DIRECT POSITION KINEMATICS OF A THREE REVOLUTE-PRISMATIC-SPHERICAL PARALLEL MANIPULATOR

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ABSTRACT

In this work, the direct position kinematics of a 3 degree-of-freedom parallel manipulator with three identical limbs, type revolute-prismatic-spherical (RPS), is analyzed. In contrast to the previous studies on this class of manipulators, the revolute joints of the proposed manipulator are actuated rather than the prismatic joints. Direct position kinematics of the manipulator leads to a system of three nonlinear equations in three unknowns that are reduced to a univariate polynomial of degree eight and two quadratic equations in sequence using Sylvester dialytic elimination method. In addition, to show the efficiency of the presented method a numerical example is provided.

Keywords: *Parallel Manipulators; Direct Kinematics; Sylvester Dialytic Elimination Method; Analytical Solution*.

1. INTRODUCTION

Parallel manipulators are the mechanisms composed of a moving platform connected to a fixed base by at least two kinematic chains (legs). The most studied type of parallel manipulator is without doubt the so-called general Gough–Stewart platform, a fully parallel manipulator introduced by Gough as a universal tire-testing machine almost five decades ago [1,2] and proposed as a flight simulator by Stewart in 1965 [3].

However, in many industrial applications, such as some assembly operations, parallel manipulators with fewer degrees of freedom than six can be successfully used instead of the general Gough–Stewart platform. The Delta robot, invented by Clavel, is a typical example of such applications. With this in mind, a significant amount of research has been devoted to the study of parallel manipulators with fewer than six degrees of freedom; see for instance [4–9]. In fact, this class of parallel manipulators offers significant advantages such as the simpler mechanical assembly, a larger workspace than a general Gough–Stewart platform and a simpler direct position kinematics.

Direct position kinematics of parallel manipulators lead to systems of nonlinear equations which are very difficult to solve. Solution approaches for such equations can be divided into two classes: numerical (iterative) methods and analytic methods.

There are different numerical methods that can deal with simultaneous non-linear equations such as Newton or conjugate gradient method commonly used as iterative methods [10], Homotopy continuation method as an improved iterative method [11], and other new numerical methods [12,13].

On the other hand, there are some analytical methods that have effectively used to solve the systems of nonlinear equations. Two well known analytical techniques which are used for solving polynomial systems are Bezout's elimination method and Sylvester's dialytic elimination method. In these methods a set of polynomials of multiple variables are reduced into a polynomial of only one variable. Many scholars have used these methods for solving direct position kinematics of parallel manipulators, e.g. [14-16].

The 3-RPS parallel manipulators constitute a class of parallel manipulators with fewer degrees of freedom than six. A 3-RPS parallel manipulator, see Fig. 1, is a mechanism where the moving platform is connected to the fixed platform by means of three limbs. Each limb is composed by a lower body and an upper body connected to each other by means of a prismatic joint. The moving platform is connected at the upper bodies via three distinct spherical joints while the lower bodies are connected to the fixed platform by means of three distinct revolute joints.

The 3-RPS parallel manipulator, in which the prismatic joints are actuated, was introduced by Hunt [17] and has been the motive of an exhaustive research field where a great number of contributions, encompassing a wide range of topics, such as kinematic and dynamic analyses, synthesis, singularity analysis, extensions to hyper-redundant manipulators, etc., see for instance [18–21].

But, by actuating the revolute joints, a new 3-RPS parallel manipulator is achieved while R denotes the actuated revolute joint. To the best knowledge of the author, no study has been done for this type of 3-RPS manipulators. This paper analyzes the direct position kinematics of such a manipulator in an analytical form using the Sylvester dialytic elimination method.

2. DIRECT POSITION KINEMATICS

Direct position kinematics of the 3-RPS parallel manipulator consists of finding the pose (position and orientation) of the moving platform with respect to the fixed base while the rotation angle of the revolute actuators θ_i (i=1, 2, 3) is given. Clearly this problem is equivalent to the computation of coordinates of the centers of the spherical joints B_i (i=1, 2, 3), attached at the moving platform, see Fig. 2.

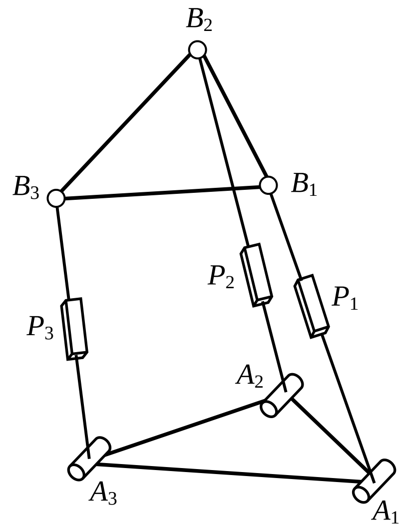


Figure 1. The 3-RPS parallel manipulator.

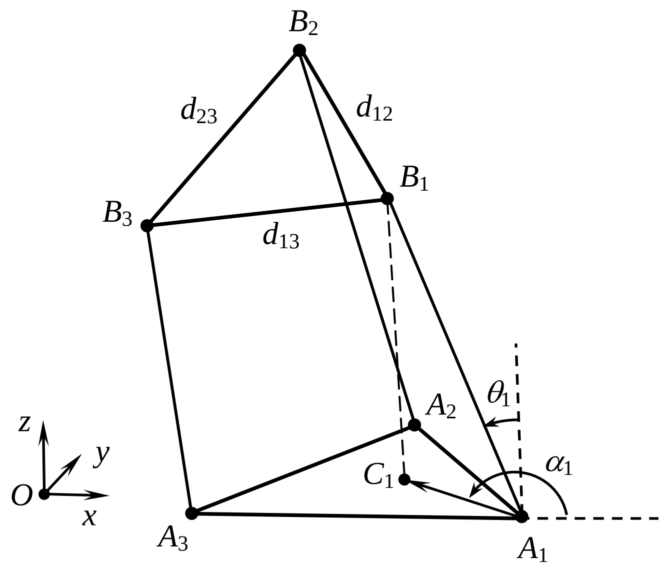


Figure 2. Geometric parameters of the 3-RPS parallel manipulator.

With regard to Fig. 2, the reference coordinate frame $O\{x, y, z\}$ is attached to the fixed base while the z-axis is perpendicular to the plane passing through A_i (i=1, 2, 3). Coordinate of the points A_i (i=1, 2, 3) can be written as

$$A_i = (u_i, v_i, w_i) \tag{1}$$

where u_i, v_i and w_i are the known values. The smallest angle between positive side of the z-axis and the line passing through the ith leg is denoted by θ_i (i=1, 2, 3). Also α_i (i=1, 2, 3) is the angle from positive side of the x-axis to the line segment $\overline{B_iC_i}$ where C_i is the projection of the point B_i on O-xy plane.

Considering legs of the manipulator as three 3D lines, the parametric equations of the lines can be expressed as

Line A_1B_1: $\begin{cases} x_1 = a_1 s + u_1 \\ y_1 = b_1 s + v_1 \\ z_1 = c_1 s + w_1 \end{cases}$ (2a)

Line A_2B_2: $\begin{cases} x_2 = a_2 t + u_2 \\ y_2 = b_2 t + v_2 \\ z_2 = c_2 t + w_2 \end{cases}$ (2b)

Line A_3B_3: $\begin{cases} x_3 = a_3 p + u_3 \\ y_3 = b_3 p + v_3 \\ z_3 = c_3 p + w_3 \end{cases}$ (2c)

In which

$$a_i = \sin\theta_i \cos\alpha_i$$
$$b_i = \sin\theta_i \sin\alpha_i$$
$$c_i = \cos\theta_i$$

and s, t and p are three independent variables.
Geometric constraints due to the manipulator architecture can be expressed as follows

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 = d_{12}^2 \tag{3a}$$
$$(x_1 - x_3)^2 + (y_1 - y_3)^2 + (z_1 - z_3)^2 = d_{13}^2 \tag{3b}$$
$$(x_2 - x_3)^2 + (y_2 - y_3)^2 + (z_2 - z_3)^2 = d_{23}^2 \tag{3c}$$

where $d_{ij} = \overline{B_i B_j}$ (i=1, 2, 3). Introducing Eqs. (2) into Eqs. (3) leads to

$$m_2 s^2 + m_1 s + m_0 = 0 \tag{4a}$$
$$n_2 s^2 + n_1 s + n_0 = 0 \tag{4b}$$
$$M_2 p^2 + M_1 p + M_0 = 0 \tag{4c}$$

in which

$$m_2 = \Delta_1,\ m_1 = \Delta_2 t + \Delta_3,\ m_0 = \Delta_4 t^2 + \Delta_5 t + \Delta_6$$
$$n_2 = \Delta_7,\ n_1 = \Delta_8 p + \Delta_9,\ n_0 = \Delta_{10} p^2 + \Delta_{11} p + \Delta_{12}$$
$$M_2 = \Delta_{13},\ M_1 = \Delta_{14} t + \Delta_{15},\ M_0 = \Delta_{16} t^2 + \Delta_{17} t + \Delta_{18}$$

And Δ_i (i=1,…, 18) are the coefficients depending on the geometric parameters of manipulator, see appendix A.

Equations (4a) and (4b) are two quadratic equations in s. The parameters s can be eliminated from these two equations by Sylvester dialytic elimination method. Taking (4a)×n_2–(4b)×m_2 and (4a)×n_0–(4b)×m_0 respectively and rewriting the resultant equations in matrix form, we have

$$\mathbf{K}_1 \begin{bmatrix} s \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{5}$$

where

$$\mathbf{K}_1 = \begin{bmatrix} n_2 m_1 - m_2 n_1 & n_2 m_0 - m_2 n_0 \\ m_2 n_0 - n_2 m_0 & n_0 m_1 - m_0 n_1 \end{bmatrix}$$

Equation (5) is valid if and only if det($\mathbf{K}_1$)=0. Thus

$$(n_2 m_1 - m_2 n_1)(n_0 m_1 - m_0 n_1) + (n_2 m_0 - m_2 n_0)^2 = 0 \tag{6}$$

Substitution of the parameters m_i and n_i (i=0, 1, 2) into Eq. (6) yields a fourth-degree polynomial in p as

$$N_4 p^4 + N_3 p^3 + N_2 p^2 + N_1 p + N_0 = 0 \tag{7}$$

In which

$$N_4 = D_1$$
$$N_3 = D_2 t + D_3$$
$$N_2 = D_4 t^2 + D_5 t + D_6$$
$$N_1 = D_7 t^3 + D_8 t^2 + D_9 t + D_{10}$$
$$N_0 = D_{11} t^4 + D_{12} t^3 + D_{13} t^2 + D_{14} t + D_{15}$$

Coefficients D_i (i=1,…, 15) are presented in appendix B. Equations (4c) and (7) constitutes a system of two nonlinear equations in unknown p. once again the parameter p can be eliminated from these two equations using Sylvester dialytic elimination method. The term p^4 can be eliminated multiplying Eq. (4c) by $N_4 p^2$ and Eq. (7) by M_2. Subtraction of the obtained expressions results in

$$(M_2 N_3 - N_4 M_1) p^3 + (M_2 N_2 - N_4 M_0) p^2 + M_2 N_1 p + M_2 N_0 = 0 \tag{8}$$

The procedure is repeated by multiplying Eqs. (4c) and (7) in $(N_4 p + N_3) p^2$ and $M_2 p_1 + M_1$ respectively. Subtraction of the obtained equations yields

$$(M_2 N_2 - N_4 M_0) p^3 + (M_2 N_1 + N_2 M_1 - N_3 M_0) p^2 + (M_2 N_0 - N_1 M_1) p + M_1 N_0 = 0 \tag{9}$$

In addition, multiplying Eq. (4c) and (7) by p results in

$$M_2 p^3 + M_1 p^2 + M_0 p = 0 \tag{10}$$

Equations (4c) and (8)–(10) can br written in a matrix form as

$$\mathbf{K}_2 \begin{bmatrix} p^3 \\ p^2 \\ p \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{11}$$

Where

$$\mathbf{K}_2 = \begin{bmatrix} M_2 N_3 - N_4 M_1 & M_2 N_2 - N_4 M_0 & M_2 N_1 & M_2 N_0 \\ M_2 N_2 - N_4 M_0 & M_2 N_1 + N_2 M_1 - N_3 M_0 & M_2 N_0 - N_1 M_1 & M_1 N_0 \\ M_2 & M_1 & M_0 & 0 \\ 0 & M_2 & M_1 & M_0 \end{bmatrix}$$

Again Eq. (11) is valid if and only if det($\mathbf{K}_2$)=0. Equating determinant of $\mathbf{K}_2$ to zero leads to a polynomial of degree eight as follows

$$G_8 t^8 + G_7 t^7 + G_6 t^6 + G_5 t^5 + G_4 t^4 + G_3 t^3 + G_2 t^2 + G_1 t + G_0 = 0 \tag{12}$$

where G_i depends on kinematic parameters of the manipulator. The detailed expressions for G_i are not given here, because they are too large to serve any useful purpose. Equation (12) admits eight real solutions for t. For each acceptable value of t, Eqs. (4a) and (4c) yield at most two solutions for s and p respectively. Finally introducing the above values of t, p and s into the Eqs. (2) leads to the coordinates of points B_i (i=1, 2, 3). What is important to point it out that only the coordinates of points B_i (i=1, 2, 3) are admissible for which Eqs. (3) are satisfied.
The above discussion shows that the 3-<u>R</u>PS parallel manipulator can have at most 32 solutions for the direct position kinematics.

3. CASE STUDY

In this section, to show the efficiency of the proposed method, direct position kinematics of the 3-<u>R</u>PS parallel manipulator is solved with the followings values: A_1=(0, 0, 0), A_2=(25, 0, 0), A_3=(12.5, 21.65, 0), d_{ij}=20 (i, j=1, 2, 3), θ_i=25 (i=1, 2, 3), α_1=30, α_2=150, α_1=270 in which the units of distance and angle are centimeter and degree respectively.

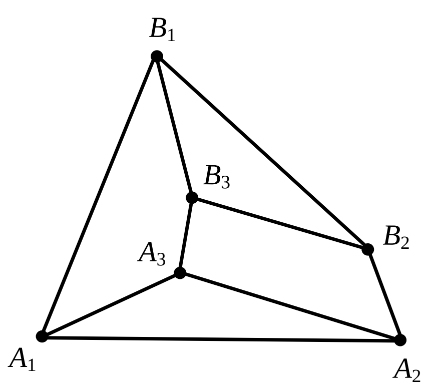
After calculating the parameters Δ_i (i=1,…, 18) and D_i (i=1,…, 15), and introducing them into Eq. (12), the values of coefficients G_i are obtained which are presented in Table 1.
Now solving Eq. (12) yields eight solutions for t. For each t, the parameters p and s, and Consequently coordinates of the points B_i (i=1, 2, 3) are computed. Taking into account Eqs. (3), seven real and acceptable solutions are found which are presented in Table 2. In addition, graphical representations of the solutions are presented in Fig. 3.

Table 1 Coefficients G_i (i = 0,…, 8) obtained for the case study

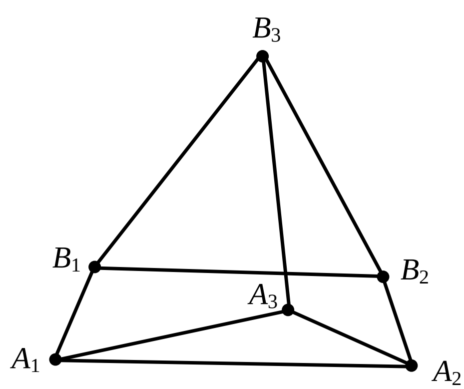
Coefficient	value	Coefficient	value
G_8	0.5005142605	G_3	–484102080.4
G_7	–136.7514833	G_2	4496468839
G_6	15145.09364	G_1	$-0.2071508511\times10^{11}$
G_5	–870363.4849	G_0	$0.3726148668\times10^{11}$
G_4	27663495.04		

Table 2 Seven solutions for direct position kinematics of the 3-<u>R</u>PS parallel manipulator.

No.	B_1	B_2	B_3
1	(7.84862080, 4.53140333, 19.4352516)	(22.5038776, 1.44113689, 6.18105607)	(12.5, 18.7593964, 6.19891942)
2	(2.50388742, 1.44562007, 6.20028452)	(22.5038776, 1.44113689, 6.18105607)	(12.5, 12.5867836, 19.4361302)
3	(2.49981681, 1.44326991, 6.19020462)	(17.1422655, 4.53666508, 19.4578193)	(12.5, 18.7636718, 6.18975061)
4	(22.4494745, 12.9612101, 55.5908098)	(2.45107331, 13.0186288, 55.8370795)	(12.5, -4.2717652, 55.5894048)
5	(22.4998143, 12.9902738, 55.7154645)	(7.85864319, 9.89656696, 42.4465126)	(12.5, -4.3303361, 55.7150105)
6	(22.4494745, 12.9612101, 55.5908098)	(2.45107331, 13.0186288, 55.8370795)	(12.5, 1.7148480, 42.7510713)
7	(17.2639771, 9.96736183, 42.7501528)	(2.45107331, 13.0186288, 55.8370795)	(12.5, -4.2717652, 55.5894048)



No. 1



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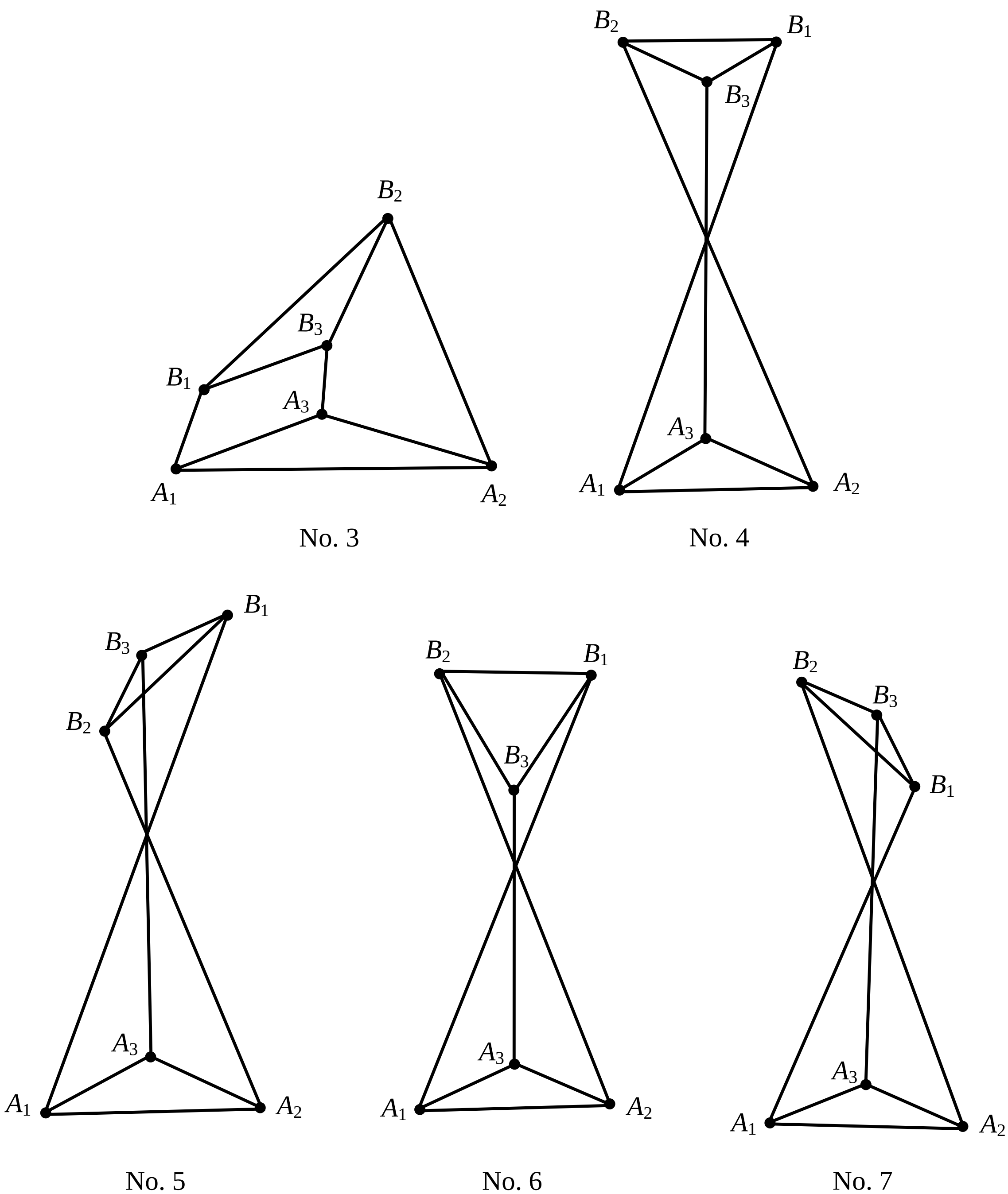

Figure 3. Schematic models of the solutions obtained for direct position kinematics of the 3-RPS manipulator.

4. CONCLUSION

In this paper, the direct position kinematics problem of the 3-RPS parallel manipulator was solved. Representing the legs of the manipulator through three parametric equations, a system of three nonlinear equations in three unknowns was obtained. The system of equations was reduced to a univariate polynomial of degree eight and two quadratic equations in sequence, recursively using the Sylvester dialytic elimination method. At the end, a numerical example was provided and seven solutions were gained for the direct position kinematics of manipulator.

APPENDIX A

Coefficients Δ_i (i=1,..., 18)

$$\Delta_1 = a_1^2 + b_1^2 + c_1^2$$
$$\Delta_2 = -2(c_1 c_2 + a_1 a_2 + b_1 b_2)$$
$$\Delta_3 = 2(a_1 u_1 - b_1 v_2 + b_1 v_1 - c_1 w_2 - a_1 u_2 + c_1 w_1)$$
$$\Delta_4 = c_2^2 + a_2^2 + b_2^2$$
$$\Delta_5 = 2(-v_1 b_2 + b_2 v_2 - w_1 c_2 + a_2 u_2 - u_1 a_2 + c_2 w_2)$$
$$\Delta_6 = -d_{12}^2 + u_1^2 - 2u_1 u_1 + v_1^2 - 2v_1 v_2 + u_2^2 + w_2^2 + w_1^2 - 2w_1 w_2 + v_2^2$$
$$\Delta_7 = a_1^2 + b_1^2 + c_1^2$$
$$\Delta_8 = -2(a_1 a_3 + c_1 c_3 + b_1 b_3)$$
$$\Delta_9 = 2(c_1 w_1 - a_1 u_3 - c_1 w_3 + b_1 v_1 + a_1 u_1 - b_1 v_3)$$
$$\Delta_{10} = c_3^2 + b_3^2 + a_3^2$$
$$\Delta_{11} = 2(-u_1 u_3 + a_3 u_3 - v_1 b_3 + c_3 w_3 + v_3 b_3 - w_1 c_3)$$
$$\Delta_{12} = -2u_1 u_3 + u_3^2 - d_{13}^2 + u_1^2 + w_1^2 + v_1^2 - 2w_1 w_3 - 2v_1 v_3 + v_3^2 + w_3^2$$
$$\Delta_{13} = c_3^2 + b_3^2 + a_3^2$$
$$\Delta_{14} = -2(b_3 b_2 + a_3 a_2 + c_3 c_2)$$
$$\Delta_{15} = 2(a_3 u_3 + c_3 w_3 - a_3 u_2 + b_3 v_3 - b_3 v_2 - c_3 w_2)$$
$$\Delta_{16} = c_2^2 + a_2^2 + b_2^2$$
$$\Delta_{17} = 2(-u_3 a_2 - v_3 b_2 + a_2 u_2 + b_2 v_2 + c_2 w_2 - w_3 c_2)$$
$$\Delta_{18} = u_3^2 - d_{23}^2 + u_2^2 + v_2^2 + w_2^2 - 2u_3 u_2 + v_3^2 - 2w_3 w_2 + w_3^2 - 2v_3 v_2$$

APPENDIX B

Coefficients D_i (i=1,..., 15)

$$D_1 = \Delta_1^2 \Delta_{10}^2$$
$$D_2 = -\Delta_1 \Delta_8 \Delta_{10} \Delta_2$$
$$D_3 = -\Delta_1 \Delta_8 \Delta_{10} \Delta_3 + 2\Delta_1^2 \Delta_{10} \Delta_{11}$$
$$D_4 = (\Delta_1 \Delta_8^2 \Delta_4 + \Delta_7 \Delta_2^2 \Delta_{10} - 2\Delta_7 \Delta_4 \Delta_1 \Delta_{10})$$
$$D_5 = (-2\Delta_7 \Delta_5 \Delta_1 \Delta_{10} - \Delta_1 \Delta_8 \Delta_{11} \Delta_2 - \Delta_1 \Delta_9 \Delta_{10} \Delta_2$$
$$+ 2\Delta_7 \Delta_2 \Delta_{10} \Delta_3 + \Delta_1 \Delta_8^2 \Delta_5)$$
$$D_6 = -\Delta_1 \Delta_8 \Delta_{11} \Delta_3 - 2\Delta_7 \Delta_6 \Delta_1 \Delta_{10} + 2\Delta_1^2 \Delta_{10} \Delta_{12} + \Delta_1 \Delta_8^2 \Delta_6 + \Delta_1^2 \Delta_{11}^2$$
$$+ \Delta_7 \Delta_3^2 \Delta_{10} - \Delta_1 \Delta_9 \Delta_{10} \Delta_3$$
$$D_7 = -\Delta_7 \Delta_2 \Delta_8 \Delta_4$$
$$D_8 = (\Delta_7 \Delta_2^2 \Delta_{11} - \Delta_7 \Delta_3 \Delta_8 \Delta_4 - 2\Delta_7 \Delta_4 \Delta_1 \Delta_{11} + 2\Delta_1 \Delta_8 \Delta_9 \Delta_4 - \Delta_7 \Delta_2 \Delta_8 \Delta_5)$$
$$D_9 = (-\Delta_1 \Delta_9 \Delta_{11} \Delta_2 - \Delta_7 \Delta_3 \Delta_8 \Delta_5 - 2\Delta_7 \Delta_5 \Delta_1 \Delta_{11} + 2\Delta_7 \Delta_2 \Delta_{11} \Delta_3 - \Delta_7 \Delta_2 \Delta_8 \Delta_6 - \Delta_1 \Delta_8 \Delta_{12} \Delta_2$$
$$+ 2\Delta_1 \Delta_8 \Delta_9 \Delta_5)$$
$$D_{10} = -\Delta_1 \Delta_9 \Delta_{11} \Delta_3 - \Delta_7 \Delta_3 \Delta_8 \Delta_6 - \Delta_1 \Delta_8 \Delta_{12} \Delta_3 + 2\Delta_1^2 \Delta_{11} \Delta_{12} + \Delta_7 \Delta_3^2 \Delta_{11}$$
$$- 2\Delta_7 \Delta_6 \Delta_1 \Delta_{11} + 2\Delta_1 \Delta_8 \Delta_9 \Delta_6$$
$$D_{11} = \Delta_7^2 \Delta_4^2$$

$$D_{12} = (2\Delta_7^2\Delta_4\Delta_5 - \Delta_7\Delta_2\Delta_9\Delta_4)$$
$$D_{13} = (2\Delta_7^2\Delta_4\Delta_6 - \Delta_7\Delta_3\Delta_9\Delta_4 + \Delta_7\Delta_2^2\Delta_{12} + \Delta_1\Delta_9^2\Delta_4$$
$$+ \Delta_7^2\Delta_5^2 - \Delta_7\Delta_2\Delta_9\Delta_5 - 2\Delta_7\Delta_4\Delta_1\Delta_{12})$$
$$D_{14} = (\Delta_1\Delta_9^2\Delta_5 - \Delta_7\Delta_2\Delta_9\Delta_6 + 2\Delta_7\Delta_2\Delta_{12}\Delta_3 - \Delta_7\Delta_3\Delta_9\Delta_5$$
$$+ 2\Delta_7^2\Delta_5\Delta_6 - 2\Delta_7\Delta_5\Delta_1\Delta_{12} - \Delta_1\Delta_9\Delta_{12}\Delta_2)$$
$$D_{15} = -\Delta_1\Delta_9\Delta_{12}\Delta_3 + \Delta_7^2\Delta_6^2 - \Delta_7\Delta_3\Delta_9\Delta_6 + \Delta_1\Delta_9^2\Delta_6$$
$$+ \Delta_7\Delta_3^2\Delta_{12} + \Delta_1^2\Delta_{12}^2 - 2\Delta_7\Delta_6\Delta_1\Delta_{12}$$

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