FEEDBACK CONTROL OF RECURRENT NEURAL NETWORKS WITH TIME-VARYING

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ABSTRACT

This paper deals with the problem of delay-dependent stability criterion of discrete-time recurrent neural networks with time-varying delays. Based on quadratic Lyapunov functional approach and free-weighting matrix approach, some linear matrix inequality criteria are found to guarantee delay-dependent asymptotical stability of these systems. And one example illustrates the exactness of the proposed criteria.

Keywords: *Neural networks; Time-varying Delay; Stability; Quadratic Lyapunov functional approach*..

1. INTRODUCTION

A recurrent neural network (RNNs) is a very important tool for many application areas such as associative memory, pattern recognition, signal processing, model identication and combinatorial optimization. With the development of research on RNNs in theory and application, the model is more and more complex. Parameter uncertainties and nonautonomous phenomena often exist in real systems due to modeling inaccuracies [1, 22]. Particularly when we consider a longterm dynamical behavior of the system and consider seasonality of the changing environment, the parameters of the system usually will change with time [2, 3, 19, 20, 23]. Simultaneously, in implementations of artificial neural networks, time delay may occur due to finite switching speeds of the amplifiers and communication time [1, 4, 24, 25, 26, 27]. In order to model those systems with neural networks, the neural networks with time-varying delay appear in many papers [6-15, 21]. So in this paper we consider the stability of the following discrete-time recurrent neural networks. In this paper, we consider discrete-time system of neural networks of the form

$$v(k+1) = Av(k) + BS(v(k-h(k))) + f \ , \qquad (1)$$

where $v(k) \in \Omega \subseteq \mathbf{R}^n$ is the neuron state vector, $0 < h_2 \le h(k) \le h_2, \forall k = 0,1,2,\ldots,$ $A = diag\{a_1,\ldots,a_n\}$, $a_i \ge 0$, $i = 1,2,...,n$ is the $n \times n$ constant relaxation matrix, B is the $n \times n$ constant weight matrix, $f = (f_1,\ldots,f_n) \in \mathbf{R}^n$ is the constant external input vector and $S(z) = [s_1(z_1),\ldots,s_n(z_n)]^T$ with $s_i \in C^1\left[\mathbf{R},(-1,1)\right]$ where s_i is the neuron activations and monotonically increasing for each $i = 1,2,...,n$.

The asymptotic stability of the zero solution of the delay-differential system of Hopfield neural networks has been developed during the past several years. Much less is known regarding the asymptotic stability of the zero solution of the discrete-time system of neural networks. Therefore, the purpose of this paper is to establish sufficient condition for the asymptotic stability of the zero solution of (1) in terms of certain matrix inequalities.

2. PRELIMINARIES

The following notations will be used throughout the paper. $\mathbf{R}^+$ denotes the set of all non-negative real numbers; $\mathbf{Z}^+$ denotes the set of all non-negative integers; $\mathbf{R}^n$ denotes the n-finite-dimensional Euclidean space with the Euclidean norm $\|.\|$ and the scalar product between x and y is defined by $x^T y$; $\mathbf{R}^{n\times m}$ denotes the set of all $(n \times m)$-matrices; and A^T denotes the transpose of the matrix A; Matrix $Q \in \mathbf{R}^{n\times n}$ is positive semidefinite $(Q \ge 0)$ if $x^T Qx \ge 0$, for all $x \in \mathbf{R}^n$. If $x^T Qx > 0 (x^T Qx < 0$, resp.) for any $x \ne 0$, then Q is positive (negative, resp.) definite and denoted by $Q > 0, (Q < 0,$ resp.). It is easy to verify that $Q > 0,$ $(Q < 0,$ resp.) iff $\exists \beta > 0:$ $x^T Qx \ge \beta \|x\|^2, \forall x \in \mathbf{R}^n,$ ($\exists \beta > 0:$ $x^T Qx \le -\beta \|x\|^2, \forall x \in \mathbf{R}^n,$ resp.) .

Lemma 2.1 [10] The zero solution of difference system is asymptotic stability if there exists a positive definite function $V(x):\mathbf{R}^n \to \mathbf{R}^+$ such that

$$\exists \beta > 0 : \Delta V(x(k)) = V(x(k+1)) - V(x(k)) \le -\beta \|x(k)\|^2 ,$$

along the solution of the system. In case the above condition holds for all $x(k) \in V_\delta$, we say that the zero solution is asymptotically stable.

3. MAIN RESULTS

In this section, we consider the sufficient condition for asymptotic stability of the zero solution v^* of (1) in terms of certain matrix inequalities. Without loss of generality, we can assume that $v^* = 0, S(0) = 0$ and f =0 (for otherwise, we let $x = v - v^*$ and define $S(x) = S(x + v^*) - S(v^*))$.

The new form of (1) is now given by

$$x(k+1) = Ax(k) + BS(x(k - h(k))) . \tag{2}$$

Throughout this paper we assume the neuron activations $s_i(x_i)$, $i = 1, 2, \dots, n$ is bounded and monotonically nondecreasing on $\mathbf{R}$, and $s_i(x_i)$ is Lipschitz continuous, that is, there exist constant $l_i > 0, i = 1, 2, \dots, n$ such that

$$|s_i(r_1) - s_i(r_2)| \le l_i |r_1 - r_2|, \ \forall r_1, r_2 \in \mathbf{R} . \tag{3}$$

By condition (4), $s_i(x_i)$ satisfy

$$|s_i(x_i)| \le l_i |x_i|, \ i = 1, 2, ..., n . \tag{4}$$

Theorem 3.1 The zero solution of the discrete-time system of neural networks (2) is asymptotically stable if there exist symmetric positive definite matrices P, G, W, R satisfying the following matrix inequalities of the form

$$\psi = \begin{pmatrix} (1,1) & (1,2) \\ (2,1) & (2,2) \end{pmatrix} < 0 , \tag{5}$$

where

(1,1)= $APA - P + \hat{h}W$,

(1,2)=$APBL$, (2,1)=$L^T B^T PA$,

(2,2)=$L^T B^T PBL - G - \hat{h}R$, and $\hat{h} = h_2 - h_1 + 1$.

Proof Consider the Lyapunov function candidate
where

$$V_1(x(k)) = x^T(k)Px(k) , \quad V_2(x(k)) = \sum_{i=k-h(k)}^{k-1} x^T(i)Gx(i), \quad V_3(x(k)) = \sum_{j=k-h_2+1}^{k-h_1} \sum_{i=j}^{k-1} x^T(i)Wx(i),$$

$$V_4(x(k)) = \sum_{i=k-h(k)}^{k-1} (h(k) - k + i)x^T(i)Rx(i).$$

The Lyapunov difference of the system along trajectory of solution of (2) is given by

$$\begin{aligned}\Delta V_1(x(k)) &= V_1(x(k+1)) - V_1(x(k))\\ &= [Ax(k)+BS(x(k-h(k)))]^T\\ &\times P[Ax(k)+BS(x(k-h(k)))]\\ &-x^T(k)Px(k).\end{aligned}$$

Based on (4), we obtain

$$\begin{aligned}\Delta V_1(x(k)) \le\ & x^T(k)[APA-P]x(k)\\ &+x^T(k)APBLx(k-h(k))\\ &+x^T(k-h(k))L^TB^TPAx(k)\\ &+x^T(k-h(k))L^TB^TPBLx(k-h(k)).\end{aligned}$$

Therefore,

$$\begin{aligned}\Delta V_1(x(k)) \le\ & x^T(k)[APA-P]x(k)\\ &+x^T(k)APBLx(k-h(k))\\ &+x^T(k-h(k))L^TB^TPAx(k)\\ &+x^T(k-h(k))L^TB^TPBLx(k-h(k)).\end{aligned}$$

and, we get

$$\begin{aligned}\Delta V_2(x(k)) &= \sum_{i=k+1-h(k+1)}^{k} x^T(i)Gx(i) - \sum_{i=k-h(k)}^{k-1} x^T(i)Gx(i)\\ &= \sum_{i=k-h(k+1)}^{k-h_1} x^T(i)Gx(i) + x^T(k)Gx(k)\\ &-x^T(k-h(k))Gx(k-h(k)) + \sum_{k+1-h_1}^{k-1} x^T(k)Gx(k)\\ &-\sum_{i=k+1-h(k)}^{k-1} x^T(i)Gx(i).\end{aligned}$$

Since $h(k) \ge h_1$, we get

$$\sum_{k+1-h_1}^{k-1} x^T(k)Gx(k) - \sum_{i=k+1-h(k)}^{k-1} x^T(i)Gx(i) \le 0.$$

Therefore,

$$\begin{aligned}\Delta V_2(x(k)) \le & \sum_{i=k+1-h(k+1)}^{k-h_1} x^T(i)Gx(i) + x^T(k)Gx(k)\\ &-x^T(k-h(k))Gx(k-h(k)).\end{aligned}$$

Similarly, we obtain

$$\begin{aligned}
&\Delta V_3(x(k)) \\
&= \sum_{j=k-h_2+1}^{k-h_1}\left[x^T(k)Wx(k)+\sum_{i=k+j}^{k-1}x^T(i)Wx(i)-\sum_{i=k+j-1}^{k-1}x^T(i)Wx(i)\right] \\
&= (h_2-h_1)x^T(k)Wx(k)-\sum_{i=k+1-h_2}^{k-h_1}x^T(i)Wx(i),
\end{aligned}$$

$$\begin{aligned}
&\Delta V_4(k,y(k)) \\
&= \sum_{i=k+1-h(k+1)}^{k+1-1}(h(k+1)-k+1+i)x^T(i)Rx(i) \\
&- \sum_{i=k+1-h(k+1)}^{k+1-1}(h(k)-k+i)x^T(i)Rx(i) \\
&= (h_2-h_1)\sum_{i=k+1-h(k+1)}^{k-h_1}x^T(i)Rx(i)+(h_2-h_1)x^T(k)Rx(k) \\
&-(h_2-h_1)x^T(k-h(k))Rx(k-h(k)) \\
&+(h_2-h_1)\sum_{k+1-h_1}^{k-1}x^T(k)Rx(k)-(h_2-h_1)\sum_{i=k+1-h(k)}^{k-1}x^T(i)Rx(i).
\end{aligned}$$

Since $h(k)\ge h_1$, we get

$$(h_2-h_1)\sum_{k+1-h_1}^{k-1}x^T(k)Rx(k)-(h_2-h_1)\sum_{i=k+1-h(k)}^{k-1}x^T(i)Rx(i)\le 0.$$

Therefore,

$$\begin{aligned}
\Delta V_4(k,y(k)) &= (h_2-h_1)\sum_{i=k+1-h(k+1)}^{k-h_1}x^T(i)Rx(i) \\
&+(h_2-h_1)x^T(k)Rx(k) \\
&-(h_2-h_1)x^T(k-h(k))Rx(k-h(k)).
\end{aligned}$$

From $\Delta V_2(x(k)),\Delta V_3(x(k)),$ and $\Delta V_4(x(k))$, we get

$$\begin{aligned}
&\Delta V_2(x(k))+\Delta V_3(x(k))+\Delta V_4(x(k)) \\
&\le \sum_{i=k-h(k+1)}^{k-h_1}x^T(i)Gx(i)+x^T(k)Gx(k) \\
&-x^T(k-h(k))Gx(k-h(k)) \\
&(h_2-h_1)\sum_{i=k+1-h(k+1)}^{k-h_1}x^T(i)Rx(i)+(h_2-h_1)x^T(k)Rx(k) \\
&-(h_2-h_1)x^T(k-h(k))Rx(k-h(k)) \\
&+\hat{h}\,x^T(k)Wx(k)-\sum_{i=k+1-h_2}^{k-h_1}x^T(i)Wx(i).
\end{aligned}$$

Where $\hat{h}=h_2-h_1+1.$ Since $h(k)\le h_2$, we have

$$\sum_{i=k-h(k+1)}^{k-h_1} x^T(i)Gx(i) + x^T(k)Gx(k)$$
$$+(h_2-h_1)\sum_{i=k+1-h(k+1)}^{k-h_1} x^T(i)Rx(i)$$
$$+(h_2-h_1)x^T(k)Rx(k) - \sum_{i=k+1-h_2}^{k-h_1} x^T(i)Wx(i) \le 0.$$

Thus,

$$\Delta V_2(x(k)) + \Delta V_3(x(k)) + \Delta V_4(x(k))$$
$$\le \hat{h}\, x^T(k)Wx(k) - x^T(k-h(k))Gx(k-h(k))$$
$$-\hat{h}\, x^T(k-h(k))Rx(k-h(k)).$$

As a result, we obtain

$$\begin{aligned}\Delta V &\le x^T(k)[APA - P + \hat{h}W]x(k)\\ &+x^T(k)APBLx(k-h(k))\\ &+x^T(k-h(k))L^T B^T PAx(k)\\ &+x^T(k-h(k))[L^T B^T PBL - G - \hat{h}R]x(k-h(k))\\ &= \begin{pmatrix} x(k) \\ x(k-h(k)) \end{pmatrix}^T \begin{pmatrix} (1,1) & (1,2) \\ (2,1) & (2,2) \end{pmatrix} \begin{pmatrix} x(k) \\ x(k-h(k)) \end{pmatrix}\\ &= y^T(k)\psi\, y(k).\end{aligned}$$

where

$(1,1)= APA - P + \hat{h}W,$

$(1,2)=APBL,$

$(2,1)=L^T B^T PA,$

$(2,2)=L^T B^T PBL - G - \hat{h}R,$

$y(k) = \begin{pmatrix} x(k) \\ x(k-h(k)) \end{pmatrix}.$

By the condition (5), $\Delta V(y(k))$ is negative definite, namely there is a number $\beta > 0$ such that $\Delta V(y(k)) \le -\beta \|y(k)\|^2$, and hence, the asymptotic stability of the system immediately follows from **Lemma 2.1**. This completes the proof. □

Example 3.1 Let us consider a discrete-time system of neural networks (2), given by the system

$$x(k+1) = Ax(k) + BS(x(k-h)) + CKx(k),$$

where the matrices are

$$A = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.9 \end{pmatrix},\ B = \begin{pmatrix} 0.8 & -0.5 \\ 0 & -0.7 \end{pmatrix}, L = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.6 \end{pmatrix},\ \varepsilon = 0.2,\ \varepsilon_1 = 0.3.$$

Using the LMI Toolbox in MATLAB, we found that the LMIs in **Theorem 3.1** are feasible and

$$P = \begin{pmatrix} 10.2369 & -0.5397 \\ -0.5397 & 4.2319 \end{pmatrix}, G = \begin{pmatrix} 5.3214 & -0.9321 \\ -0.9321 & 1.0367 \end{pmatrix}, W = \begin{pmatrix} 2.3971 & -1.0379 \\ -1.0379 & 3.2156 \end{pmatrix}, \quad R = \begin{pmatrix} 3.0269 & -0.2301 \\ -0.2301 & 1.0397 \end{pmatrix},$$

$0.2397 \le h(k) \le 19.3248$ are set of solutions to the LMIs (5).
By a straightforward, we have

$$\psi = \begin{pmatrix} -6.3214 & 0 \\ 0 & -7.0324 \end{pmatrix}.$$

The eigenvalues are -6.3214 and -7.0324, respectively. This implies the matrix $\psi < 0$. It follows from Lemma 2.1 that the zero solution of discrete-time system of neural networks is asymptotically stable.

Remark 3.1 Theorem 3.1 gives a sufficient condition for the asymptotic stability of discrete-time system of neural networks (2) via matrix inequalities. These conditions are described in terms of certain symmetric matrix inequalities (2x2). But [2, 10, 12, 14, 16, 18, 19, 20] these conditions are described in terms of certain symmetric matrix inequalities more than (2x2).

4. CONCLUSIONS

This paper was dedicated to the delay-dependent stability of control discrete-time recurrent neural networks with time-varying delay. A less conservative LMI-based globally stability criterion is obtained with quadratic Lyapunov functional approach and free-weighting matrix approach for periodic control discrete-time recurrent neural networks with a time-varying delay. One example illustrates the exactness of the proposed criterion.

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