PHASE CONJUGATION AND PHONON ECHOES IN OPTICS AND ACOUSTICS

Arthur Ekpekpo
Department of Physics, Delta State University, Abraka, Nigeria
*E-mail: arthurekpekpo@yahoo.com

ABSTRACT
The phonon – echo phenomena is explained in terms of phase conjugation which is a concept widely used in non – linear optics. This point of view provides a unified description for backward echoes both in optics and in acoustics.

1. INTRODUCTION

The phonon – echo phenomenon was independently discovered by two soviet groups in 1971 [10, 5]. In its simplest form (two – pulse echo) the phenomenon can be described as follows. A piezo – electric sample is irradiated at time t= 0 by a short electromagnetic pulse at frequency f= $^{w}/_{2\prod}$, later, at time t= τ, a second pulse at the same frequency irradiates again the sample; as a result, at time t= 2τ a signal, still at frequency f, is coherently radiated by the sample. Since the response to two pulses is not the sum of the responses to each pulse separately, this is a non-linear effect. Because of the analogy with the famous exponential achieved in 1950 by Hahn on nuclear spins [4], such an effect is called "echo". Following the same analogy, the exponential decrease of the echo signal when the time τ is increased is characterized by a relaxation time called T_2.

The earliest experiments [10] gave same indications that the mechanism motion of the sample plays a role in the effect. However the real proof of the occurrence of phonons was only obtained in 1974 [1], when it was shown that the sound velocity is a relevant quantity of the problem. Since this experiment the name "Phonon echo" is generally used.

After its appearance in 1971, the phonon-echo technique has known numerous developments: Three-pulse echoes giving an interesting memory effect, echoes in powders, applications to the study of phase transition e.t.c. Several review articles are published [3] and [2] on the subject.

During the period when the phonon echo technique was developed (generally by people who were already familiar with acoustics) opticians, using their powerful lasers, were also studying non-linear effects. Among these effects there is, for instance, the frequency mixing which is used in particular for frequency doubling. In this case two electromagnetic waves at frequency f interact to give an electromagnetic wave at frequency $2f$, this is a three-wave interaction. Another interesting process is the interaction of four waves, all at the same frequency and with wave vectors opposite in pairs (degenerate four – wave mixing, or DFWM).

The interest of the latter interaction lies in the possibility of obtaining a "wave vector reversal" or equivalently a "time reversal". This effect may be used to correct a light beam aberrated by the inhomogeneities of the medium in which it propagates. The wave-vector reversal can be included into more general concept of "phase conjugation" (in short: PC). The first observations of PC were obtained by two soviet groups [6] and [7], using stimulated Brillouin Scattering (SBS), an effect resulting from the coupling of two electromagnetic waves with one acoustic wave. PC due to DFWM was observed later [8] and [9].

The main purpose of this article is to show that the two phenomena (phonon echo on the one hand and optical phase conjugation on the other hand) which have grown up in parallel, but in mutual ignorance are indeed quite similar.

THEORETICAL CONSIDERATIONS AND CALCULATIONS

Let Q(x, y, ƶ, t) be a function of spatial coordinates and time which can be written in the form

$$Q(x, y, ƶ, t) = \mathrm{Re}\,[u(x, y, ƶ, t)\, l^{iwt}] \qquad (1)$$

The phase conjugation operation is the transformation of Q(x, y, ƶ, t) into Q(x, y, z, t) with

$$\overline{Q}(x, y, ƶ, t) = \mathrm{Re}\,[u^{u}(x, y, ƶ, t)\, l^{iwt}] \qquad (2)$$

$\overline{Q}$ differs form Q by the complex conjugation of the spatial factor, while the temporal factor is unchanged. Since the real part of the product is finally taken, it would be equivalent to conjugate the temporal factor l^{iwt}, in other words, phase conjugation is equivalent to "time reversal".

If we consider the special case of a plane wave propagating along the ƶ – axis the spatial factor can be written as U(x, y, ƶ) = V exp ¡(Ψ – qƶ) with V real. The initial wave is therefore

$$Q(x, y, ƶ, t) = V\,\mathrm{Re}\,[\exp ¡(\omega t - qƶ + \Psi)] \qquad (3)$$

The conjugated wave is

$$\overline{Q}(x, y, ƶ, t) = V\,\mathrm{Re}\,[\exp ¡(\omega t + qƶ - \Psi))] \qquad (4)$$

The initial wave is propagating in the – z direction and its phase is Ψ. On the contrary, the conjugated wave is propagating in the Z direction with an opposite phase – Ψ. Such a simple example provides us with a physical idea of the mathematical operation of phase conjugation (PC).

This concept can be applied to more complex cases. An example of a divergent beam emitted by a point source is very striking. After reflection in an ordinary mirror, this beam continues to diverge. On the contrary, if the divergent one which comes back to the emitting point

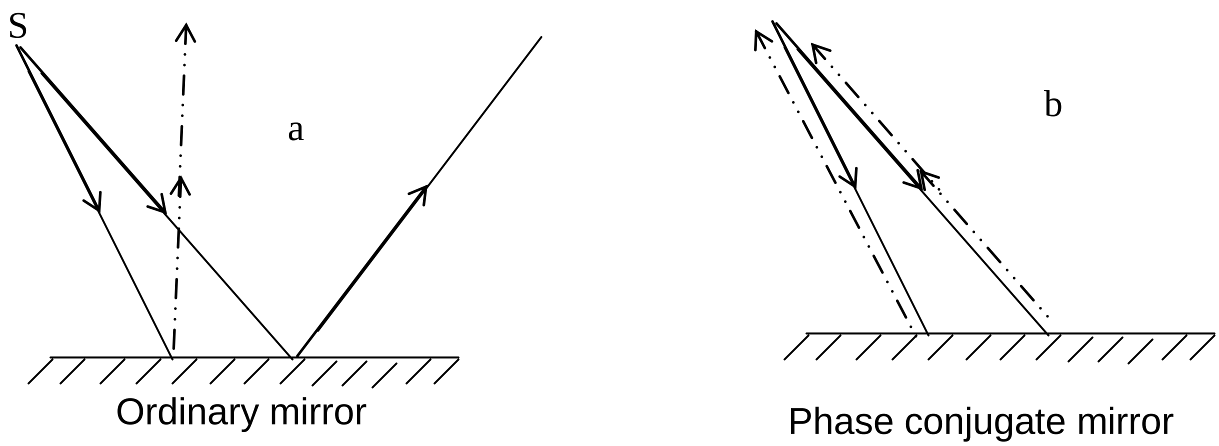

Figure 1 – *Reflection of a divergent beam on: (a) an ordinary mirror (b) a phase conjugate mirror*

A coupling of the form $X^{(4)} E_1(\omega, k)\, E_2(\omega, -\vec{k})\, E_3(\omega, \vec{q})\, E_4(\omega, -\vec{q})$ between the electric fields of four electromagnetic waves having all the same frequency is responsible for PC in optics through

1. DFWM.

Indeed, the occurrence of this non-linear interaction is very natural. It originates from one of the terms of the expansion of the electric energy density of a medium in increasing powers of the electric field.

$$U_{electric} = X^{(2)} E^2 + X^{(3)} E^3 + X^{(4)} E^4 + \dots \qquad (5)$$

In this equation the upper index (n) refers to the rank of the tensor $X^{(n)}$, it allows to immediately know if $X^{(n)}$ is equal to zero or not (if n is odd and if the medium has a centre of symmetry, $X^{(n)}$ vanishes). Our notation is a shorthand for a more complicated one:

$$\frac{1}{2!} \quad \frac{1}{3!} \quad \frac{1}{4!}$$

$$X^4 E^{(4)} = \sum_{\alpha\beta\gamma\partial} X_{\alpha\beta\gamma\partial} E_\alpha E_\beta E_\gamma E_\partial \qquad (6)$$

where $\alpha, \beta, \gamma, \partial$ = 1, 2,3 or x, y,z.

A similar expansion can be written for the elastic energy density in increasing powers of the elastic strain E.

$$\frac{1}{2!} \quad \frac{1}{3!} \quad \frac{1}{4!}$$

$$U_{elastic} = C^{(4)} E^2 + C^{(6)} E^3 + C^{(9)} E^4 + \dots \qquad (7)$$

We notice that all the tensors $C^{(n)}$ appearing in this expansion have an even rank and therefore they are never equal to zero. From the analogy between the proceedings two expansions we can guess that acoustic DFWM nay exist at least, owing to the C^4 terms.

Besides the purely electric and the purely elastic contribution to the energy density it also exists as an electro acoustic contribution. Its expansion begins as

$$U_{electro\text{-}acoustic} = \alpha^{(1)} E\, E + g^{(4)} E^2 E + \beta^{(5)} E\, E^2 + \gamma^{(6)} E^2 E^2 + \dots \qquad (8)$$

The fourth term is of fundamental importance for DFWM. It is at the origin of an electro acoustic interaction of the form:

$$\gamma^{(6)} E_1(\omega, \vec{k})\, E_2(\omega, -\vec{k})\, E_3(\omega, \vec{q})\, E_4(\omega, -\vec{q}) \quad (9)$$

The roles are distributed among these actors as follows: the elastic wave ε_3 is the probe while the elastic wave E_4 is the signal, the two electromagnetic waves E_1 and E_2 are the pump.

It is useful to do the following comments:

I. Because tensor $^{(6)}$ is of the even rank. It does not vanish whatever the symmetry of the medium is.

II. This interaction exists in the bulk of the sample because the sum of the wave vectors of the four waves is zero (this property is related to the translational in variance of the medium.

We consider now the third term which can give the following interaction

$$\beta^{(5)} E_{12}(2\omega, \vec{k})\, E_3(\omega, \vec{q})\, E_4(\omega, \vec{q}^1) \quad (10)$$

Two comments have to be done

I. Tensor $\beta^{(5)}$ is of an odd rank and it vanishes for any medium having a centre of symmetry.

II. This interaction is efficient in the bulk of the sample only if the condition

III.
$$\vec{K} + \vec{q} + \vec{q}^1 = 0,$$

for the wave-vectors is fulfilled. However we know that an order of magnitude for the sound velocity is v = 3 x $10^3 m/s^{-1}$ while the light velocity is of the order of c = 3 x 10^8m/s. Consequently, at the same frequency the wave-vector $\vec{k}$ is much smaller that the wave-vector $\vec{q}$ $k/\vec{q} = 10^{-5}$. Then $\vec{k}$ may be neglected (and taken equal to 0) and the interaction between the three waves becomes

$$\beta^{(5)} E_{12}(2\omega, o)\, E_3(\omega, \vec{q})\, E_4(\omega, -\vec{q}) \quad (11)$$

In this three wave mixing (TWM) the probe E_3 and the signal E_4 are acoustic while the pump is the electromagnetic wave E_{12}.

Thus we know three types of interactions which describe wave mixing in optics, in acoustic and in electro acoustics. We emphasize their similarities by remarking that in all of them we have:

1) a probe (E_3 or E_3) with frequency f = w/2π and wave-vector $\vec{q}$.

2) a signal (E_4 or E_4) with the same frequency 2f and is uniform in space.

Before we show that the previous three interactions are able to produce PC we give some physical insight onto the electro acoustic coefficients $\beta^{(5)}$ and $^{(6)}$.

For this purpose we write down again the energy density expansion:

$$U_{total} = \frac{1}{2}X^{(2)}E^2 + \propto^{(3)} E\,\varepsilon + \frac{1}{2}C^{(4)}\varepsilon^2 + \frac{1}{6}X^{(3)}E^3 + g^{(4)} E^2\varepsilon + \beta^{(5)}E\,\varepsilon^2 + \frac{1}{6}\,C^{(6)}\varepsilon^3 + \frac{1}{24}X^{(4)}E^4 + --- \;\gamma^{(5)}E^2\varepsilon^2 + --- + \frac{1}{24}C^{(8)}\varepsilon^4 + --- \quad (12)$$

We first consider coefficient $\beta^{(5)}$. We can write either

$$\propto^{(3)} E\,\varepsilon + \beta^{(5)}E\,\varepsilon^2 \left[= \alpha^{(3)} + \beta^{(5)}\varepsilon\right] E\,\varepsilon$$

or

$$\frac{1}{2}c^{(4)}\varepsilon^2 + \beta^{(5)}E\,\varepsilon^2 = \frac{1}{2}\left[c^{(4)} + 2\beta^{(5)}E\right]\varepsilon^2 \quad (13)$$

$\beta^{(5)}$ appears to be either the derivative with respect to E of a generalized (strain dependent) piezoelectric coefficient or half of the derivative with respect to E of a generalized (electric field dependent) elastic constant. A method for direct measurement of $\beta^{(5)}$ (simply written β) is deduced from the last point of view. The sound velocity is:

$$V(E) = \sqrt{\frac{C(E)}{\rho}} = \sqrt{\frac{C + 2\beta E}{\rho}} = V(o)\left[1 + \frac{\beta E}{C}\right] \quad (14)$$

where V(o) is the sound velocity without electric field we see that the linear variation of sound velocity when a static electric field is applied is directly related to β. The numerical values of β typically range around 1 or 10 $NV^{-1}m^{-1}$ [11]

Coefficient $\gamma^{(6)}$ may be examined in an analogous manner, We can write either

$$\frac{1}{2}X^{(2)}E^2 + \gamma^{(6)}E^2\mathcal{E}^2 = \frac{1}{2}\left[X^{(2)} + 2\gamma^{(6)}E^2\right]E^2 \qquad (15)$$

At this moment we have to make a remark on the elastic strain tensor £. Its components are generally written

$$\mathcal{E}ij = \frac{1}{2}\left[\frac{\eth ui}{\eth xj} + \frac{\eth uj}{\eth xi}\right] \qquad (16)$$

where the vector u is the displacement. But we must keep in mind that such an equation is a linearized one. When this approximation is used the dominant terms are generally obtained.

We consider the one dimensional case (plane acoustic waves propagating along the x – axis).our starting equations are:

(i) the equation of motion

$$\rho\ddot{u} = \frac{\eth\sigma}{\eth x}, \qquad (17)$$

where u is the displacement, σ is the elastic stress (defined as ðv/ðE = σ) and E is the elastic strain (equal to ðu/ðx).

(ii) the matter equation giving σ in function of c and E; it is obtained from the energy density (only the relevant terms are written):

$$u = \frac{1}{2}c\mathcal{E}^2 + \beta E\mathcal{E}^2 \qquad (18)$$

which immediately gives

$$\sigma = \frac{\eth u}{\eth\mathcal{E}} = c\mathcal{E} + 2\beta E\mathcal{E} \qquad (19)$$

.

Here c stands for $c^{(4)}$ and β for $\beta^{(5)}$. Using this last equation, the equation of motion becomes:

$$\rho\ddot{u} - c\frac{\eth^2 u}{\eth x^2} = 2\beta\frac{\eth}{\eth x}(E\mathcal{E}) \qquad (20)$$

This is a **wave** equation with a source term on the right. It is implied that all the quantities (u, E, E) appearing in this equation are real quantities.

What should we like to find? We hope that if an acoustic wave propagating in the +x direction interacts with a convenient electric field, then an acoustic wave propagating in the – x direction is generated. This means that the product of an elastic strain

$$\mathcal{E}_3 = \frac{1}{2}\left[\bar{\mathcal{E}}_3 \ell^{i(\omega t - qx)} + \bar{\mathcal{E}}_3^* \ell^{-i(\omega t - qx)}\right] \qquad (21)$$

and of an elastic field

$$E_{12} = \frac{1}{2}\left[\bar{E}_3 \ell^{i2\omega t} + \bar{E}^*_{12} \ell^{-i2\omega t}\right] \qquad (22)$$

Put in the right – hand side of the wave equation is able to generate an elastic strain which may be written as :

$$E_4 = \frac{1}{2}\left[\bar{\mathcal{E}}_4 \ell^{i(\omega t + qx)} + \bar{\mathcal{E}}^*{}_4 \ell^{-i(\omega t + qx)}\right] \quad (23)$$

After a differentiation with respect to x, the equation of motion turns out to be:

$$\frac{1}{v^2}\ \frac{\partial^2 \mathcal{E}_4}{\partial t^2}\ \frac{-\partial^2 \mathcal{E}_4}{\partial x^2} = \frac{\beta}{2c}\frac{\partial^2}{\partial x^2}\left[\bar{E}_{12}\bar{\mathcal{E}}^* \ell^{i(\omega t + qx)} + c.c\right] \quad (24)$$

It is not possible to rigorously solve this equation. Consequently we use the following approximations:
(i) The pump amplitude E_{12} is assumed to be constant
(ii) The amplitudes of the probe E_3 (t) and of the signal E_4 (t) which are time dependent are assumed to have only variations which are slow in comparison with oscillations at frequency $\omega/2\pi$.
This is the so-called slowly varying envelope approximation:

$$\frac{\partial^2 \bar{\mathcal{E}}}{\partial t^2} << \frac{\omega \partial \bar{\mathcal{E}}}{\partial t} << \omega^2 \bar{\mathcal{E}} \quad (25)$$

Separating the two complex conjugate parts of the equation of motion we get:

$$\frac{1}{v^2}\left[\frac{\partial^2 \bar{\mathcal{E}}_4}{\partial t^2} + 2i\omega \frac{\partial \bar{\mathcal{E}}_4}{\partial t} - \omega^2 \bar{\mathcal{E}}_4\right] + q^2 \bar{\mathcal{E}}_4 = -q\frac{2\beta}{c}\ \bar{E}_{12}\bar{\mathcal{E}}_3{}^* \quad (26)$$

In the left hand side the last two terms cancel (because $\omega = qv$, while the first is henceforth neglected (because of the slowly varying envelope approximation). we obtain

$$\frac{\partial \bar{\mathcal{E}}_4}{\partial t} = i\omega \frac{\beta}{2c}\ \bar{E}_{12}\bar{\mathcal{E}}_3{}^* \quad (27)$$

Infact, the signal and the probe have very similar roles. Physically they can be distinguished by the initial values of their amplitudes when the process begins: at time t = 0 the signal amplitude is $E_4(0) = 0$, while the probe amplitude has a finite value $E_3(0) = E_4 0$.
Besides this difference, they have the same behaviours and consequently they satisfy similar equations:

$$\left.\begin{aligned} \frac{\partial \bar{\mathcal{E}}_4}{\partial t} &= i\omega \frac{\beta}{2c}\ \bar{E}_{12}\bar{\mathcal{E}}_3{}^* \\ \frac{\partial \bar{\mathcal{E}}_3}{\partial t} &= i\omega \frac{\beta}{2c}\ \bar{E}_{12}\bar{\mathcal{E}}_4{}^* \end{aligned}\right\} \quad (28)$$

On substitution of one of these equations in the other conjugate one we obtain

$$\left.\begin{aligned} &\frac{\partial^2 \bar{\mathcal{E}}_4}{\partial t^2} - \left(\frac{\omega\beta}{2c}\right)^2 \left|\bar{E}_{12}\right|^2 \bar{\mathcal{E}}_3 = 0 \\ &\frac{\partial^2 \bar{\mathcal{E}}_4}{\partial t^2} \end{aligned}\right\}$$

$$-\left(\frac{\omega\beta}{2c}\right)^2 \left|\bar{E}_{12}\right|^2 \bar{\mathcal{E}}_4 = 0 \quad (29)$$

With $s = \frac{\beta}{2c}\bar{E}_{12}$ the solutions of these equations are :

$$\left.\begin{aligned}\bar{\mathcal{E}}_3(t) &= A_3 \sinh(s\omega t) + B_3 \cosh(s\omega t)\\ \bar{\mathcal{E}}_4(t) &= A_4 \sinh(s\omega t) + B_4 \cosh(s\omega t)\end{aligned}\right\} \quad (30)$$

The initial condition $\bar{\mathcal{E}}_3(0) = \bar{\mathcal{E}}_3^0$, $\bar{\mathcal{E}}_4(0) = 0$, impose $B_4 = 0$ and $B_3 = \bar{\mathcal{E}}_3^0$ and also $A_3 = 0$ and $A_4 = i\,\bar{\mathcal{E}}_3^{0*}$.

Therefore, with our initial conditions the solutions are:

$$\left.\begin{aligned}\bar{\mathcal{E}}_3(t) &= \bar{\mathcal{E}}_3^0 \cosh(s\omega t)\\ \bar{\mathcal{E}}_4(t) &= \bar{\mathcal{E}}_3^0 \sinh(s\omega t)\end{aligned}\right\} \quad (31)$$

This result deserves some comments:

(i) Owing to the non – linear interaction between electric field and the strain the initial elastic wave (probe) is amplified while a backward propagating elastic wave (the signal) is generated.

(ii) The acoustic energy density of an elastic wave is proportional to $|\bar{\mathcal{E}}|^2$. Here we find

Find $|\mathcal{E}_3|^2 - |\mathcal{E}_4|^2 = |\bar{\mathcal{E}}_3^{\circ}|^2$ = constant (32)

This means that an equal power is transferred from the pump to the signal and to the probe. A quantum description of this feature can be given (Billmann, A et al 1973): a photon (2ω, o) is splitted into a phonon $(\omega, \vec{q})$ and a phonon $(\omega, -\vec{q})$; in this process, energy and momentum are evidently conserved.

In the lowest order we get:

$$\left\{\begin{aligned}\bar{\mathcal{E}}_3(t) &= \bar{\mathcal{E}}_3^0\\ \bar{\mathcal{E}}_4(t) &= i\bar{\mathcal{E}}_3^{0*} s\omega t\end{aligned}\right. \quad (33)$$

If Δt is the duration of the interaction, (that is of the electric field pulse) it comes that for t ≥ Δt

$$\bar{\mathcal{E}}_3(t) = \bar{\mathcal{E}}_3^0; \quad \mathcal{E}_4(t) = i\mathcal{E}_3^{0*}\, s\omega\Delta t$$

Our aim is achieved. Starting from

$$Q(x, t) = \mathrm{Re}\left[(\bar{\mathcal{E}}_3 \ell^{qx})\, \ell^{i\omega t}\right] \quad (34)$$

we obtain another function proportional to

$$\bar{Q}(x, t) = \mathrm{Re}\left[(\mathcal{E}_3^* \ell^{iqx})\, \ell^{i\omega t}\right] \quad (35)$$

which is the phase conjugate of the previous one. We are thus able to achieve a PC process through WM. The efficiency Γ of the PC process is

$$\Gamma = |\bar{\mathcal{E}}_4/\bar{\mathcal{E}}_3^*| = \frac{1}{2}\frac{\beta}{c}\bar{E}_{12}\, \omega\Delta t \text{ (TWM)} \quad (36)$$

For an experiment achieved on a quartz crystal, the numerical values are

$\bar{E}_{12} = 10^5 Vm^{-1}$; $\beta = 1NV^{-1}m^{-1}$; $\Delta t = 10^{-6}s$; $c = 7.5 \times 10^{10} Nm^{-2}$; $w = 2\pi \times 10^{10} rads^{-1}$

and it follows: Γ = 5% (or -26dB).

A similar result is easily obtained for the electro acoustic DFWM. In this case the efficiency of the PC process is given by

$$\Gamma = |\mathcal{E}_4/\mathcal{E}_3^*| = \frac{1}{2}\frac{\gamma}{c}\bar{E}_1\bar{E}_2\, \omega\Delta t \text{ (DFWM)}, \quad (37)$$

Where γ stands for $\gamma^{(6)}$

3. RESULTS AND DISCUSSION

In the above discussion, we have shown that TWM (and DFWM too) can produce phase conjugation or equivalently "time reversal" on a monochromatic wave.

The first electric pulse generates through Piezoelectric effect on the sample surface (x = 0) an elastic pulse which propagates in the bulk of the sample. Monochromatic waves follow from the Fourier analysis.

Let

$$E(t) = \frac{1}{2}\bar{E}^{o}\left[\ell^{i\omega_o t} + \ell^{-i\omega_o t}\right] f_1(t) \qquad (38)$$

be the electric field of the first pulse. f_1 (t) is the envelope function. ω_o 2π is the central frequency of the pulse. $\bar{E}^{o}$ is a real amplitude.

In all what follows we write f_1 (ω) the fourier transformation of the function f_1 (t). Consequently a monochromatic component of the strain E_3(x, t) generated by this pulse has an amplitude $\mathcal{E}_3{}^{0}(\omega)$ defined by equation

$$\int d\omega\, \bar{\mathcal{E}}_3{}^{0}(\omega)\ell^{i\omega t}\, \ell^{-iqx}$$

and given by

$$\bar{\mathcal{E}}_3{}^{0}(\omega) = \frac{1}{2}\,\omega \bar{E}^{o}\, f_1(\omega) + [\eth(\omega - \omega_o) + \eth(t - \tau)] \qquad (39)$$

ω is a constant which includes the efficiency of the piezoelectric transduction.

The electric field of the second electric pulse is:

$$E_{12}(t) = \frac{1}{2} E^{o}_{12}\, [\ell^{i2\omega ot} + \ell^{-i2\omega ox}]\, [f_2(t) + \eth(t - \tau)] \qquad (40)$$

f2 (t) is the envelope function of this second pulse.

Conventionally the envelope function f1 (t) and f2 (t) both start at t = 0; for the second pulse the convolution with ð(t - τ) translates f2 (t) into f2 (t - τ). This electric field is supposed to be uniformly applied in space. The Fourier component Ê (2ω) of this field is:

$$\hat{E}_{12}(2\omega) = - E_{12}{}^{o}\;\; f_2(2\omega) * [\eth(2\omega - 2\omega_o) + \eth(2\omega + 2\omega_o)]\, \ell^{-i2\omega t} \qquad (41)$$

We are now able to write down explicitly the equation giving the backward wave packet $\mathcal{E}_4(x, t)$

$$\frac{1}{v^2}\;\frac{\eth^2 \mathcal{E}_4}{\eth t^2} \;-\; \frac{\eth^2 \mathcal{E}_4}{\eth x^2} \;=\; \frac{2\beta}{c}\;\frac{\eth^2}{\eth x^2}\left[\mathcal{E}_3(x, t)\, E_{12}(t)\right] \qquad (42)$$

It is found [3] and [2] (we do not reproduce the calculation here) that this surface is proportional to

$$\int d\omega\, f_1 *(\omega) f_2(2\omega)\ell^{i\omega(t - 2\tau)} = f_1(-t) * f_2({}^{t}/_{2}) * \eth(t - 2\tau) \qquad (43)$$

Finally, the electric field Ee (t) obtained by Piezoelectric transduction from $\mathcal{E}_3$(o, t) (this is the echo electric field) is proportional to

$$E_e(t) \approx |\omega|^2 E^{o} E_{12}{}^{o}\, \frac{\beta}{c}\left\{f_1(-t) * f_2({}^{t}/_{2}) * \eth(t - 2\tau)\right\} \cos w_o t \qquad (44)$$

In addition to already mentioned approximations (one-dimensional medium, constant amplitude pump, slowly varying envelope, e.t.c) we must emphasize that we have completely neglected alternation. It can be taken into account phenomenological through a relaxation factor exp (-2π/T2) where T2 is a relaxation time. We find that the sample radiates an electric field of central frequency wo/2π and an envelope

$$f_1\,(-t) * f_2\,({}^{t}/_{2}) * ð(t - 2\tau) \tag{45}$$

Usually, the functions f1 (t) and f2 (t) are constant in intervals $0 \le t \le \Delta t1$ and $0 \le t \le \Delta t2$ respectively, and equal to 0 elsewhere. it is a well know property of convolution that fe (-t) = f1 (-t) ∗ f2 (t/2) is different of 0, only in an interval of width Δte.= Δt1 + 2Δt2.

It is easy to show that the echo signal begins at time 2τ – Δt1 (and not 2τ). This is due to the minus sign in f1 (-t) which, in turn comes from the conjugate f1 ∗(ω). This is a signature of PC which has been experimentally checked. It has also been checked accurately (see figure 1) that the equation

Δte.= Δt1 + 2Δt2 is very well satisfied.

Sb Sl crystal v= 9.3 GHz T= 4.2 K

CURVES $\Delta t_e .= \Delta t_1 + 2\Delta t_2$.

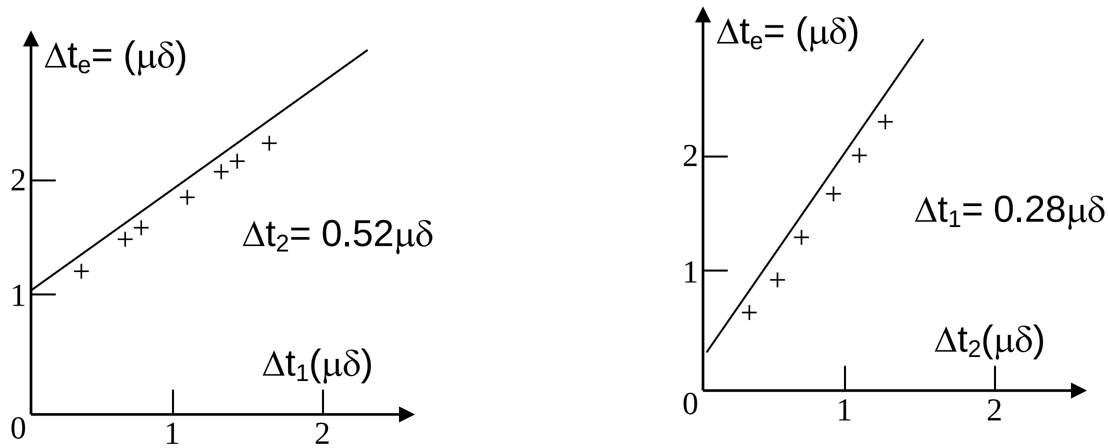

Figure 2 – *Experimental width (Δt_e) of a two-pulse phonon echo as a function of the widths of pulse 1(Δt_1) and pulse 2 (Δt_2). The full line shows the expected variation according to the equation $\Delta t_e .= \Delta t_1 + 2\Delta t_2$*

The echo amplitude must be proportional to the amplitude of the first pulse and proportional to the amplitude of the second (at $2\omega_o$ for TWM) or to the square of this amplitude (at ω_o for DFWM). This is experimentally observed, at least if the pulse powers are not too strong.

4. CONCLUSION

We may conclude that at the end of a simple calculation we have obtained a theoretical expression which describes quite well the experiment results. We have clearly shown in this article that the phase conjugation concept is useful as well in optics (for DFWM) as in acoustics (for backward phonon echoes.

5. REFERENCE:

[1]. Billmann, A, Frenois C, Joffrin, J, Levelut, A and Ziolklewicz, S., (1973) – J. Physique 34, 453.
[2]. Fossheim, K and Holt R. M., (1982); In Physical Acoustics, Vol. 16 Academic Press
[3]. Joffrin, J and Levelut, A; (1976) – New Directions in physical Acoustics LXIII Corsoed by Soc. Italiana difistica, Bologun, Italy)
[4]. Hahn, E. (1950), - Phys. Rev. 80, 580.
[5]. Kopvilleum, U., Smolyakov, B. P and Sharipov, R. Z, (1972) – Sov. Phys. JETP Lett, 13, 398.
[6]. Zeldovich, B. Y., Popovichev, V. L; Ragulskii V. V; and Falzullov, F, S; (1972) – Sov. Phys. JETP 15, 109. Theory Phy. 1, 27
[7]. Nosach O. Y, Popvichev V. I, Ragulskii V. V. and Faizullov F.S (1972) – Sov. Phys. JETP 16, 435
[8]. Hellwarth, R. W (1977) – J. opt. soc. Amer. 67, 1.
[9]. Bloom, D. M and Bjorklund G. C (1977) – Appl. phys. lett. 31, 592
[10]. Popov S. N nad Krainik, N. N (1971) – sov. phys. sol. st. 12, 2440
[11]. Hkuska K and Kazda V, (1968) – Czech, J. Phys. B 18, 500