A DETERMINISTIC INVENTORY MODEL FOR DETERIORATING ITEMS WITH ON-HAND INVENTORY DEPENDENT, VARIABLE TYPE DEMAND RATE

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ABSTRACT
A model for deterministic perishable items that follows variable type demand rate with infinite time horizon, constant deterioration and without shortages is considered. An optimal production policy is derived with maximization of profit. The result is illustrated with numerical example**.**

Keyword: *Variable demand, on-hand inventory, shortages, replenishment cost.*

AMS Classification No: $90B05$

1. INTRODUCTION
In recent trends businessmen have shown an increasing awareness of the need for precision in the field of inventory control of deteriorating items. In general, deterioration is defined as damage, spoilage, decay, obsolescence, evaporation, pilferage etc., that result in decrease of usefulness of the original one. It is reasonable to note that a product may be understood to have a lifetime which ends when utility reaches zero. The decrease or loss of utility due to decay is usually a function of the on-hand inventory. For items such as steel, hardware, glassware and toys, the rate of deterioration is so low that there is little need for considering deterioration in the determination of the economic lot size. But some items such as blood, fish, strawberry, alcohol, gasoline, radioactive chemical, medicine and food grains (i.e., paddy, wheat, potato, onion etc.) deteriorate remarkably overtime.
Whitin [10] considered an inventory model for fashion goods deteriorating at the end of a prescribed storage period. Ghare and Scharder [4] developed an EOQ model with an exponential decay and a deterministic demand. Thereafter, Covert and Philip[2] and Philip[5] extended EOQ (Economic Order Quantity) models for deterioration which follows Weibull distribution. Wee [9] developed EOQ models to allow deterioration and an exponential demand pattern. In last two decades the economic situation of most countries have changed to an extent due to sharp decline in the purchasing power of money, that it has not been possible to ignore the effects of time value of money. Data and Pal[3], Bose et al.[2] have developed the EOQ model incorporating the effects of time value of money, a linear time dependent demand rate. Further Sana [8] considered the money value and inflation in a new level.
In the present paper demand rate is considered as constant to a fixed time, after then it varies linearly with time. Sahu et al[6] and Samal et al [7] have established models where the demand rate is dependent on the on-hand inventory. In what follows in the present paper we consider a model by taking variable type of demand which behaves differently in the given time horizon such as the demand rate is constant for a certain fixed time and after that period it varies linearly with time.

2. FUNDAMENTAL ASSUMPTIONS AND NOTATIONS
Following assumptions are made for the proposed model:

i. Demand rate is variable with respect to time.
ii. Single inventory will be used.
iii. Lead time is zero.
iv. Shortages are not allowed.
v. Replenishment rate is infinite but size is finite.
vi. Time horizon is finite.
vii. There is no repair of deteriorated items will occur during the cycle.

Following notations are made for the given model:

$I(t)$ = On hand inventory at time t .

$R(t)$ = Demand.

$\theta =$ The constant deterioration rate where $0 \le \theta < 1$.
$I_0 =$ Inventory at time $t = 0$.
$s =$ Selling price per unit.
$c =$ Unit cost of the item.
$Q =$ On-hand inventory.
$h =$ Holding cost per unit item per unit time.
$r =$ Replenishment cost per replenishment which is a constant.
$T =$ Duration of a cycle.
$R_0 =$ Initial demand rate.
$a =$ Rate of change of demand with respect to t .

3. FORMULATION

In this model we consider the rate of demand $R(t)$ to be a constant up to a certain time $t = t_1$ and after which it varies linearly with time. If $I(t)$ be the on hand inventory at time $t \ge 0$, then at time $t + \Delta t$, the on-hand inventory will be

$I(t + \Delta t) = I(t) - R(t)\Delta t - \theta \cdot I(t) \cdot \Delta t$.

Dividing by Δt and then taking as $\Delta t \to 0$ we get

(3.1). $$\frac{dI}{dt} = -R(t) - \theta \cdot I(t) \ .$$

We define $R(t)$ as

(3.2). $$R(t) = R_0 + a.(t - t_1).H(t - t_1) \ ,$$

where $H(t - t_1) = 1$ for $t \ge t_1$ and 0 for $t \le t_1$.

Now from equation (3.2) we have

(3.3). $$\frac{dI}{dt} = -R_0 - \theta \cdot I(t) \ , \qquad \text{for } 0 \le t \le t_1 \ ,$$

(3.4). $$\frac{dI}{dt} = -R_0 - a(t - t_1) - \theta \cdot I(t) , \qquad \text{for } t_1 \le t \le T \ .$$

Solving the equation (3.3) using the initial condition $I = I_0$ at $t = 0$ we have

(3.5). $$I = -\frac{R_0}{\theta} + e^{-\theta t}\left(I_0 + \frac{R_0}{\theta} \right) , \qquad \text{for } 0 \le t \le t_1 \ .$$

Again using $I = I_1$ at $t = t_1$

(3.6). $$I_1 = -\frac{R_0}{\theta} + e^{-\theta t_1}\left(I_0 + \frac{R_0}{\theta} \right)$$

and consequently

(3.7) $$t_1 = -\frac{1}{\theta} \cdot \ln\left(\frac{R_0 + I_1\theta}{R_0 + I_0\theta} \right) \ .$$

Solving the equation (3.4) using the condition $I = 0$ at $t = T$ we have

(3.8). $$I = \frac{a}{\theta}(T - t) \ . \ \text{for } t_1 \le t \le T$$

But using $I = I_1$ at $t = t_1$

(3.9). $$I_1 = \frac{a}{\theta}\left(T - t_1\right) \ .$$

Using equation (3.7) in (3.9) we have

(3.10). $$T = \frac{\theta \cdot I_1}{a} - \frac{1}{\theta}\ln\left(\frac{R_0 + I_1\theta}{R_0 + I_0\theta}\right).$$

Now average on-hand inventory is given by

$$Q = \int_0^T I dt = \int_0^{t_1} I dt + \int_{t_1}^T I dt .$$

Now substituting the corresponding values of I we have

(3.11). $$Q = \frac{1}{\theta}\left(I_0 + \frac{R_0}{\theta}\right)\cdot\left(1 - e^{-\theta t_1}\right) - \frac{R_0 t_1}{\theta} + \frac{a}{2\theta}\left(T - t_1\right)^2 .$$

Now eliminating the values of t_1 and T we have

(3.12). $$Q = \frac{I_0 - I_1}{\theta} + \frac{\theta \cdot I_1^2}{2a} + \frac{R_0}{\theta^2}\ln\left(\frac{R_0 + I_1\theta}{R_0 + I_0\theta}\right).$$

Now the profit function $\phi(I_0)$ is given by

(3.13). $$\phi(I_0) = \frac{1}{T}\left((s - c - r)I_0\right) - \frac{h}{T}\cdot\left(\frac{I_0 - I_1}{\theta} + \frac{\theta \cdot I_1^2}{2a} + \frac{R_0}{\theta^2}\ln\left(\frac{R_0 + I_1\theta}{R_0 + I_0\theta}\right)\right).$$

The necessary condition for $\phi(I_0)$ to attain maximum is $\frac{d[\phi(I_0)]}{dP} = 0$, which gives

(3.14). $$I_0 = \frac{R_0(s - c - r)}{h - \theta(s - c - r)}.$$

Using Equation (3.14)

(3.15). $$\frac{d^2\phi(I_0)}{dI_0^{\,2}} = \frac{-h \cdot R_0}{T(R_0 + I_0\theta)^2} < 0 .$$

It will give a global maximum for profit function $\phi(I_0)$.

4. NUMERICAL EXAMPLE

We have considered the values of parameters in appropriate units as follows:

i) $a = 50, t_1 = 0.20, I_0 = 350, R_0 = 300, s = 30, c = 10, h = 0.7, r = 12, \theta = 0.2$

Using the decision rule we get T= 0.65185 years and $\phi(I_0) = 2481.465$ per year.

ii) $a = 45, t_1 = 0.20, I_0 = 350, R_0 = 300, s = 30, c = 10, h = 0.6, r = 12, \theta = 0.2$

Using the decision rule we get T= 0.77517 years and $\phi(I_0) = 2086.6945$ per year.

5. CONCLUSION

From the studied physical model, it is common belief that the demand rate of various inventories is remained constant up to a time t_1 after which the demand rate varies with time. During the period $[0,t_1]$, the demand rate is maintained at a constant level but after that period the amount of inventory decreases continuously with time but the effect of deterioration is maintained through out the cycle. Hence the inventory level decreases due to the combined effect of demand as well as deterioration. The above model can also be studied under shortages, backlogging and backordering. .

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