THE SEMI NORMED SPACE DEFINED BY ORLICZ SPACE OF ENTIRE SEQUENCE

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ABSTRACT
In this paper we introduce the sequence spaces $\Gamma_M(p,\sigma,q,s)$, $\wedge_M(p,\sigma,q,s)$ using an modulus function *M* and defined over a semi normed space (X,q), semi normed by *q and* study some properties of these sequence spaces. We also obtain some inclusion relations.

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1. INTRODUCTION

A complex sequence, whose k^{th} term is x_k is denoted by $\{x_k\}$ or simply *x*. A sequence $x=\{x_k\}$ is said to be analytic, if $\sup_{(k)}|x_k|^{1/k}<\infty$. The vector space of all analytic sequences will be denoted by $\wedge$. A sequence *x* is entire sequence if $\lim_{k\to\infty}|x_k|^{1/k}=0$. The vector space of all entire sequences will be denoted by Γ.

Let σ be a one-one mapping of the set of positive integers into itself such that $\sigma^m(n)=\sigma(\sigma^{m-1}(n)), m=1,2,...$ A continuous linear functional Φ on $\wedge$ is said to be an invariant mean or a σ-mean if and only if

(i) $\Phi(x)\geq 0$ when the sequence $x=(x_n)$ has $x_n\geq 0$ for all *n*.

(ii) $\Phi(e)=1$ where $e=(1,1,1,....)$ and

(iii) $\Phi(\{x_{\sigma(n)}\})=\Phi(\{x_n\})$ for all $x\in\wedge$.

For certain kinds of mappings σ, every invariant mean Φ extends the limit functional on the space C of all real convergent sequences in the sense that $\Phi(x)=\lim x$ for all $x\in C$ consequently $C\subset V_\sigma$, where V_σ the set of analytic sequences all of whose σ-means are equal.

If $x=(x_n)$ set $Tx=(Tx_n)^{1/n}=(x_{\sigma(n)})$. It can be shown that

$$V_\sigma=\left\{x=(x_n):\lim_{m\to\infty}t_{mn}(x_n)^{1/n}=Le \text{ uniformly in } n, L=\sigma-\lim_{n\to\infty}(x_n)^{1/n}\right\}$$

where

$$(1.1)\qquad t_{mn}(x)=\frac{(x_n+Tx_n+...+T^m x_n)^{1/n}}{m+1}.$$

Orlicz [7] used the idea of Orlicz function to construct the space (L^M). Lindenstrauss and *Tzafriri* [4] investigated Orlicz sequence spaces in more detail, and they proved that every Orlicz sequence space ℓ_M contains a subspace isomorphic to $\ell_p(1\leq p<\infty)$. Subsequently different classes of sequence spaces are defined by *Parashar* and *Choudhary* [8], *Mursaleen* et al [6], *Bektas* and *Altin* [1], *Tripathy* et al. [2]. *Rao* and *Subramanian* [9] and many others. The *Orlicz* sequence spaces are the special cases of *Orlicz* spaces studied in *Ref* [ ].

Recall ([7], [3]) that an Orlicz function is a function $M:[0,\infty)\to[0,\infty)$ which is continuous, non-decreasing and convex with $M(0)=0, M(x)>0$ for $x>0$ and $M(x)\to\infty$ as $x\to\infty$. If convexity of Orlicz function M is replaced by $M(x+y)\le M(x)+M(y)$, then this function is called modulus function, defined and discussed by *Ruckle* [10] and *Maddox* [5].

Let $(\Omega, \sum, \mu)$ be a finite measure space. We denote by $E(\mu)$ the space of all (equivalence classes of) $\sum$-measurable functions *x* from Ω into $[0,\infty)$. For an Orlicz function *M*, we define on $E(\mu)$ a convex functional I_M by

$$I_M(x)=\int_\Omega M(x(t))\,d\mu,$$

and an Orlicz space $L^M(\mu)$ by

$$L^M(\mu)=\{x\in E(\mu): I_M(\lambda x)<+\infty \ \textit{for some}\ \lambda>0\}$$

(For detail see [7], [3]).
Lindenstrauss and *Tzafriri* [4] used the idea of Orlicz function to construct Orlicz sequence space

$$\ell_M=\left\{x\in w:\sum_{k=1}^{\infty}M\left(\frac{|x_k|}{\rho}\right)<\infty,\ \textit{for some}\ \rho>0\right\}$$

where w = {all complex sequences}.
The space ℓ_M with the norm

$$\|x\|=\inf\left\{\rho>0:\sum_{k=1}^{\infty}M\left(\frac{|x_k|}{\rho}\right)\le 1\right\},$$

becomes a Banach space which is called an Orlicz sequence space. For $M(t)=t^p, 1\le p<\infty$, the spaces ℓ_M coincide with the classical sequence space ℓ_p.

Definition 1. Let M be a modulus function. The space consisting of all those sequences *x* in w such that $M\left(\frac{|x_k|^{1/k}}{\rho}\right)\to 0$ *as* $k\to\infty$ for some arbitrarily fixed $\rho>0$ is called the Orlicz space of entire sequence. It isdenoted by Γ_M. Clearly $\left\{M\left(\frac{|x_k|^{1/k}}{\rho}\right)\right\}$ is a null sequence.The space Γ_M is a metric space with the metric

$$d(x,y)=\sup_{(k)} M\left(\frac{|x_k-y_k|^{1/k}}{\rho}\right)$$

for all $x=\{x_k\}$ and $y=\{y_k\}$ in Γ_M.

Lemma-1. If *M* is a convex function and $M(0)=0$, then $M(\lambda x)\le\lambda M(x)$ for all λ with $0<\lambda<1$.

Definition-2. For the modulus function M, we define the Orlicz Space of bounded sequence, usually denoted by λ_M as the space consisting of all those sequences *x* in w such that

$$\left(\sup_{(k)}\left(M\left(\frac{|x_k|^{1/k}}{\rho}\right)\right)\right)<\infty,$$

for some arbitrarily fixed $\rho>0$ is denoted by $\wedge_M$, *M* being a modulus function. In other words

$$\left(\sup_{(k)}\left(M\left(\frac{|x_k|^{1/k}}{\rho}\right)\right)\right)$$

is bounded sequence..

Definition-3. Let p ,q be semi norms on a vector space X. Then p is said to be stronger than q if , for any sequence (x_n) in X with $p(x_n)\to 0$, implies $q(x_n)\to 0$. If each is stronger than the other there are said to be equivalent.

Lemma-2. Let p and q be semi norms on a linear space X. Then p is stronger than q if and only if there exists a constant M such that $q(x)\le Mp(x)$ for all $x\in X$.

Definition-4. A sequence space E is said to be solid or normal if for every $(x_k)\in E$ and for all sequence of scalars (α_k) with $|\alpha_k|\le 1$, $(\alpha_k x_k)\in E$.

Definition-5. A sequence space E is said to be monotone if it contains the canonical pre-images of all its step spaces.

Remark-1. From the above two definitions it is clear that a sequence E is solid implies that it is monotone.

Definition-6. A sequence E is said to be convergence free if $(y_k)\in E$ whenever $(x_k)\in E$ and $x_k=0$ implies that $y_k=0$.

Lemma-3. Let $p=(p_k)$be sequence of positive real numbers with $0<p_k<\sup p_k=G$ and let $D=Max(1,2^{G-1})$. Then for $a_k,b_k\in C$, the set of complex numbers for all $k\in N$ we have

$$|a_k+b_k|^{1/k}\le D\left\{|a_k|^{1/k}+|b_k|^{1/k}\right\} \tag{1.2}$$

Let (X,q) be a semi normed space over the field C of complex numbers with the semi norm q. We denote $\wedge(X)$ as the spaces of all analytic sequences defined over X.

We define the following sequence spaces:

$$\wedge_M(p,\sigma,q,s)=\left\{x\in\wedge(X):\sup_{(n,k)}k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]^{p_k}<\infty,\ uniformly\ in\ n,\rho\ge 0,s\ge 0\right\}$$

$$\Gamma_M(p,\sigma,q,s)=\left\{x\in\Gamma_M(X):\ k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]\to 0,\ as\ k\to\infty\ ,\ uniformly\ in\ n,\rho\ge 0,s\ge 0\right\}$$

2. MAIN RESULTS

In this paper we study some of the topological property of orlicz space of entire sequences.

Theorem-1. If M is a modulus function, then $\Gamma_M(p,\sigma,q,s)$ are linear spaces over the set of complex numbers.

Proof. Let $x,y\in\Gamma_M(p,\sigma,q,s)$ and $\alpha,\beta\in C$. In order to prove the result we need to find some ρ_3 such that

$$k^{-S}\left[M\left(q\left(\frac{\left|\alpha x_{\sigma^k(n)}+\beta y_{\sigma^k(n)}\right|^{1/k}}{\rho_3}\right)\right)\right]^{p_k} \to 0 \ as \ k\to\infty, \qquad \text{uniformly} \qquad n,$$

$\rho_3 \geq 0, s \geq 0$. Since $x, y \in \Gamma_M(p,\sigma,q,s)$, there exists a positive numbers ρ_1 and ρ_2 such that

$$(2.1) \qquad k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho_1}\right)\right)\right]^{p_k} \to 0 \ as \ k\to\infty,$$

uniformly in $n, s \geq 0$ and

$$(2.2) \qquad k^{-S}\left[M\left(q\left(\frac{\left|y_{\sigma^k(n)}\right|^{1/k}}{\rho_2}\right)\right)\right]^{p_k} \to 0 \ as \ k\to\infty,$$

uniformly in $n, s \geq 0$. Let ρ_3 be such that

$$\frac{1}{\rho_3} = \min\left\{\frac{1}{|\alpha|\rho_1}, \frac{1}{|\beta|\rho_2}\right\}.$$

Since *M* is a non-decreasing modulus function, we have

$$k^{-S}\left[M\left(q\left(\frac{\left|\alpha x_{\sigma^k(n)}+\beta y_{\sigma^k(n)}\right|^{1/k}}{\rho_3}\right)\right)\right]^{p_k} \leq k^{-S}\left[M\left(q\left(\frac{|\alpha|\left|x_{\alpha^k(n)}\right|^{1/k}}{\rho_3}+\frac{|\beta|\left|y_{\alpha^k(n)}\right|^{1/k}}{\rho_3}\right)\right)\right]^{p_k}$$

$$\leq k^{-S}\left[M\left(q\left(\frac{\left|x_{\alpha^k(n)}\right|^{1/k}}{\rho_1}+\frac{\left|y_{\alpha^k(n)}\right|^{1/k}}{\rho_2}\right)\right)\right]^{p_k}$$

$$\leq k^{-S}\left[M\left(q\left(\frac{\left|x_{\alpha^k(n)}\right|^{1/k}}{\rho_1}\right)\right)+M\left(q\left(\frac{\left|y_{\alpha^k(n)}\right|^{1/k}}{\rho_2}\right)^{p_k}\right)\right]$$

$$\leq Dk^{-S}\left[M\left(q\left(\frac{\left|x_{\alpha^k(n)}\right|^{1/k}}{\rho_1}\right)\right)\right]^{p_k}+Dk^{-S}\left[M\left(q\left(\frac{\left|y_{\sigma^k(n)}\right|^{1/k}}{\rho_2}\right)\right)\right]^{p_k} \to 0 \ as \ k\to\infty$$

uniformly in $n, s \geq 0$. Thus, for $x, y \in \Gamma_M(p,\sigma,q,s)$ *and* $\alpha, \beta \in C$

$$(\alpha x+\beta y)\in\Gamma_M(p,\sigma,q,s).$$

Hence $\Gamma_M(p,\sigma,q,s)$ is a linear.

This completes the proof.

Theorem-2. $\Gamma_M(p,\sigma,q,s)$ are paranormed spaces with

$$g^*(x) = \inf\left\{\rho^{p_m/H} : \sup_{k\geq 1} k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right] \leq 1, \textit{ uniformly in } n,\ \rho > 0\right\}$$

where $H = \max\left(\sup_{(k)} p_k\right)$.

Proof. Clearly $g(x) = g(-x)$ and $g(\theta) = 0$, where θ is the zero sequence of *X*. Let $(x_k), (y_k) \in \Gamma_M(p,\sigma,q,s)$ and ρ_1 and $\rho_2 > 0$ be such that

$$\sup_{k\geq 1} k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho_1}\right)\right)\right] \leq 1,$$

uniformly in *n* and

$$\sup_{k\geq 1} k^{-S}\left[M\left(q\left(\frac{\left|y_{\sigma^k(n)}\right|^{1/k}}{\rho_1}\right)\right)\right] \leq 1,$$

uniformly in *n*..Then we have

$$\sup_{k\geq 1} k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)} + y_{\sigma^k(n)}\right|^{1/k}}{\rho_1+\rho_2}\right)\right)\right] \leq \left(\frac{\rho_1}{\rho_1+\rho_2}\right)\sup_{k\geq 1} k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho_1}\right)\right)\right] +$$

$$\left(\frac{\rho_2}{\rho_1+\rho_2}\right)\sup_{k\geq 1} k^{-S}\left[M\left(q\left(\frac{\left|y_{\sigma^k(n)}\right|^{1/k}}{\rho_2}\right)\right)\right]$$

≤ 1 uniformly in *n*.

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Hence

$$g(x+y) \leq \inf\left\{(\rho_1+\rho_2)^{p_m/H} : m \in N\right\}$$

$$\leq \inf\left\{(\rho_1)^{p_m/H} : \sup_{k\geq 1} k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho_1}\right)\right)\right] \leq 1, \rho_1 > 0, \textit{ uniformly in } n, m \in N\right\} +$$

$$\inf\left\{(\rho_2)^{p_m/H} : \sup_{k\geq 1} k^{-S}\left[M\left(q\left(\frac{\left|y_{\sigma^k(n)}\right|^{1/k}}{\rho_2}\right)\right)\right] \leq 1, \rho_{21} > 0, \textit{ uniformly in } n, m \in N\right\}$$

Hence $g(x+y) = g(x) + g(y)$.Thus g satisfies the triangle inequality.

Next, for $0 < \lambda < 1$

$$g(\lambda x) = \inf \left\{ (\rho)^{p_m/H} : \sup_{k\geq 1} k^{-S}\left[M\left(q\left(\frac{\left|\lambda x_{\sigma^k(n)}\right|^{1/k}}{\rho} \right)\right)\right] \leq 1, \rho > 0, \, uniformly \ in \ n, m \in N \right\}$$

$$= \inf \left\{ (|\lambda|\ r)^{p_m/H} : \sup_{k\geq 1} k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{r} \right)\right)\right] \leq 1, r > 0, \, uniformly \ in \ n, m \in N \right\}$$

where $r = \dfrac{\rho}{|\lambda|}$.

Hence $\Gamma_M(p,\sigma,q,s)$ is a paranormed space.

This completes the proof.

Theorem-3. Let M_1 and M_2 be two modulus function. Then

$$\Gamma_{M_1}(p,\sigma,q,s) \cap \Gamma_{M_2}(p,\sigma,q,s) \subseteq \Gamma_{M_1+M_2}(p,\sigma,q,s).$$

Proof. Let $x \in \Gamma_{M_1}(p,\sigma,q,s) \cap \Gamma_{M_2}(p,\sigma,q,s)$.

Then there exists ρ_1 and ρ_2 such that

$$k^{-S}\left[\Gamma_{M_1}\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho_1} \right)\right)\right]^{p_k} \to 0 \ as \ k \to \infty, \text{ uniformly in } n. \tag{2.3}$$

and

$$k^{-S}\left[\Gamma_{M_2}\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho_2} \right)\right)\right]^{p_k} \to 0 \ as \ k \to \infty, \text{ uniformly in } n. \tag{2.4}$$

Let ρ be such that $\dfrac{1}{\rho} = \min\left(\dfrac{1}{\rho_1}, \dfrac{1}{\rho_2} \right)$. Then we have

$$k^{-S}\left[\Gamma_{M_1+M_2}\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho} \right)\right)\right]^{p_k} \leq k^{-S}\ D\left[\Gamma_{M_1}\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho_1} \right)\right)\right]^{p_k} + k^{-S}\, D\left[\Gamma_{M_2}\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho_2} \right)\right)\right]^{p_k}$$

$$\to 0 \ as \ k \to \infty,$$

uniformly in *n*, by (2.3) and (2.4)

Therefore $x \in \Gamma_{M_1+M_2}(p,\sigma,q,s)$.

This completes the proof.

Theorem-4. Let *M* be a modulus function then $\Gamma_M(p,\sigma,q,s) \subset \wedge_M(p,\sigma,q,s)$.

Proof. Let $x \in \Gamma_M(p,\sigma,q,s)$. Then we have

$$k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]^{p_k} \to 0 \text{ as } k \to \infty, \text{ uniformly in } n, \rho \geq 0.$$

Let (ℓ_k) be a sequence of scalars such that $|\ell_k| \leq 1$ for all $k \in N$.

$$k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]^{p_k} = k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)} - \ell + \ell\right|^{1/k}}{\rho}\right)\right)\right]^{p_k}$$

$$\leq Dk^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)} - \ell\right|^{1/k}}{\rho}\right)\right)\right]^{p_k} + Dk^{-S}\left[M\left(q\left(\frac{\left|\ell y_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]^{p_k}$$

$$\leq Dk^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)} - \ell\right|^{1/k}}{\rho}\right)\right)\right]^{p_k} + |\ell| Dk^{-S}\left[\sup_{(n,k)}\left(M\left(q\left(\frac{\left|\sum_{k=1}^{\infty} y_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right)\right]^{p_k}$$

$\to 0$, *as* $k \to \infty$, uniformly in n.

Thus we get $x \in \wedge_M(p,\sigma,q,s)$.

This completes the proof.

Theorem-5. For any two sequences $p = (p_k)$ and $t = (t_k)$ of positive real numbers and for any two semi norms q_1 and q_2 on *X* we have

$$\Gamma_M(p,\sigma,q_1,s) \cap \Gamma_M(t,\sigma,q_2,s) \neq \phi.$$

Proof. The proof follows as the zero element θ belongs to each of the classes of sequences involved in the intersection.

Remark- 2. Let *M* be a modulus function and let q_1 and q_2 be two semi norms on *X*, we have

(i) $\Gamma_M(p,\sigma,q_1,s) \cap \Gamma_M(p,\sigma,q_2,s) \subseteq \Gamma_M(p,\sigma,q_1+q_2,s)$

(ii) If q_1 is stronger than q_2 then $\Gamma_M(p,\sigma,q_1,s) \subseteq \Gamma_M(p,\sigma,q_2,s)$

(iii) If q_1 is equivalent to q_2 than $\Gamma_M(p,\sigma,q_1,s) = \Gamma_M(p,\sigma,q_2,s)$.

Theorem-6.

(i) Let $0 \leq p_k \leq r_k$ and $\left\{\frac{r_k}{p_k}\right\}$ be bounded. Then $\Gamma_M(r,\sigma,q,s) \subset \Gamma_M(p,\sigma,q,s)$

(ii) $s_1 \leq s_2$ implies $\Gamma_M(r,\sigma,q,s_1) \subset \Gamma_M(p,\sigma,q,s_2)$.

Proof. Let $x \in \Gamma_M(r,\sigma,q,s)$. Then

$$k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]^{r_k} \to 0 \; as \; k \to \infty .$$

Let

$$t_{kn} \;=\; k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]^{r_k} \; and \; \lambda_{kn} = \frac{r_{kn}}{p_{kn}} .$$

Since $p_k \le r_k$, we have $0 \le \lambda_k \le 1$. Take $0 < \lambda < \lambda_k$. Let us define

$$u_{kn} = t_{kn}(t_{kn} \ge 1)$$
$$= 0(t_{kn} < 1) \, ,$$

and

$$v_{kn} = 0(t_{kn} \ge 1)$$
$$= t_{kn}(t_{kn} < 1)$$
$$t_{kn} = u_{kn} + v_{kn}, \; t_{kn}^{\lambda_k} = u_{kn}^{\lambda_k} + v_{kn}^{\lambda_k}$$

Now it follows that

$$u_k^{\lambda_k} \le u_{kn}^{\lambda_k} + t_{kn} \; and \; v_{kn}^{\lambda_k} \le v_{kn}^{\lambda}$$

That is. $t_{kn}^{\lambda_k} = u_{kn}^{\lambda_k} + v_{kn}^{\lambda_k}$

$$t_{kn}^{\lambda_k} \le t_{kn} + v_{kn}^{\lambda}$$

$$\Rightarrow k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]^{\lambda_k} \le k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]^{r_k}$$

$$k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]^{p_k/r_k} \le k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]^{r_k}$$

$$k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]^{p_k} \le k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]^{r_k}$$

But

$$k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]^{r_k} \to 0 \; as \; k \to \infty$$

Hence

$$k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]^{p_k} \to 0 \ as \ k\to\infty .$$

Hence $x \in \Gamma_M(p,\sigma,q,s)$. Thus $x\in\Gamma_M(r,\sigma,q,s)$ *implies* $x\in\Gamma_M(p,\sigma,q,s)$. Hence $.\Gamma_M(r,\sigma,q,s)\subset\Gamma_M(p,\sigma,q,s)$

This completes the proof.

Proof.(ii). Since

$$k^{-S_2}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]^{p_k} \le k^{-S_2}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]^{p_k}$$, for all *k* and *n*..

Thus $x \in \Gamma_M(p,\sigma,q,s_1)$ *implies* $x \in \Gamma_M(p,\sigma,q,s_2)$

Hence $\Gamma_M(p,\sigma,q,s_1)\subset\Gamma_M(p,\sigma,q,s_2)$.

This completes the proof.

Theorem-7. The space $\Gamma_M(p,\sigma,q,s)$ are solid and as such are monotone.

Proof. Let $(x_k)\in\Gamma_M(p,\sigma, q,s)$ and (α_k) be a sequence of scalars such that $|\alpha_k|\le 1$ for all $k\in N$. Then

$$k^{-S}\left[M\left(q\left(\frac{\left|\alpha_k \ x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]^{p_k} \le k^{-S}\left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]^{p_k} \text{ for all } k\in N$$

$$\left[M\left(q\left(\frac{\left|\alpha_k \ x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]^{p_k} \le \left[M\left(q\left(\frac{\left|x_{\sigma^k(n)}\right|^{1/k}}{\rho}\right)\right)\right]^{p_k} \text{ for all } k\in N$$

This completes the proof.

Theorem-8. The space $\Gamma_M(p,\sigma,q,s)$ is not convergence free.

Proof: Let $M(x)=x$ *for* $x\in[0,\infty)$; $s=0$; $p_k=1$ for *k* even and $p_k=2$ for *k* odd.

Let $X=X$, $q(x)=|x|$ *and* $\sigma(n)=n+1$ for all $n\in N$. Then we have

$$\sigma^2(n)=\sigma(\sigma(n))=\sigma(n+1)=(n+1)+1=n+2$$

and

$$\sigma^3(n)=\sigma(\sigma^2(n))=\sigma(n+2)=(n+2)+1=n+3 .$$

Therefore $\sigma^k(n)=(n+k)$ for all $n,k\in N$. Consider the sequences (x_k) and (y_k) defined as

$$x_k=\left(\frac{1}{k}\right)^k \ and \ (y_k)^k = k^k \text{ for all } k\in N$$

Then $|x_k|^{1/k} = \frac{1}{k}$ *and* $|y_k|^{1/k} = k$ for all $k \in N$ .Hence

$$\frac{1}{\rho}\left|\left(\frac{1}{(n+k)}\right)^{n+k}\right|^{p_k} \to 0 \ as \ k \to \infty .$$

Therefore $(x_k) \in \Gamma_M(p,\sigma)$.But $\frac{1}{\rho}\left|(n+k)^{n+k}\right|^{p_k} \neq 0 \ as \ k \to \infty$. Hence $(y_k) \notin \Gamma_M(p,\sigma)$.Hence the space $\Gamma_M(p,\sigma,q,s)$ is not convergence free.

This completes the proof.

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