ON COMMON FIXED POINT THEOREMS FOR MULTIVALUED MAPPINGS IN INTUITIONISTIC FUZZY METRIC SPACES

Kamal Wadhwa[1] & Hariom Dubey[2]
[1]Govt. Narmada Mahavidyalaya, Hoshangabad (M.P.) India
[2]Technocrats Institute of technology, Bhopal, (M.P.) India
E-mail: omsatyada@gmail.com

ABSTRACT

The aim of this paper is to obtain some common fixed point theorems in an intuitionistic fuzzy metric space for two pairs of occasionally weakly semi compatible hybrid mappings.

Keywords: *Intuitionistic fuzzy metric space, occasionally weakly semi-compatible pairs.*

Mathematics Subject Classification: 47H10, 54H25

1. INTRODUCTION

As a generalization of fuzzy sets introduced by Zadeh [16], Atanassov [4] introduced the concept of intuitionistic fuzzy sets. Recently, using the idea of intuitionistic fuzzy sets, Park [9] introduced the notion of intuitionistic fuzzy metric spaces with the help of continuous t-norm and continuous t-conorms as a generalization of fuzzy metric spaces due to George and Veeramani [5]. Jungck and Rhoades [6] gave more generalized concept weak compatibility then compatibility.Al-Thagafi and Shahzad [3] weakened the concept of weakly compatible maps by giving the concept of occasionally weakly compatible maps.More recently Abbas and Rhoades [1] extended the definition of o.w.c. maps to the setting of set-valued maps. Recently, many authors have studied fixed point theory in intuitionistic fuzzy metric spaces ([1],[6],[7],[11],[13]).In this paper we obtain common fixed point theorems for hybrid pairs of single and multivalued occasionally weakly semi-compatible mappings.

We begin by briefly recalling some definitions and notions from fixed point theory literature that we will use in the sequel.

Definition 2.1 [22] - A binary operation $*:[0,1]\times[0,1]\to[0,1]$ is a continuous t-norms if $*$ satisfying conditions:

(i) $*$ is commutative and associative;
(ii) $*$ is continuous;
(iii) $a*1=a$ for all $a\in[0,1]$;
(iv) $a*b\le c*d$ whenever $a\le c$ and $b\le d$ for all $a,b,c,d\in[0,1]$.

Definition 2.2 A binary operation $\Diamond:[0,1]\times[0,1]\to[0,1]$ is continuous t-conorm if $\Diamond$ is satisfying the following conditions:

(i) $\Diamond$ is commutative and associative;
(ii) $\Diamond$ is continuous;
(iii) $a \Diamond 0 = a$ for all $a\in[0,1]$;
(iv) $a \Diamond b \le c \Diamond d$ whenever $a \le c$ and $b \le d$ for all $a,b,c,d\in[0,1]$.

Definition 2.3 A 5-tuple $(X,M,N,*,\Diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t-conorm and M,N are fuzzy sets on $X^2\times(0,\infty)$ satisfying the following conditions:

(i) $M(x,y,t)+N(x,y,t)\le 1$ for all $x,y\in X$ and $t>0$;

(ii) $M(x,y,0)=0$ for all $x,y\in X$;

(iii) $M(x,y,t)=1$ for all $x,y\in X$ and $t>0$ if and only if $x=y$;

(iv) $M(x,y,t)=M(y,x,t)$ for all $x,y\in X$ and $t>0$;

(v) $M(x,y,t)*M(y,z,s)\le M(x,z,t+s)$ for all $x,y,z\in X$ and s, $t>0$;

(vi) for all $x,y\in X$, $M(x,y,.):[0,\infty)\to[0,1]$ is left continuous;

(vii) $\lim_{t\to\infty} M(x,y,t)=1$ for all $x,y\in X$ and $t>0$;

(viii) $N(x,y,0)=1$ for all $x,y\in X$;

(ix) $N(x,y,t)=0$ for all $x,y\in X$ and $t>0$ if and only if $x=y$;

(x) $N(x,y,t)=N(y,x,t)$ for all $x,y\in X$ and $t>0$;

(xi) $N(x,y,t)\Diamond N(y,z,s)\ge N(x,z,t+s)$ for all $x,y,z\in X$ and s, $t>0$;

(xii) for all, $x,y\in X, N(x,y,.):[0,\infty)\to[0,1]$ is right continuous;

(xiii) $\lim_{t\to\infty} N(x,y,t)=0$ for all $x,y\in X$;

(M,N) is called an intuitionistic fuzzy metric on X . The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t , respectively.

Remark An intuitionistic fuzzy metric spaces with continuous t-norm $*$ and Continuous t -conorm $\Diamond$ defined by $a*a\ge a$, $a\in[0,1]$ and $(1-a)\Diamond(1-a)\le(1-a)$ for all $a\in[0,1]$, Then for all $x,y\in X, M(x,y,*)$ is non-decreasing and, $N(x,y,\Diamond)$ is non-**increasing.**

Definition:2.4 Let $(X,M,N,*,\Diamond)$ be an intuitionistic fuzzy metric space and $f:X\to X, F:X\to B(X)$ .The hybrid pairs (f,F) is said to be occasionally weakly compatible(o.w.c.) iff there exists some point $x\in X$ such that $fx\in Fx$ and $fFx\subseteq Ffx$.

Definition2.5: Let $(X,M,N,*,\Diamond)$ be an intuitionistic fuzzy metric space and $f:X\to X, F:X\to B(X)$ .The hybrid pairs (f,F) is said to be occasionally weakly semi- compatible(o.w.s.c.) iff there exists some point $x\in X$ such that $fx\in Fx$ and $f^2x\in Ffx$.

It is clear that owc hybrid pair is owsc pair, but not the converse in view of the following example.

Example:2.6 Let $X=[0,1]$ and Define $fx=1-x$ and $Fx=[0,\frac{1}{2}]$.

Then $f(\frac{1}{2})=\frac{1}{2}\in F(\frac{1}{2}),\ fF(\frac{1}{2})=[\frac{1}{2},1]\not\subset Ff(\frac{1}{2})=[0,\frac{1}{2}]$ and $f^2(\frac{1}{2})=\frac{1}{2}\in Ff(\frac{1}{2})$.

Thus the hybrid pair (f,F) is owsc, but not owc.

Definition2.7 [13] Let $CB(X)$ be the set of all nonempty bounded and closed subsets of X ,we define the functions;

$M^{\nabla}(a,B,t)=\max\{M(a,b,t):b\in B\}$

If $a\in B$ then from above definition $M^{\nabla}(a,B,t)=1$

$N^{\Delta}(B,y,t)=\min\{N(b,y,t);b\in B\}$

Definition 2.8 [11] Let $CB(X)$ be the set of all nonempty bounded and closed subsets of X ,we define the functions;

$\delta(A,B,t)=\inf\{M(a,b,t):a\in A,b\in B\}$.

3. MAIN RESULTS

Theorem1: Let $(X,M,N,*,\Diamond)$ be a intuitionistic fuzzy metric space and $f,g:X\to X, F,G:X\to CB(X)$ be mappings satisfying

(i) the pairs (f,F) and (g,G) are o.w.s.c.

(ii) . $\int_0^{M^{\nabla}(fx,Gy,qt)}\varphi(t)dt \le \int_0^{\min\{M(fx,gy,t),M^{\nabla}(fx,Fx,t),M^{\nabla}(gy,Gy,t)\}}\varphi(t)dt$ and

$$\int_0^{N^{\Delta}(fx,Gy,qt)}\varphi(t)dt \ge \int_0^{\max\{N(fx,gy,t),N^{\Delta}(fx,Fx,t),N^{\Delta}(gy,Gy,t)\}}\varphi(t)dt$$

for all $x,y\in X$, where $q>1$, $\varphi:R^{+}\to R$ is a Lebesgue-integrable mapping which is summable, nonnegative and such that $\int_0^{\varepsilon}\phi(t)dt>0$ for each $\varepsilon>0$.

Proof: Since the pairs (f,F) and (g,G) are occasionally weakly semi-compatible,there exist $u,v\in X$ such that $fu\in Fu, f^2u\in Ffu, gv\in Gv$ and $g^2v\in Ggv$.

Suppose $fu\ne gv$.From (ii),we have

$$\int_0^{M(fu,gv,qt)}\varphi(t)dt \le \int_0^{M^{\nabla}(fu,Gv,qt)}\varphi(t)dt \le \int_0^{\min\{M(fu,gv,t),M^{\nabla}(fu,Fu,t),M^{\nabla}(gv,Gv,t)\}}\varphi(t)dt$$

$$=\int_0^{\min\{M(fu,gv,t),1,1\}}\varphi(t)dt$$

$$=\int_0^{M(fu,gv,t)}\varphi(t)dt \quad \text{....(I)}$$

and

$$\int_0^{N(fu,gv,qt)}\varphi(t)dt \ge \int_0^{N^{\Delta}(fu,Gv,qt)}\varphi(t)dt \ge \int_0^{\max\{N(fu,gv,t),N^{\Delta}(fu,Fu,t),N^{\Delta}(gv,Gv,t)\}}\varphi(t)dt$$

$$=\int_0^{\max\{N(fu,gv,t),0,0\}}\varphi(t)dt$$

$$= \int_0^{N(fu,gv,t)} \varphi(t)dt \qquad \text{....(II)}$$

(I) and (II) implies that $fu = gv$

Suppose $f^2u \neq fu$ .i.e. $f^2u \neq gv$ .Since $fu \in Fu$ and $f^2u \in Ffu$,from (ii),we have

$$\int_0^{M(f^2u,gv,qt)} \varphi(t)dt \leq \int_0^{M^{\nabla}(f^2u,Gv,qt)} \varphi(t)dt \leq \int_0^{\min\{M(f^2u,gv,t),M^{\nabla}(f^2u,Ffu,t),M^{\nabla}(gv,Gv,t)\}} \varphi(t)dt$$

$$\int_0^{M(f^2u,fu,qt)} \varphi(t)dt \leq \int_0^{\min\{M(f^2u,fu,t),M^{\nabla}(f^2u,Ffu,t),M^{\nabla}(gv,Gv,t)\}} \varphi(t)dt$$

$$\int_0^{M(f^2u,fu,qt)} \varphi(t)dt \leq \int_0^{\min\{M(f^2u,fu,t),1,1\}} \varphi(t)dt$$

$$\int_0^{M(f^2u,fu,qt)} \varphi(t)dt \leq \int_0^{M(f^2u,fu,t)} \varphi(t)dt \qquad \text{.......(III)}$$

$$\int_0^{N(f^2u,fu,qt)} \varphi(t)dt \geq \int_0^{\max\{N(f^2u,fu,t),N^{\Delta}(f^2u,Ffu,t),N^{\Delta}(gv,Gv,t)\}} \varphi(t)dt$$

$$\int_0^{N(f^2u,fu,qt)} \varphi(t)dt \geq \int_0^{\max\{N(f^2u,fu,t),N^{\Delta}(f^2u,Ffu,t),N^{\Delta}(gv,Gv,t)\}} \varphi(t)dt$$

$$\int_0^{N(f^2u,fu,qt)} \varphi(t)dt \geq \int_0^{\max\{N(f^2u,fu,t),0,0\}} \varphi(t)dt$$

$$\int_0^{N(f^2u,fu,qt)} \varphi(t)dt \geq \int_0^{N(f^2u,fu,t)} \varphi(t)dt \qquad \text{........(IV)}$$

(II) and (IV) implies that $f^2u = fu$

Thus $fu = f^2u \in Ffu$ .Hence fu is a common fixed point of f and F .Similarly we can show that gv is common fixed point of g and G since $fu = gv$,it follows that fu is common fixed point of f, g, F and G .Uniqueness of common fixed point follows easily from (ii).

Theorem 2: Let $(X, M, N, *, \Diamond)$ be a fuzzy metric space and $f, g : X \to X, F, G : X \to CB(X)$ be mappings satisfying

(i) the pairs (f, F) and (g, G) are occasionally weakly semi-compatible,

(ii) $$\int_0^{\delta(Fx,Gy,qt)} \varphi(t)dt \geq \int_0^{\inf\{M(fx,gy,t),M^{\nabla}(fx,Fx,t),M^{\nabla}(gy,Gy,t),M^{\nabla}(fx,Gy,t),M^{\nabla}(gy,Fx,t)\}} \varphi(t)dt$$

(iii) $$\int_0^{\delta(Fx,Gy,qt)} \varphi(t)dt \le \int_0^{\max\{N(fx,gy,t),N^{\Delta}(fx,Fx,t),N^{\Delta}(gy,Gy,t),N^{\Delta}(fx,Gy,t),N^{\Delta}(gy,Fx,t)\}} \varphi(t)dt$$

for all $x, y \in X$, where $0 < q < 1$, $\varphi : R^{+} \to R$ is a Lebesgue-integrable mapping which is summable, nonnegative and such that $\int_0^{\varepsilon} \phi(t)dt > 0$ for each $\varepsilon > 0$.

Proof: Since the pairs (f, F) and (g, G) are occasionally weakly semi-compatible,there exist $u, v \in X$ such that $fu \in Fu,\ f^2u \in Ffu,\ gv \in Gv$ and $g^2v \in Ggv$.

Suppose $fu \ne gv$.From (ii),we have

$$\int_0^{M(fu,gu,qt)} \varphi(t)dt \ge \int_0^{\delta(Fu,Gv,qt)} \varphi(t)dt \ge \int_0^{\inf\{M(fu,gv,t),M^{\nabla}(fu,Fu,t),M^{\nabla}(gv,Gv,t),M^{\nabla}(fu,Gv,t),M^{\nabla}(gv,Fu,t)\}} \varphi(t)dt$$

$$\int_0^{M(fu,gu,qt)} \varphi(t)dt \ge \int_0^{\inf\{M(fu,gv,t),1,1,M(fu,gv,t),M(gv,fu,t)\}} \varphi(t)dt$$

$$\int_0^{M(fu,gv,qt)} \varphi(t)dt \ge \int_0^{M(fu,gv,t)} \varphi(t)dt \qquad \text{....(I)}$$

and

$$\int_0^{N(fu,gv,t)} \varphi(t)dt \le \int_0^{\delta(Fu,Gv,qt)} \varphi(t)dt \le \int_0^{\max\{N(fu,gv,t),N^{\Delta}(fu,Fu,t),N^{\Delta}(gv,Gv,t),N^{\Delta}(fu,Gv,t),N^{\Delta}(gv,Fu,t)\}} \varphi(t)dt$$

$$\int_0^{N(fu,gv,t)} \varphi(t)dt \le \int_0^{\max\{N(fu,gv,t),0,0,N(fu,gv,t),N(gv,fu,t)\}} \varphi(t)dt$$

$$\int_0^{N(fu,gv,qt)} \varphi(t)dt \le \int_0^{N(fu,gv,t)} \varphi(t)dt \qquad \text{.....(II)}$$

(I) and (II) implies that $fu = gv$

Suppose $f^2u \ne fu$ .i.e. $f^2u \ne gv$ .Since $fu \in Fu$ and $f^2u \in Ffu$,from (ii),we have

$$\int_0^{M(f^2u,gv,t)} \phi(t)dt \ge \int_0^{\delta(Ffu,Gv,qt)} \varphi(t)dt \ge \int_0^{\inf\{M(f^2u,gv,t),M^{\nabla}(f^2u,Ffu,t),M^{\nabla}(gv,Gv,t),M^{\nabla}(f^2u,Gv,t),M^{\nabla}(gv,Ffu,t)\}} \varphi(t)dt$$

$$\int_0^{M(f^2u,gv,t)} \phi(t)dt \ge \int_0^{\inf\{M(f^2u,gv,t),1,1,M(f^2u,gv,t),M(gv,f^2u,t)\}} \varphi(t)dt$$

$$\int_0^{M(f^2u,gv,t)} \phi(t)dt \geq \int_0^{M(f^2u,gv,t)} \varphi(t)dt \qquad \text{……(III)}$$

And

$$\int_0^{N(f^2u,gv,t)} \varphi(t)dt \leq \int_0^{\delta(Ffu,Gv,qt)} \varphi(t)dt \leq \int_0^{\max\left\{N(f^2u,gv,t),N^{\Delta}(f^2u,Ffu,t),N^{\Delta}(gv,Gv,t),N^{\Delta}(f^2u,Gv,t),N^{\Delta}(gv,Ffu,t)\right\}} \varphi(t)dt$$

$$\int_0^{N(f^2u,gv,t)} \varphi(t)dt \leq \int_0^{\max\left\{N(f^2u,gv,t),0,0,N(f^2u,gv,t),N(gv,f^2u,t)\right\}} \varphi(t)dt$$

$$\int_0^{N(f^2u,gv,t)} \varphi(t)dt \leq \int_0^{N(f^2u,gv,t)} \varphi(t)dt \qquad \text{……(IV)}$$

(II) and (IV) implies that

$$f^2u = fu$$

Thus $fu = f^2u \in Ffu$ .Hence fu is a common fixed point of f and F .Similarly we can show that gv is common fixed point of g and G since $fu = gv$,it follows that fu is common fixed point of f, g, F and G .Uniqueness of common fixed point follows easily from (ii).

4. REFERENCES:

[1]. Abbas,M.,Rhoades, B.E, Common Fixed Point Theorems for Hybrid pairs of Occasionally Weakly Compatible Mappings Satisfying Generalized Contractive Condition of Intregal type .Fixed Point Theory and Applications , 2007,Article ID 54101,9 pages.

[2]. C. Alaca, D. Turkoglu, and C. Yildiz, Fixed point in intuitionistic fuzzy metric spaces,Chaos, Solitons and Fractals 29 (2006), 1073{1078.

[3]. Al-Thagafi, M.A, Shahzad, N., Generalized I-nonexpansive self maps and invariant approximations.Acta.Math.Sinica. 24(5)(2008), 867-876

[4]. K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986), 87{96.

[5]. A. George and P. Veeramani, On some results in fuzzy metric spaces, Fuzzy sets andSystems 64 (1994), 395{399.

[6]. G. Jungck and B. E. Rhoades, Fixed point for set valued functions without continuity,Ind. J. Pure & Appl. Math. 29 (1998), no. 3, 227{238.

[7]. I. Kubiaczyk and S. Sharma, Common fixed point in fuzzy metric space, J. Fuzzy Math.(to appear).

[8]. S. Kutukcu, A common fixed point theorem for a sequence of self maps in intuitionisticfuzzy metric spaces, Commun. Korean Math. Soc. 21 (2006), no. 4, 679{687.

[9]. J. H. Park, Intuitionistic fuzzy metric spaces, Chaos, Solitons and Fractals 22 (2004),1039{1046.

[10]. B. Schweizer and A. Sklar, Statistical spaces, Paci¯c J. Math. 10(1960),313{334.

[11]. S. Sedghi "Common fixed point theorems for multivalued contractions" International Mathematical Forum, 2, 2007, no. 31, 1499-1506.

[12]. S. Sharma, On fuzzy metric space, Southeast Asian Bull. Math. 6 (2002), no. 1, 145{157.

[13]. Sushil Sharma, Servet Kutuku, and R.S.Rathore,commun." Common fixed point for multivalued mappings in intuitionistic fuzzy metric spaces" Korean Math.Soc.22(2007), No.3,pp. 391-399.

[14]. D. Turkoglu, C. Alaca, and C. Yildiz, Compatible maps and compatible maps of type (®) and (¯) in intuitionistic fuzzy metric spaces, Demonstratio Math. 39 (2006), no. 3,671{684).

[15]. D. Turkoglu, C. Alaca, Y. J. Cho, and C. Yildiz, Common ¯xed point theorems inintuitionistic fuzzy metric spaces, J. Appl. Math. & Computing 22 (2006), no. 1-2,411{424.

[16]. L. A. Zadeh, Fuzzy sets, Inform. and Control 8 (1965), 338{353).