DESIGN CENTERING AND REGION APPROXIMATION USING SEMIDEFINITE PROGRAMMING

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ABSTRACT

The design centering problem seeks for the optimal values for the system designable parameters that maximize the production yield (probability of satisfying the design specifications by the manufactured systems).A new method for design centering and region approximation for a convex and bounded feasible region is introduced. The method finds iteratively a sequence of increasing-volume ellipsoids enclosing tightly selective sets of feasible points. These ellipsoids are found using semidefinite programming problem and known as Löwner-John ellipsoids. The sequence of Löwner-John ellipsoids is well definedin the method to converge to the minimum volume ellipsoid containing the feasible region. The center of the final ellipsoiddefines a design center for the proposed design problem and the ellipsoid itself is considered as a region approximation for the feasible region. Are-use of system simulations is performedin order to minimize the overall computational effort. Numerical and practical examples are considered to show the effectiveness of the new method.

KEYWORDS: *Design Centering, Yield Optimization,Semidefiniteprogramming, Löwner-John Ellipsoid,Minimum Volume Ellipsoid,Region Approximation.*

1. INTRODUCTION

Design centering problem [1] is a crucial for optimization issues and system design. The problem treats the optimal values for the system designable parameters that maximize the probability of ensuring that the behavior of the manufactured system remains within specification bounds [2], this probability measure is called the production yield [1]. In optimal system design, we have some designable parameters$x \in \mathcal{R}^n$, and system performance measures $f_i(x), i = 1,2,\dots,m,$ which are functions of the designable parameters. A region in the parameter space called thefeasible region $\mathcal{R}_f$ is defined by specifying bounds on the performance measures,

$$\mathcal{R}_f = \{x \in \mathcal{R}^n:\ f_i(x) \geq 0, i = 1,2,\dots,m\}. \tag{1}$$

Naturally, the performance measures are evaluated through numerical system simulations.Throughout the context, the feasible region will be assumed to be convex and bounded.

To simulate the statistical fluctuations which affect the system parameters during the manufacturing process, system designable parameters are considered as random variableswith joint probability density function $p(x;\mu,\Sigma)$, where $\mu$is the mean vector of $x,$ and $\Sigma$is the variance-covariance matrix. Without loss of generality, we can assume that the random vector x has a normal distribution [3].
The production yield can be defined as,[4]:

$$Y(\mu,\Sigma) = \int_{\mathcal{R}_f} p(x;\mu,\Sigma)dx. \tag{2}$$

Thus the design centering problem (yield maximization against designable system parameters) is,

$$max_\mu \left[Y(\mu) = \int_{\mathcal{R}_f} p(x;\mu)dx\right] \tag{3}$$

Methods dealing with the design centering problem can be divided, in general, into two approaches. The first approach is a geometric approach where the yield function is implicitly maximized by approximating the feasible region using a symmetric convex body, e.g. a hypersphere or hyperellipsoid[2-12].The center of this body is taken as the design center. The second approach is a statistical approach which optimizes the yield function directly using statistical estimation of the yield values [13-16]. Statistical Methods have the advantage of dealing with ill-shaped regions, but they are usually computationally expensive.

In this paper a new method for design centering and region approximation is introduced.The method finds the minimum volume ellipsoid containing the feasible region [17-19].To achieve this task, random samples are generated in the design parameter space using Latin-Hypercube [20]. A sequence of minimum volume ellipsoids enclosing selective sets of the generated feasible samples is obtained iteratively. These ellipsoids are called Löwner-John ellipsoids [19-21]. An information exchange takes placebetween each Löwner-John ellipsoid and the next random generation to guarantee the convergence of the ellipsoids sequence to the minimum volume ellipsoidenclosing the feasible region. The center of the obtained ellipsoid is considered as a design center for the proposed design problem, and the ellipsoid itself is considered as an ellipsoidal approximation for the feasible region.

The paper is organized as follows. In section (2), the mathematical formulation of the Löwner-John ellipsoid problem will be described in details. The proposed method with an algorithm will be given in section (3). In section (4), numerical and practical examples are shown. Finally, the conclusions for our results are summarized in section (5).

2. LÖWNER-JOHN ELLIPSOID PROBLEM

In this section, the mathematical formulation for the Löwner-John ellipsoid will be considered. The aim of this formulation is to reach the standard form for the determinant maximization problem discussed in [17]. For brevity, we will call this problem by $MinVE$.

Consider a set X_f of p points in n-dimensional space, i.e., $X_f = \{v^{(1)}, v^{(2)}, \ldots\ldots, v^{(p)}\} \subset \Re^n$, and it is required to find the minimum volume ellipsoid enclosing the set X_f.

Define the ellipsoid

$$\mathcal{E}(B,q) = \{x \in \Re^n : (x-q)^T B^{-1}(x-q) \le 1\}, \tag{4}$$

where$q \in \Re^n$ is the center of the ellipsoid, and $B \in S\Re^n$, $B \succ 0$, where $S\Re^n$ denotes the space of $n \times n$ symmetric matrices, and the notation $X \succ 0$ ($X \succcurlyeq 0$) means that X is positive definite (positive semidefinite) matrix.

We can assume that $B = QQ^T$, where Q is nonsingular. Without loss of generality, we can assume that $Q \in S\Re^n$, $Q \succ 0$,[22].

For the points of the set X_f to be enclosed by an ellipsoid $\mathcal{E}$, they must satisfy

$$\left(v^{(j)} - q\right)^T (QQ^T)^{-1}\left(v^{(j)} - q\right) \le 1, j = 1,2, \ldots, p. \tag{5}$$

The volume of the ellipsoid is given by [22]:

$$Vol(\mathcal{E}) = V_0 \, det(Q), \tag{6}$$

whereV_0 is the volume of the unit hypersphere in the n-dimensional space.

Hence, the problem of finding the minimum volume ellipsoid enclosing the set X_f, can be formulated as the following determinant minimization problem,

$$\begin{aligned} &min_{q,Q} logdetQ \\ &subjectto\left(v^{(j)} - q\right)^T (QQ^T)^{-1}\left(v^{(j)} - q\right) \le 1, j = 1, \ldots, p, \\ &\qquad Q = Q^T, Q \succ 0. \end{aligned} \tag{7}$$

Consider the constraint equation of the last problem,

$$(Q^{-1}v^{(j)} - Q^{-1}q)^T (Q^{-1}v^{(j)} - Q^{-1}q) \le 1, j = 1,2, \ldots, p. \tag{8}$$

In order to formulate this equation into Linear Matrix Inequality problem[18], change of variables is used as follows, let$A = Q^{-1}$&$b = -Q^{-1}q = -Aq$,

$$\begin{aligned} &\therefore (Av^{(j)} + b)^T (Av^{(j)} + b) \le 1, j = 1,2, \ldots, p, \\ &i.e. \left\|Av^{(j)} + b\right\| \le 1, j = 1,2, \ldots, p. \end{aligned} \tag{9}$$

So, the $MinVE$ problem expressed by Eq. (7) can be considered as the following determinant maximization problem [17-19],

$$\begin{aligned} &max_{b,A} logdetA \\ &\text{subject to } \begin{pmatrix} I_n & Av^{(j)} + b \\ \left(Av^{(j)} + b\right)^T & 1 \end{pmatrix} \succcurlyeq 0, j = 1,2, \ldots, p, \\ &\qquad A = A^T, A \succ 0, \end{aligned} \tag{10}$$

whereI_n denotes the $n \times n$ identity matrix.

Let, $A_j = \begin{pmatrix} A_{1j} & A_{2j} & \cdots & A_{nj} \end{pmatrix}^T$, where$A_j$ isthe j-th column of A,

$$b = (b_1 \quad b_2 \quad \ldots \quad b_n)^T, \qquad \text{and } v^{(j)} = \left(v_1^{(j)} \quad v_2^{(j)} \quad \ldots \quad v_n^{(j)}\right)^T.$$

Define the following vectors and matrices, $y_{MinVE} \in \Re^{\frac{n(n+3)}{2}}, e^{(i)} \in \Re^{(n+1)}, \hat{e}^{(i)} \in \Re^n,\ E^{(k)} \in S\Re^{(n+1)}$, and $\hat{E}^{(k)}, \hat{E}^{(i,k)} \in S\Re^n$, as follows,

$$y_{MinVE} = (-b_1 - b_2 \ldots - b_n \vdots -A_{11} - A_{12} \ldots - A_{1n} \vdots -A_{22} - A_{23} \ldots - A_{2n} \vdots -A_{33} \ldots - A_{3n} \vdots \cdots \ldots \vdots -A_{nn})^T, \tag{11}$$

$$e^{(i)} = (0 \ 0 \ \ldots 1^i \ldots 0)^T, \qquad i = 1, \cdots, n+1, \tag{12}$$

$$\hat{e}^{(i)} = (0 \ 0 \ \ldots 1^i \ldots 0)^T, \ \ i = 1, \cdots, n, \tag{13}$$

where the notation 1^i means that the i-th element of the vector $= 1$,

$$E^{(k)} = e^{(k)} e^{(n+1)^T} + e^{(n+1)} e^{(k)^T}, k = 1, \cdots, n, \tag{14}$$

$$\hat{E}^{(k)} = \hat{e}^{(k)} \hat{e}^{(k)^T}, k = 1, \cdots, n, \tag{15}$$

$$\hat{E}^{(i,k)} = \hat{e}^{(i)} \hat{e}^{(k)^T} + \hat{e}^{(k)} \hat{e}^{(i)^T}, i = 1, \cdots, n-1;\ k = i+1, \cdots, n. \tag{16}$$

Also, define the following block diagonal matrices ${F_b}^{(K)}, F_A^{(K)},\ F_A^{(i,k)} \in S\Re^{(n+1)p}$, where,

$${F_b}^{(K)} = diag\left(E^{(k)}, E^{(k)}, \cdots, E^{(k)}\right), k = 1, \cdots, n, \tag{17}$$

$$F_A^{(K)} = \ diag\left(v_k^{(j)} E^{(k)}, j = 1, \ldots, p\right), k = 1, \cdots, n, \tag{18}$$

$$F_A^{(i,k)} = diag\left(v_k^{(j)} E^{(i)} + v_i^{(j)} E^{(k)}, j = 1, \ldots, p\right), i = 1, \cdots, n-1; k = i+1, \cdots, n. \tag{19}$$

Finally, let,

$$A(y_{MinVE}) = \sum_{k=1}^{n} (-b_k) {F_b}^{(K)} + \sum_{k=1}^{n} (-A_{kk})\, F_A^{(K)} + \sum_{i=1}^{n-1} \sum_{k=i+1}^{n} (-A_{ik}) F_A^{(i,k)}, \tag{20}$$

$$\hat{A}(y_{MinVE}) = \sum_{k=1}^{n} (-A_{kk})\, \hat{E}^{(k)} + \sum_{i=1}^{n-1} \sum_{k=i+1}^{n} (-A_{ik})\, \hat{E}^{(i,k)}, \tag{21}$$

$$C = I_{(n+1)p}, \tag{22}$$

$$\hat{C} = 0_{n \times n}\,, \tag{23}$$

$$\hat{S} = A \succ 0. \tag{24}$$

Hence, the $MinVE$ problem expressed by Eq. (10), can take the following form of the determinant maximization problem,

$$\begin{aligned} MinVE\text{Problem: } & max_{y_{MinVE}}\ \ log\ det\, \hat{S}\ , \\ & subject\ to A(y_{MinVE}) + S = C, S \succcurlyeq 0, \\ & \hat{A}(y_{MinVE}) + \hat{S} = \hat{C}, \hat{S} \succ 0. \end{aligned} \tag{25}$$

It is to be noticed that the $MinVE$ problem expressed by Eq. (25) iswritten in the standard dual form of the determinant maximization problem discussed by[17],

$$\begin{aligned} & max\ b^T y + log\ det\, \hat{S} + l\ , \\ & subject\ to\ \textstyle\sum_{k=1}^{m} y_k\, A_k + S = C, S \succcurlyeq 0, \\ & \sum_{k=1}^{m} y_k \hat{A}_k + \hat{S} = \hat{C}, \hat{S} \succ 0, \end{aligned} \tag{26}$$

where $C, A_k \in S\Re^n, \hat{C}, \hat{A}_k \in S\Re^l$, and $b \in \mathcal{R}^m$ are given data, and $S \in S\Re^n, \hat{S} \in S\Re^l$, and $y \in \mathcal{R}^m$ are the variables. This problem is anextension of the semidefinite programming problem [23]. Hence, the semidefinite programming techniques of solutions are used to solve it.

3. THE METHOD AND ALGORITHM

In this section, the new method for design centering and regionapproximation is introduced. The method finds the minimum volume ellipsoid containing the feasible region. The center of this ellipsoid is considered as the design center for the design problem and the ellipsoid itself gives a region approximation for the feasible region. An algorithm for the method will be also given.

3.1. Minimum Volume Ellipsoid Enclosing The Feasible Region

The proposed method presents a statistical-geometrical approach for design centering dependent on generating a sequence of Löwner-John ellipsoids which converges to the minimum volume ellipsoid containing the feasible region.

The method starts with an initial point $q^{(0)}$which might be infeasible.Ageneration of$N$Latin-Hypercube normally distributed samples [20]in the design parameter space is performed. The generated samples have a prescribed variance-covariance matrix $B^{(0)}$such that some points of these samples are feasible. These points are called the feasible points. Then, the Löwner-John ellipsoid containing these feasible points is constructed. The center $q$of the obtained ellipsoid is taken as a mean value for the next normally distributed random sample generation, with its ellipsoid matrix taken as a scaled variance-covariance matrix for this generation. Repeating this process, gives a sequence of increasing-volume ellipsoids tend to cover the feasible region. Finally, the minimum volume ellipsoid containing the feasible region is obtained. The obtained ellipsoid is considered as a region approximation with its center as a design center. In order to reduce the number of system simulations needed at each iteration, only generated sampleslying outside a shrunk scaled version of the current ellipsoid are tested to be feasible or not. Then, the feasible points were already tested in the last iteration are retrieved and added to the new feasible points giving us the new sample space needed to run the next iteration.

3.2. The Algorithm

Initialization

1- Definitions:

Define the feasible region $\mathcal{R}_f$ in$n$- dimensional space.

Choose proper values for the following:

- A number of samples N to be generated at each iteration, an initial starting ellipsoid center $q^{(0)} \in \Re^n$, and an initial ellipsoid matrix $B^{(0)} = R^2 I_n$, where $R \in \Re, R > 0.$.
- An offset number of points $n_\varepsilon, 0 \ll n_\varepsilon \leq N$.
- Two ellipsoidal radii $r_1, and\ r_2,\ \ 0 \leq r_1 \leq 0.98$, and $0.9 \leq r_2 \leq 1$.
- Acceptable stopping measures ϵ_q, ϵ_B, and n_f, where$0 \leq n_f \ll N$.

2- Random Samples Generation:

Generate N random samples $v_i^{(0)}$, $i = 1,2, \dots . N$, belonging to a normal distribution $N\left(q^{(0)}, B^{(0)}\right)$.

3- Feasibility Check:

For $i = 1$ to N

If $v_i^{(0)} \in \mathcal{R}_f,\ i = 1, \dots, N,$

store$x_i^{(0)}$ in the feasible set $X_f^{(0)}$.

4- Set $N^{(0)} =$ length $(X_f^{(0)})$.

5- Minimum Volume Ellipsoid Enclosing the Feasible Set:

Obtain the Löwner-John ellipsoid $\mathcal{E}_1(q^{(1)}, B^{(1)})$ enclosing the set of points $X_f^{(0)}$.

Processing

6- Let $k = 1$.

7- Draw the most distant samples were used to construct the current ellipsoid:

If $N^{(k-1)} > n_\varepsilon$

$X_{fo}^{(k-1)} = \{v \in X_f^{(k-1)}: \left(v - q^{(k)}\right)^T {B^{(k)}}^{-1} \left(v - q^{(k)}\right) \geq r_1^2\}$,

else$X_{fo}^{(k-1)} = X_f^{(k-1)}$.

8- Random Sample Generation:

Generate N random samples $v_i^{(k)}$; $i = 1,2, \dots . N$, belonging to a normal distribution $N(q^{(k)}, \frac{1}{n} B^{(k)})$

9- Optimized Feasibility Check:

If $\left(v_i^{(k)} - q^{(k)}\right)^T B^{(k)^{-1}} \left(v_i^{(k)} - q^{(k)}\right) \geq r_2^2$, and $v_i^{(k)} \in \mathcal{R}_f$, $i = 1,2, \ldots . N$,
store $v_i^{(k)}$ in the set $X_{f\varepsilon}^{(k)}$.
Let $N_{new}^{(k)} = \text{length}\ (X_{f\varepsilon}^{(k)})$.

10-Feasible Set Construction:
Set $X_f^{(k)} = X_{f\varepsilon}^{(k)} \cup X_{fo}^{(k-1)}$, $N^{(k)} = \text{length}\ (X_f^{(k)})$.
11-Minimum Volume Ellipsoid Enclosing the Feasible Set:
Obtain the Löwner-John ellipsoid $\mathcal{E}_{k+1}(q^{(k+1)}, B^{(k+1)})$ enclosing the set of points $X_f^{(k)}$.
12-Stopping Criteria:
Choose a stopping criterion from the following:

- A pre-specified number of iterations.
- A certain relative tolerance measure for the ellipsoid center $\frac{\|q^{(k+1)} - q^{(k)}\|}{\|q^{(k+1)}\|} < \epsilon_q$.
- A certain relative tolerance measure for the ellipsoid volume $\left|\frac{\det\left(B^{(k+1)}\right) - \det\left(B^{(k)}\right)}{\det\left(B^{(k+1)}\right)}\right| < \epsilon_B$.
- The number of newly added feasible points to construct the new ellipsoid $N_{new}^{(k)} < n_f$.

Else set $k = k + 1$, and go to step 7.

4. EXAMPLES

Numerical and practical examples are considered in this section. Solutions for the given examples are obtained via the free Max-det. software package SDPT3-ver 4.0 beta, of Toh et al. [24,25], which applies the primal-dual predictor-corrector path-following algorithms [26, 27]. The Helmberg-Rendl-Vanderbei-Wolkowicz / Kojima-Shindoh-Hara / Monteiro (H..K..M) direction of the Monteiro-Zhang family [28] is chosen as a search direction to solve the introduced examples.

4.1. Numerical Example

Consider the following two-dimensional nonlinearly feasible region given by the following constraints:

$$((y-1)^2 + 1)exp(1-x) \leq 7,$$
$$exp(x - 2y + 1) \leq 7,$$
$$x^2 + y^2 \leq 8.$$

An initial infeasible point $q^{(0)} = (5 \quad 5)^T$ is considered with an initial ellipsoid matrix $B^{(0)} = 25I_2$ to begin the minimum volume ellipsoid algorithm. The ellipsoidal radii are $r_1 = 0$, and $r_2 = 1$, the stopping criterion is $\epsilon_B = 10^{-5}$, and points offset $n_\varepsilon = 500$. At each iteration, Latin-Hypercube sampling technique is used to generate 500 normally distributed samples. Final result for the minimum volume ellipsoid is shown in Figure1.

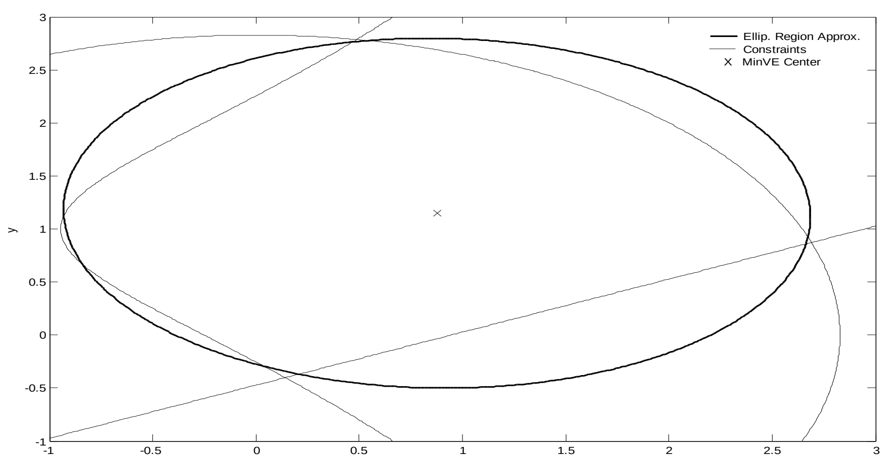

Figure1.The design center and the ellipsoidal region approximation of example 4.1

The final results are: $q_{MinVE} = \begin{pmatrix} 0.8657 \\ 1.2176 \end{pmatrix}, B_{MinVE} = \begin{pmatrix} 3.1356 & 0.1101 \\ 0.1101 & 2.6014 \end{pmatrix}$.

The results were obtained after performing 5 iterations, with 1047 function evaluations.

The yield values are evaluated at $q^{(0)},\ and\ q_{MinVE}$ via Monte Carlo method [29], using 1000 normally distributed samples for each yield evaluation. The yield values are given in Table 1, and Table 2, for the independent and correlated cases respectively.

Table 1. Yield results ofexample 4.1 for the independent parameters case

Parameter Spreads	Initial Yield	Final Yield	
	Actual Region	Actual Region	Region Approx.
(0.5,0.5)	0%	98%	99.6%
(0.6,0.8)	0%	86%	93.5%

Table 2. Yield results of example 4.1 for the correlated parameters case

Covariance Matrix	Initial Yield	Final Yield	
	Actual Region	Actual Region	Region Approx.
B_{MinVE}	0%	31.7%	40%
$B_{MinVE}/4$	0%	77.7%	86.7%
$\sigma_x^2 = 0.25, \sigma_y^2 = 0.25,$ $\rho_{xy} = 0.8$	0%	97.1%	98.9%

4.2. Two-Section CapacitivelyLoaded Line Transformer

Atwo-section capacitivelyloaded line transformer [4] shown in Figure2, is considered. The feasible region is defined using the reflection coefficient sampled at 11 frequencies $\{0.5\ GHz, 0.6\ GHz, \dots, 1.5\ GHz\}$.The magnitude of the reflection coefficient at these frequencies ≤ 0.5. The characteristic impedances $Z_1 and Z_2$ are the designable parameters. The normalized lengths $L_1, and L_2$ with respect to the quarter-wave length at the center frequency 1 $GHz$are taken as 0.9333, and 0.8001, respectively

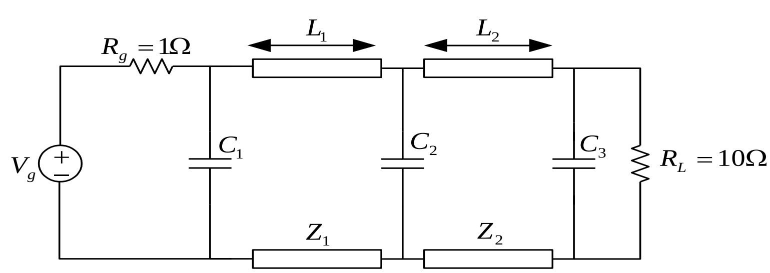

Figure2. Two-section transmission line transformer

An initial infeasible point $q^{(0)} = (2 \quad 2)^T$, is considered with an initial ellipsoid matrix$B^{(0)} = 4I_2$ to begin the minimum volume ellipsoid algorithm. At each iteration, Latin-Hypercube sampling technique is used to generate 500 normally distributed samples. The ellipsoidal radii are $r_1 = 0,$ and$r_2 = 1$, the stopping criterionis$\epsilon_q = 10^{-4}$, and points offset $n_\varepsilon = 500$.The final result for the minimum volume ellipsoidis shown in Figure3.

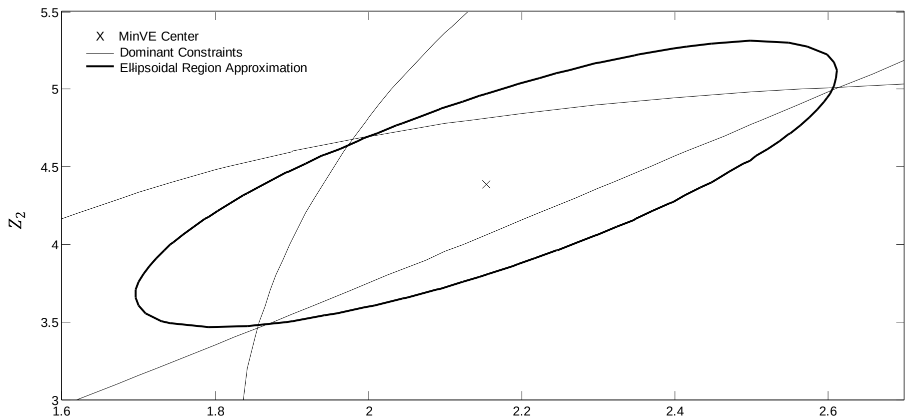

Figure3. The design center and the ellip l region approximation of example 4.2

The final results are: $q_{MinVE} = \begin{pmatrix} 2.1528 \\ 4.4057 \end{pmatrix}$, $B_{MinVE} = \begin{pmatrix} 0.2025 & 0.3132 \\ 0.3132 & 0.8086 \end{pmatrix}$.

The results were obtained after performing 5 iterations, with 1219 system simulations.
The yield values are evaluated at $q^{(0)}$, $and\ q_{MinVE}$ via Monte Carlo method using 1000 normally distributed samples for each yield evaluation. The yield values are given in Table 3, and Table 4, for the independent and correlated cases respectively.

Table 3. Yield results of example 4.2 for the independent parameters case

Parameter Spreads	Initial Yield	Final Yield	
	Actual Region	Actual Region	Region Approx.
(0.2,0.4)	0%	46.7%	68.4%
(0.2,0.4)/2	0%	83.6%	96.4%

Table 4. Yield results of example 4.2 for the correlated parameters case

Covariance Matrix	Initial Yield	Final Yield	
	Actual Region	Actual Region	Region Approx.
B_{MinVE}	0%	21.6%	39.1%
$B_{MinVE}/4$	0%	60%	86.5%

4.3. Design of CMOS Operational Amplifier

A Miller CMOS operational amplifier [30], shown in Figure4, is considered as a 3-dimensional design problem. The design parameters are the channel widths of the transistors 1,5, and 6. The widths of transistors 7, and 8 are kept constants at $100\ \mu m$, and the widths of transistors 3, and 4 are $50\ \mu m$. The channel lengths of all transistors are kept at $8\ \mu m$. The biasing current $I_B = 20\mu A$, and the compensation capacitor $C_c = 5PF$. Transistors 1, and 2 are identical.

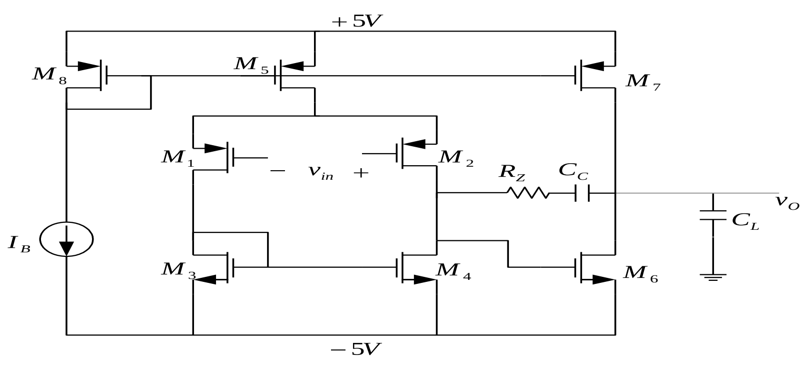

Figure4. Miller CMOS operational amplifier

The required specifications for this operational amplifier are:

$$Low frequency gain \geq 98\ dB,$$
$$Gain - bandwidth product \geq 17 Mrad/sec,$$
$$Power dissipation \leq 0.65\ mW,$$
$$Area = W_1 + W_5 + W_6 \leq 308\ \mu m.$$

An initial point $q^{(0)} = (80 \quad 80 \quad 80)^T$, is considered with an initial ellipsoid matrix $B^{(0)} = 225 I_3$ to begin the minimum volume ellipsoid algorithm. At each iteration, Latin-Hypercube sampling technique is used to generate 500 normally distributed samples. The ellipsoidal radii are $r_1 = 0.5$, and $r_2 = 1$, the stopping criterion is $n_f = 2$, and points offset $n_\varepsilon = 300$.

The final results are:

$$q_{MinVE} = (161.7425 \quad 68.7166 \quad 59.9796)^T,$$
$$B_{MinVE} = 10^3 \begin{pmatrix} 5.7952 & -2.3616 & -3.5662 \\ -2.3616 & 1.3677 & 1.4818 \\ -3.5662 & 1.4818 & 3.2189 \end{pmatrix}.$$

The results were obtained after performing 7 iterations, with 1647 circuit simulations.

The yield values are evaluated at $q^{(0)}$, *and* q_{MinVE} via Monte Carlo method using 1000 normally distributed samples for each yield evaluation. The yield values are given in Table 5, and Table 6, for the independent and correlated cases respectively.

Table 5. Yield results of example 4.3 for the independent parameters case

Parameter Spreads	Initial Yield	Final Yield	
	Actual Region	Actual Region	Region Approx.
(4,4,4)	0%	98.9%	100%
(6,6,6)	0%	92%	99.7%

Table 6. Yield results of example 4.3 for the correlated parameters case

Covariance Matrix	Initial Yield	Final Yield	
	Actual Region	Actual Region	Region Approx.
$B_{MinVE}/9$	0%	77.6%	96.9%
$B_{MinVE}/16$	0%	92.9%	99.9%

4.4. Seven-Section Capacitively Loaded Line Transformer

A seven-sectioncapacitivelyloaded line transformer [31], shown in Figure 5 is considered. The feasible region is given by the reflection coefficient sampled at 68 frequencies $\{1\ GHz, 1.1\ GHz, \ldots., 7.7\ GHz.\}$.The magnitude of the reflection coefficient at these frequencies does not have to exceed 0.07. The design parameters are the normalized lengths $L_1,\ L_2, \ldots, L_7$ with respect to the quarter-wave length at the center frequency $4.35 GHz$.

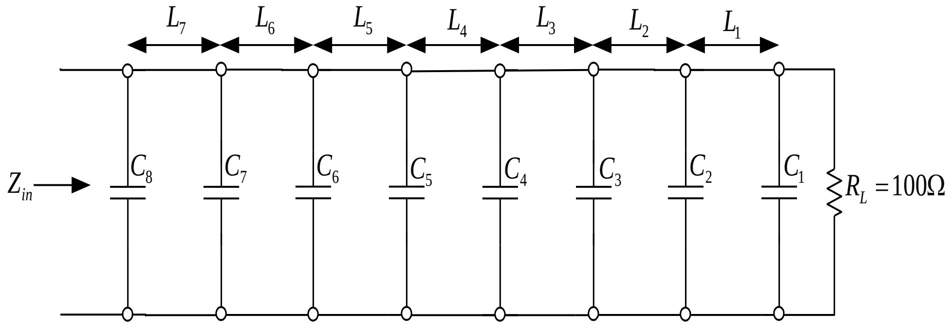

Figure5. Seven-section capcitively loaded line transformer.

The characteristic impedances are taken as:

$$(Z_1 \quad Z_2 \quad \dots \quad Z_7) = (91.94454 \quad 85.52387 \quad 78.15262 \quad 70.710678 \quad 63.97737 \quad 58.46320 \quad 54.38060)\ Ohm.$$

An initial infeasible point $q^{(0)} = (0.9 \quad 0.9 \quad 0.9 \quad 0.9 \quad 0.9 \quad 0.9 \quad 0.9)^T$ is considered with an ellipsoid matrix $B^{(0)} = 25 \times 10^{-4} I_7$, to begin the algorithm.At each iteration, Latin-Hypercube sampling technique is used to generate 500 normally distributed samples. The ellipsoidal radii are: $r_1 = 0.7, r_2 = 1$, the stopping criteria is $n_f = 2,$ and points offset $n_\varepsilon = 200.$

.The final results are:

$$q_{MinVE} = (0.8555 \quad 0.9251 \quad 0.9552 \quad 0.9619 \quad 0.9670 \quad 0.9941 \quad 0.8598)^T,$$

$$B_{MinVE} = \begin{pmatrix} 0.0217 & -0.0069 & 0.0016 & 0.0020 & -0.0016 & 0.0090 & -0.0093 \\ -0.0069 & 0.0268 & -0.0101 & 0.0018 & 0.0154 & -0.0170 & 0.0149 \\ 0.0016 & -0.0101 & 0.0280 & 0.0009 & -0.0231 & 0.0143 & 0.0006 \\ 0.0020 & 0.0018 & 0.0009 & 0.0081 & 0.0005 & 0.0010 & 0.0019 \\ -0.0016 & 0.0154 & -0.0231 & 0.0005 & 0.0329 & -0.0153 & 0.0035 \\ 0.0090 & -0.0170 & 0.0143 & 0.0010 & -0.0153 & 0.0305 & -0.0081 \\ -0.0093 & 0.0149 & 0.0006 & 0.0019 & 0.0035 & -0.0081 & 0.0308 \end{pmatrix}.$$

The results were obtained after performing 12 iterations, with 2582 system simulations.

The yield values are evaluated at $q^{(0)},\ and\ q_{MinVE}$ via Monte Carlo method using 1000 normally distributed samples for each yield evaluation. The yield values are given in Table 7, and Table 8, for the independent and correlated cases respectively.

Consider the following parameter spreads $\sigma = (0.6, 0.35, 0.16, 0.11, 0.01, 0.06, 0.05)$:

Table 7. Yield results of example 4.4 for the independent parameters case

Parameter Spreads	Initial Yield	Final Yield	
	Actual Region	Actual Region	Region Approx.
$\sigma/7$	0%	60.5%	75.1%
$\sigma/21$	0%	99.9%	100%

Table 8. Yield results of example 4.4 for the correlated parameters case

Covariance Matrix	Initial Yield	Final Yield	
	Actual Region	Actual Region	Region Approx.
$B_{MinVE}/49$	0%	99.7%	100%

5. CONCLUSIONS

In this paper, a new application for the recently developed semidefinite programming problem has been introduced. The newmethod finds a design center and an ellipsoidal region approximation for the feasible region in a simple manner. The methodadapts the Löwner-John ellipsoid in order to find the minimum volume ellipsoid enclosing the feasible region. The method has the advantage that no derivatives are calculated through its processing. The design center obtained shows sufficient increase in the yield values for all the examples considered. A relatively little number of system simulations is needed to run the algorithm. As expected, the minimum volume ellipsoidal approximation gives an optimistic yield results compared with the actual region.

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