline{X}}_c + A_3\overline{S}_c, \overline{\overline{X}}_d + A_3\overline{S}_d)$$
$$= (U\tilde{C}L_1^{\alpha}, U\tilde{C}L_2, U\tilde{C}L_3, U\tilde{C}L_4^{\alpha})$$
$$C\tilde{L} = (\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d) = (\tilde{C}L_1, \tilde{C}L_2, \tilde{C}L_3, \tilde{C}L_4)$$
$$L\tilde{C}L_{\overline{X}} = \tilde{C}L - A_3\overline{S} = (\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d) - A_3(\overline{S}_a, \overline{S}_b, \overline{S}_c, \overline{S}_d)$$
$$= (\overline{\overline{X}}_a - A_3\overline{S}_a, \overline{\overline{X}}_b - A_3\overline{S}_b, \overline{\overline{X}}_c - A_3\overline{S}_c, \overline{\overline{X}}_d - A_3\overline{S}_d)$$
$$= (L\tilde{C}L_1, L\tilde{C}L_2, L\tilde{C}L_3, L\tilde{C}L_4)$$

6.2 Control Limits for α – Cut Fuzzy $\tilde{\bar{X}}$

The control limits for α - Cut Fuzzy $\tilde{\bar{X}}$ control chart based on standard deviation are obtained as follows

$$U\tilde{C}L_{\overline{X}}^{\alpha} = (\overline{\overline{X}}_a^{\alpha}, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d^{\alpha}) + A_3(\overline{S}_a^{\alpha}, \overline{S}_b, \overline{S}_c, \overline{S}_d^{\alpha})$$
$$= (\overline{\overline{X}}_a^{\alpha} + A_3\overline{S}_a^{\alpha}, \overline{\overline{X}}_b + A_3\overline{S}_b, \overline{\overline{X}}_c + A_3\overline{S}_c, \overline{\overline{X}}_d^{\alpha} + A_3\overline{S}_d^{\alpha})$$
$$= (U\tilde{C}L_1^{\alpha}, U\tilde{C}L_2, U\tilde{C}L_3, U\tilde{C}L_4^{\alpha})$$
$$C\tilde{L}_{\overline{x}}^{\alpha} = (\overline{\overline{X}}_a^{\alpha}, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d^{\alpha}) = (\tilde{C}L_1^{\alpha}, \tilde{C}L_2, \tilde{C}L_3, \tilde{C}L_4^{\alpha})$$
$$L\tilde{C}L_{\overline{X}}^{\alpha} = (\overline{\overline{X}}_a^{\alpha}, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d^{\alpha}) - A_3(\overline{S}_a^{\alpha}, \overline{S}_b, \overline{S}_c, \overline{S}_d^{\alpha})$$
$$= (\overline{\overline{X}}_a^{\alpha} - A_3\overline{S}_a^{\alpha}, \overline{\overline{X}}_b - A_3\overline{S}_b, \overline{\overline{X}}_c - A_3\overline{S}_c, \overline{\overline{X}}_d^{\alpha} - A_3\overline{S}_d^{\alpha})$$
$$= (L\tilde{C}L_1^{\alpha}, L\tilde{C}L_2, L\tilde{C}L_3, L\tilde{C}L_4^{\alpha})$$

Where $\overline{S}_a^{\alpha} = \overline{S}_a + \alpha(\overline{S}_b - \overline{S}_a)$

$\overline{S}_d^{a} = \overline{S}_d - \alpha(\overline{S}_d - \overline{S}_c)$

6.3 α - LEVEL FUZZY MIDRANGE FOR α - CUT FUZZY $\tilde{\bar{X}}$ CONTROL CHART BASED ON STANDARD DEVIATION

The control limits and centre line for α - Cut Fuzzy $\tilde{\bar{X}}$ control chart based on standard deviation using α - Level fuzzy midrange are

$$UCL_{mr-\overline{X}}^{\alpha} = CL_{mr-\overline{x}}^{\alpha} + A_3\left(\frac{\overline{S}_a^{\alpha} + \overline{S}_d^{\alpha}}{2}\right)$$

$$CL^{\alpha}_{mr-\bar{X}} = f^{\alpha}_{mr-\bar{x}}(\tilde{C}L) = \frac{\bar{\bar{X}}^{\alpha}_{a} + \bar{\bar{X}}^{\alpha}_{d}}{2}$$

$$LCL^{\alpha}_{mr-\bar{X}} = CL^{\alpha}_{mr-\bar{x}} - A_3\left(\frac{\bar{S}^{\alpha}_{a} + \bar{S}^{\alpha}_{d}}{2}\right)$$

The definition of α - level fuzzy midrange of sample j for fuzzy $\tilde{\tilde{X}}$ control chart is

$$S^{\alpha}_{mr-\bar{X}.j} = \frac{(\bar{X}_{aj} + \bar{X}_{dj}) + \alpha\left[(\bar{X}_{bj} - \bar{X}_{aj}) - (\bar{X}_{dj} - \bar{X}_{cj})\right]}{2}$$

Then, the condition of process control for each sample can be defined as:

$$\text{Process control} = \begin{cases} incontrol;\ forLCL^{\alpha}_{mr-\bar{X}} \le S^{\alpha}_{mr-\bar{X}j} \le UCL^{\alpha}_{mr-\bar{X}} \\ out-of-control; otherwise \end{cases}$$

7. FUZZY $\bar{S}$ CONTROL CHART

The control limits for shewhart $\bar{S}$ control chart is given by

$$UCL_S = B_4\bar{S},\ CL_S = \bar{S} \text{ and } LCL_S = B_3\bar{S}$$

Where B_4 and B_3 are control chart co-efficient [11]. Then the Fuzzy $\bar{S}$ control chart limits can be obtained as follows:

$$U\tilde{C}L_S = B_4\bar{S} = B_4(\bar{S}_a, \bar{S}_b, \bar{S}_c, \bar{S}_d)$$

$$C\tilde{L}_S = \bar{S} = (\bar{S}_a, \bar{S}_b, \bar{S}_c, \bar{S}_d)$$

$$L\tilde{C}L_S = B_3\bar{S} = B_3(\bar{S}_a, \bar{S}_b, \bar{S}_c, \bar{S}_d)$$

7.1 α - CUT FUZZY $\bar{S}$ CONTROL CHART

The control limits of α - Cut Fuzzy $\bar{S}$ control chart can be obtained as follows:

$$U\tilde{C}L^{\alpha}_S = B_4\bar{S}^{\alpha} = B_4(\bar{S}^{\alpha}_a, \bar{S}_b, \bar{S}_c, \bar{S}^{\alpha}_d)$$

$$C\tilde{L}^{\alpha}_S = \bar{S}^{\alpha} = (\bar{S}^{\alpha}_a, \bar{S}_b, \bar{S}_c, \bar{S}^{\alpha}_d)$$

$$L\tilde{C}L^{\alpha}_S = B_3\bar{S}^{\alpha} = B_3(\bar{S}^{\alpha}_a, \bar{S}_b, \bar{S}_c, \bar{S}^{\alpha}_d)$$

7.2 α - LEVEL FUZZY MIDRANGE FOR α - CUT FUZZY $\bar{S}$ CONTROL CHART

The control limits of α - Level fuzzy midrange for α - Cut Fuzzy $\bar{S}$ control chart can be obtained in a similar way to α - Cut Fuzzy $\tilde{R}$ control chart

$$UCL^{\alpha}_{mr-S} = B_4 f^{\alpha}_{mr-S}(\tilde{C}L)$$

$$CL^{\alpha}_{mr-S} = f^{\alpha}_{mr-S}(\tilde{C}L) = \frac{\bar{S}^{\alpha}_a + \bar{S}^{\alpha}_d}{2}$$

$$LCL^{\alpha}_{mr-S} = B_3 f^{\alpha}_{mr-S}(\tilde{C}L)$$

The definition of α - level fuzzy midrange of sample j for fuzzy $\bar{S}$ control chart can be calculated as follows

$$S^{\alpha}_{mr-S.j} = \frac{(S_{aj} + S_{dj}) + \alpha\left[(S_{bj} - S_{aj}) - (S_{dj} - S_{cj})\right]}{2}$$

Then, the condition of process control for each sample can be defined as:

$$\text{Decision} = \begin{cases} incontrol;\ for LCL^{\alpha}_{mr-S} \le S^{\alpha}_{mr-S.j} \le UCL^{\alpha}_{mr-S} \\ out-of-control; otherwise \end{cases}$$

Numerical Example:

An application was given by Sentruk and Erginel [9] using triangular fuzzy numbers on controlling piston inner diameters in compressors. The same data have been considered with the first fifteen samples each with size 5 (the total measurements is 5 × 15 = 75). These measurements are converted into trapezoidal fuzzy numbers and given in Table 1. Fuzzy control limits are calculated according to the procedures given in the previous sections.

For $n = 5$, $A_2 = 0.577$. Where A_2 is obtained from the coefficients table for variable control charts.

Table - 1

Sample No	X_a					X_b					X_c					X_d				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	5.71	5.50	5.43	5.20	5.51	5.73	5.57	5.45	5.25	5.53	5.75	5.60	5.46	5.27	5.55	5.76	5.62	5.47	5.28	5.56
2	5.41	5.52	5.25	5.51	5.65	5.43	5.57	5.29	5.53	5.69	5.44	5.58	5.30	5.56	5.70	5.45	5.59	5.31	5.58	5.71
3	5.25	5.51	5.00	5.20	5.31	5.29	5.53	5.13	5.25	5.33	5.32	5.54	5.17	5.28	5.37	5.33	5.55	5.19	5.29	5.39
4	5.42	5.26	5.42	5.49	5.60	5.51	5.31	5.44	5.55	5.64	5.62	5.38	5.46	5.58	5.71	5.75	5.40	5.51	5.65	5.76
5	5.19	5.18	5.25	5.21	5.52	5.30	5.2	5.28	5.27	5.57	5.32	5.31	5.32	5.30	5.60	5.41	5.35	5.40	5.39	5.65
6	5.36	5.31	5.18	5.38	5.26	5.42	5.40	5.23	5.48	5.33	5.63	5.55	5.31	5.47	5.42	5.71	5.62	5.32	5.68	5.52
7	5.26	5.53	5.41	5.28	5.19	5.27	5.57	5.46	5.29	5.26	5.31	5.62	5.49	5.32	5.30	5.46	5.59	5.52	5.38	5.39
8	5.43	5.28	5.44	5.50	5.58	5.52	5.33	5.47	5.56	5.62	5.63	5.40	5.50	5.60	5.69	5.76	5.42	5.55	5.56	5.74
9	5.69	5.45	5.32	5.19	5.45	5.72	5.49	5.35	5.22	5.48	5.75	5.56	5.41	5.28	5.51	5.76	5.61	5.45	5.31	5.56
10	5.31	5.26	5.14	5.40	5.31	5.35	5.29	5.21	5.43	5.36	5.42	5.32	5.26	5.46	5.38	5.46	5.39	5.32	5.52	5.42
11	5.28	5.5	5.41	5.31	5.35	5.32	5.56	5.46	5.34	5.41	5.36	5.62	5.48	5.38	5.44	5.43	5.65	5.52	5.44	5.49
12	5.43	5.22	5.15	5.34	5.48	5.46	5.26	5.18	5.36	5.52	5.48	5.28	5.21	5.41	5.56	5.53	5.31	5.26	5.44	5.62
13	5.46	5.35	5.35	5.22	5.28	5.52	5.39	5.42	5.28	5.32	5.56	5.43	5.48	5.34	5.36	5.62	5.48	5.54	5.38	5.42
14	5.41	5.36	5.52	5.51	5.38	5.44	5.4	5.56	5.54	5.42	5.48	5.43	5.62	5.62	5.46	5.52	5.48	5.68	5.71	5.49
15	5.62	5.48	5.42	5.18	5.41	5.64	5.55	5.46	5.23	5.46	5.68	5.59	5.49	5.26	5.49	5.73	5.63	5.54	5.31	5.53

The fuzzy ranges for the above X_r ; r = a, b, c, d values for the 15 samples are calculated as follows

1. $R_{a1} = X_{Maxa1} - X_{Mind1} = 5.71 - 5.28 = 0.43$

   $R_{b1} = X_{Maxb1} - X_{Minb1} = 0.48$

   $R_{c1} = X_{Maxc1} - X_{Minc1} = 0.48$

   $R_{d1} = X_{Maxd1} - X_{Mina1} = 0.56$

2. $R_{a2} = X_{Maxa2} - X_{Mind2} = 0.34$

   $R_{b2} = X_{Maxb2} - X_{Minb2} = 0.40$

   $R_{c2} = X_{Maxc2} - X_{Minc2} = 0.40$

   $R_{d2} = X_{Maxd2} - X_{Mina2} = 0.46$

   .

   .

   .

15. $R_{a15} = X_{Maxa15} - X_{Mind15} = 0.31$

   $R_{b15} = X_{Maxb15} - X_{Minb15} = 0.41$

   $R_{c15} = X_{Maxc15} - X_{Minc15} = 0.42$

$$R_{d15} = X_{Maxd15} - X_{Mind15} = 0.55$$

Then the values for $\overline{R}_r$ and $\overline{\overline{X}}_r$ is given below, where r = a, b, c, d

$$\overline{R}_a = 0.1980,\ \overline{R}_b = 0.3260,\ \overline{R}_c = 0.3273,\ \overline{R}_d = 0.4587$$

$$\overline{\overline{X}}_a = 5.3701,\ \overline{\overline{X}}_b = 5.4173,\ \overline{\overline{X}}_c = 5.4667,\ \overline{\overline{X}}_d = 5.5067$$

Fuzzy $\tilde{\overline{\overline{X}}}$ control chart based on ranges:

By using the above $\overline{R}_r$ and $\overline{\overline{X}}_r$, the fuzzy $\tilde{\overline{\overline{X}}}$ control limits based on trapezoidal number can be calculated as follows

$$U\tilde{C}L_{\overline{X}} = C\tilde{L} + A_2\overline{R} = (\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d) + A_2(\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d)$$

= (5.37, 5.42, 5.47, 5.51) + 0.577(0.1980, 0.360, 0.3273, 0.4587)

$= (U\tilde{C}L_1, U\tilde{C}L_2, U\tilde{C}L_3, U\tilde{C}L_4)$

= (5.484, 5.627, 5.69, 5.78)

$$C\tilde{L} = (\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d) = (\tilde{C}L_1, \tilde{C}L_2, \tilde{C}L_3, \tilde{C}L_4) = (5.37, 5.42, 5.47, 5.51)$$

$$L\tilde{C}L_{\overline{X}} = C\tilde{L} - A_2\overline{R} = (\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d) - A_2(\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d)$$

= (5.37, 5.42, 5.47, 5.51) - 0.577(0.1980, 0.360, 0.3273, 0.4587)

$= (L\tilde{C}L_1, L\tilde{C}L_2, L\tilde{C}L_3, L\tilde{C}L_4)$

= (5.26, 5.21, 5.28, 5.25)

α - Cut Fuzzy $\tilde{\overline{\overline{X}}}$ control chart based on ranges:

α - Cuts in the control limits provide the ability of determining the tightness of the sampling process. α - Level can be selected according to the nature of the production process. α - level was defined as 0.65 for piston inner diameter production process.

$\overline{\overline{X}}_a^{0.65} = \overline{\overline{X}}_a + \alpha(\overline{\overline{X}}_b - \overline{\overline{X}}_a)$ = 5.3701+0.65(5.4173-5.3701) = 5.41

$\overline{\overline{X}}_d^{0.65} = \overline{\overline{X}}_d - \alpha(\overline{\overline{X}}_d - \overline{\overline{X}}_c)$ = 5.51-0.65(5.51-5.47) = 5.484

$\overline{R}_a^{0.65} = \overline{R}_a + \alpha(\overline{R}_b - \overline{R}_a)$ = 0.198+0.65(0.326-0.198) = 0.281

$\overline{R}_d^{0.65} = \overline{R}_d + \alpha(\overline{R}_d - \overline{R}_c)$ = 0.459-0.65(0.455-0.327) = 0.373

$$U\tilde{C}L_{\overline{X}}^{0.65} = (\overline{\overline{X}}_a^{0.65}, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d^{0.65}) + A_2(\overline{R}_a^{0.65}, \overline{R}_b, \overline{R}_c, \overline{R}_d^{0.65})$$

= (5.41, 5.42, 5.47, 5.48)+0.577(0.281, 0.326, 0.327, 0.373)

$= (U\tilde{C}L_1^{0.65}, U\tilde{C}L_2, U\tilde{C}L_3, U\tilde{C}L_4^{0.65})$

= (5.57, 5.61, 5.66, 5.70)

$$C\tilde{L}_{\overline{x}}^{0.65} = (\overline{\overline{X}}_a^{0.65}, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d^{0.65})$$

$= (\tilde{C}L_1^{0.65}, \tilde{C}L_2, \tilde{C}L_3, \tilde{C}L_4^{0.65})$

= (5.41, 5.42, 5.47, 5.48)

and $L\tilde{C}L_{\overline{X}}^{0.65} = (\overline{\overline{X}}_a^{0.65}, \overline{\overline{X}}_b, \overline{\overline{X}}_c, \overline{\overline{X}}_d^{0.65}) - A_2(\overline{R}_a^{0.65}, \overline{R}_b, \overline{R}_c, \overline{R}_d^{0.65})$

= (5.41, 5.42, 5.47, 5.48) - 0.577(0.281, 0.326, 0.327, 0.373)

$= (L\tilde{C}L_1^{0.65}, L\tilde{C}L_2, L\tilde{C}L_3, L\tilde{C}L_4^{0.65})$

= (5.248, 5.232, 5.22, 5.21)

α - Level fuzzy midrange for α - Cut Fuzzy $\tilde{\overline{\overline{X}}}$ control chart based on ranges:

Control limits using α - level fuzzy midrange for α - Cut Fuzzy $\tilde{\overline{\overline{X}}}$ control chart have been obtained by the following way

$$UCL^{0.65}_{mr-\bar{X}} = CL^{0.65}_{mr-\bar{x}} + A_2\left(\frac{\bar{R}^{0.65}_a + R^{0.65}_d}{2}\right) = 5.445 + 0.577\left(\frac{0.281 + 0.327}{2}\right) = 5.62$$

$$CL^{0.65}_{mr-\bar{X}} = f^{0.65}_{mr-\bar{x}}(\tilde{C}L) = 5.445$$

$$LCL^{0.65}_{mr-\bar{X}} = CL^{0.65}_{mr-\bar{x}} - A_2\left(\frac{\bar{R}^{0.65}_a + R^{0.65}_d}{2}\right) = 5.445 - 0.577\left(\frac{0.281 + 0.327}{2}\right) = 5.27$$

Fuzzy $\tilde{R}$ control chart:

$$U\tilde{C}L_R = D_4(\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d)$$

= 2.114 (0.198, 0.326, 0.327, 0.459) = (0.419, 0.689, 0.691, 0.970)

$C\tilde{L}_R = (\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d)$ = (0.198, 0.326, 0.327, 0.459)

and $L\tilde{C}L_R = D_3(\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d)$ = 0(0.198, 0.326, 0.327, 0.459) = (0, 0, 0, 0)

Where $D_4 = 2.114$, n =5. D_4 and D_3 are obtained from the coefficients table for variable control charts.

α - Cut Fuzzy $\tilde{R}$ control chart:

The control limits of α - cut fuzzy $\tilde{R}$ control chart based on trapezoidal fuzzy numbers are stated by using

$$U\tilde{C}L^{0.65}_R = D_4(\bar{R}^{0.65}_a, \bar{R}_b, \bar{R}_c, \bar{R}^{0.65}_d) = 2.114(0.281, 0.326, 0.327, 0.373)$$

= (0.594, 0.689, 0.691, 0.789)

$$C\tilde{L}^{0.65}_R = (\bar{R}^{0.65}_a, \bar{R}_b, \bar{R}_c, \bar{R}^{0.65}_d) = (0.281, 0.326, 0.327, 0.373)$$

and $L\tilde{C}L^{0.65}_R = D_3(\bar{R}^{\alpha}_a, \bar{R}_b, \bar{R}_c, \bar{R}^{\alpha}_d) = 0\ (0.281, 0.326, 0.327, 0.373)$ = (0, 0, 0, 0)

α - Level fuzzy midrange for α - Cut Fuzzy $\tilde{R}$ control chart:

The control limits of α - Level fuzzy midrange for α - Cut Fuzzy $\tilde{R}$ control chart based on fuzzy Trapezoidal number can be calculated as follows

$$UCL^{0.65}_{mr\ R} = D_4 f^{\alpha}_{mr\ R}(\tilde{C}L) = 0.4314$$

$$CL^{0.65}_{mr-R} = f^{0.65}_{mr-R}(\tilde{C}L) = 0.3265$$

$$LCL^{\alpha}_{mr-R} = D_3 f^{\alpha}_{mr-R}(\tilde{C}L) = 0$$

The values of $S^{\alpha}_{mr\ \bar{X}.j}$ and $S^{\alpha}_{mr\ R.j}$ have been calculated by using the formula of α - Level fuzzy midrange for α - Cut Fuzzy $\tilde{\tilde{X}}$ control chart based on ranges and α - Level fuzzy midrange for α - Cut Fuzzy $\tilde{R}$ control chart respectively and the values are given in Table 2. As shown in table 2, the process was in control with respect to $S^{\alpha}_{mr\ \bar{X}.j}$ and $S^{\alpha}_{mr\ R.j}$ for each sample. so, these control limits can be used to control the production process.

Table – 2

Control limits using α - Level fuzzy midrange for α - Cut Fuzzy $\tilde{\tilde{X}}$ control chart based on ranges and α - Level fuzzy midrange for α - Cut Fuzzy $\tilde{R}$ control chart

Sample no	$S^{\alpha}_{mr\ \bar{X}.j}$	$5.27 \leq S^{\alpha}_{mr\ \bar{X}.j} \leq 5.62$	$S^{\alpha}_{mr\ R.j}$	$0 \leq S^{\alpha}_{mr\ R.j} \leq 0.4314$
1	5.512	In Control	0.418	In Control
2	5.454	In Control	0.400	In Control
3	5.288	In Control	0.403	In Control
4	5.522	In Control	0.321	In Control
5	5.351	In Control	0.335	In Control
6	5.424	In Control	0.319	In Control
7	5.391	In Control	0.311	In Control
8	5.533	In Control	0.310	In Control
9	5.478	In Control	0.402	In Control
10	5.351	In Control	0.301	In Control
11	5.436	In Control	0.312	In Control
12	5.374	In Control	0.345	In Control
13	5.411	In Control	0.311	In Control
14	5.450	In Control	0.217	In Control
15	5.475	In Control	0.412	In Control

CONCLUSION

This paper shows that the fuzzy set theory is suitable to identify the signals in the variable control charts, because its gives some flexibility to the control limits. Since the plotted values are close to the control limits may cause false alarms with traditional control charts, fuzzy observations and fuzzy control limits can provide more flexibility for controlling a process. In this paper, α - Cut fuzzy $\tilde{\tilde{X}} - \tilde{R}$ and $\tilde{\tilde{X}} - \tilde{S}$ control charts are proposed based on fuzzy trapezoidal number. The α - Level fuzzy midrange transformation techniques are used to illustrate applications in a production process. The methodology can be extended for the other three measures.

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