# GAURSAT FUNCTION FOR AN ELASTIC PLATE WEAKENED BY A CURVILINEAR HOLE IN THE PRESENCE OF HEAT

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**ABSTRACT**
In this work, Complex Variable method is used to obtain the complex potential functions, Goursat functions, for an infinite elastic plate weakened by a curvilinear hole. This curvilinear hole is conformally mapped outside a unit circle $\gamma$ by means of a general rational mapping function $z = c\omega(\zeta)$ doesn't vanish or become infinite outside the unit circle. Many applications for the first and second fundamental problems are investigated and discussed. Also many special and new cases are derived from the work.

**keywords** : *An infinite plate, complex variable method, Goursat functions, first And second fundamental problem.*

## 1. INTRODUCTION AND FORMULATION OF THE PROBLEM

The boundary value problems for isotropic homogeneous perforated infinite plates have been discussed by several authers [1-6].Some of them used Laurant's theorem to express the solution in the form of power series, see [1,6]. Others used complex variable method to obtain the solution in the form of two complex functions, Gaursat functions. Consider a thin infinite plate of thickness h with a curvilinear hole C, where the origin lies inside the hole, conformally mapped outside the domain of a unit circle $\gamma$ by means of a rational mapping function $z = c\omega(\zeta)$, subject to the condition $z'(\zeta)$ does not vanish or become infinite outside a unit circle $\gamma$, $\zeta = e^{i\psi}$, $0 \le \psi \le 2\pi$. If a heat $\Theta = qy$ is following uniformly in the direction of the negative $y-axis$ , where the increasing temperature $\Theta$ is assumed to be constant across the thickness of the plate i.e. $\Theta = \Theta(x, y)$, and is the constant temperature gradient. Here, we take the $x-axis$ to be the horizontal axis which is perpendicular to the $y-axis$. The uniform flow of heat is distributed by the presence of an insulated curvilinear hole C, and heat equation satisfies the relation

$$(i)\ \nabla^2\Theta = 0 \quad ,\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{1.1}$$

$$(ii)\ \frac{\partial\Theta}{\partial n} = 0 \quad r = r_0 \tag{1.2}$$

where $n$ is the unit vector perpendicular to the surface. Neglecting the variation of the strain and the stress with respect to the thickness of the plate the thermoelastic potential $\Theta$ satisfies the formula, see [6]

$$\nabla^2\Phi = (1+\nu)\alpha\Theta, \tag{1.3}$$

where $\alpha$ is a scalar presents the coefficient of the thermal expansion and $\nu$ is Poisson's ratio. Assume the faces of the plate are free of applied loads.

It is known that, see [?] , in the thermoelastic problem, the first and second boundary value problems are equivalent to finding two analytic functions $\phi(z)$ and $\psi(z)$ of one complex argument $z = x + iy$ .These functions must satisfy the boundary conditions

$$K\phi_1(t) - t\overline{\phi_1'} - t\overline{\psi_1(t)} = f(t), \tag{1.4}$$

where for the first boundary value problem $K = -1$, $f(t)$ is a given function of stresses. While for the second boundary value problem $K = k > 1$, $f(t)$ is a given function of displacement, is called the thermal conductivity of the material and $t$ denote the affix of a point on the boundary.

The formula (4) for the first and second boundary value problems, respectively, take the following form

$$\phi_1(t) + t\overline{\phi_1'(t)} + \overline{\varphi_1(t)} = \frac{\partial\Phi}{\partial x} + i\frac{\partial\Phi}{\partial y} + \frac{1}{2G}\int_0^s [iX(t) - Y(t)dt + c, \tag{1.5}$$

$$k\phi_1(t)\overline{\phi_1^{'}(t)}+\overline{\varphi_1(t)}=u+iv-\frac{\partial\Phi}{\partial x}+i\frac{\partial\Phi}{\partial y} \quad (1.6)$$

where the applied stresses $X(s)$ and $Y(s)$ are prescribed on the boundary of the plane, s is the length measured from arbitrary point, u and v are the displacement components, $G$ is the shear modulus, and $\Phi$ represents the thermoelastic potential function. Also, here the applied stresses $X(s)$ and $Y(s)$ must satisfy the following, see [2]

$$X(s)=\sigma_{xx}\frac{dy}{ds}-\sigma_{xy}\frac{dx}{ds} \quad (1.7)$$

$$Y(s)=\sigma_{yx}\frac{dy}{ds}-\sigma_{yy}\frac{dx}{ds} \quad (1.8)$$

where $\sigma_{xx},\sigma_{yy}$ and $\sigma_{xy}$ are called the stress components. In the thermoelastic problems of the plate, the stress components are given by [5 ]

$$\sigma_{xx}-\sigma_{yy}+2i\sigma_{xy}=2G\left[\frac{\partial^2\Phi}{\partial x^2}-\frac{\partial^2\Phi}{\partial y^2}+2i\frac{\partial^2\Phi}{\partial x\partial y}\right]-4G\left[z\overline{\phi^{''}(z)}+\overline{\psi^{'}(z)}\right]$$

$$\sigma_{xx}+\sigma_{yy}=4G\left[\phi^{'}(z)+\overline{\phi^{'}(z)}-\lambda\Theta A\right] \quad (1.9)$$

where $\lambda=\alpha(\frac{1+\nu}{2})$ is the coefficient of heat transfer and $\nu$ is the Poison's ratio.

A very powerful method for solving the thermoelastic problems makes use of the conformal mapping to reduce the problem for any given region whose boundary $c$ satisfies certain regularity conditions, to corresponding problem for region having a unit circle.

In terms of the rational mapping function $z=c\omega(\zeta)$ where $c>0$, and $\omega^{'}(\zeta)$ does not vanish or becomes infinite for $|\zeta|>1$ then the infinite region outside a class contour may be conformally mapped outside the unit circle $\gamma$. The two complex potential functions $\phi_1(z)$ and $\psi_1(z)$ in the form, see [ 3 ]

$$\phi_1(z)=-\frac{S_x+iS_y}{2\pi(1+k)}ln\zeta+c\Gamma\zeta+\phi(\zeta) \quad (1.10)$$

$$\psi_1(z)=\frac{k(S_x-iS_y)}{2\pi(1+k)}ln\zeta+c\Gamma^{+}\zeta+\psi(\zeta) \quad (1.11)$$

where $S_x,S_y$ are the components of the resultant vector of all external forces acting on the boundary, $\Gamma,\Gamma^{*}$ represent the stresses at infinity, generally complex, $\phi(\zeta)$ and $\psi(\zeta)$ are single value analytic functions within the region outside the unit circle $\gamma$ and $\phi(\infty)=\psi(\infty)=0$ which means that, $\phi(\zeta)$ and $\psi(\zeta)$ are homomorphic functions at infinity. It will be assumed that $\Gamma=\overline{\Gamma}$ and $S_x=S_y=0$ for the first boundary value problem. The rational mapping $z=c\omega(\zeta)$ maps the boundary $C$ of the given region occupied by the middle plane of the plate in the $z-plane$ onto the unit circle $\gamma$ in the $\zeta-plane$. Curvilinear coordinates $(\rho,\theta)$ are thus introduce into the $z-plane$ which are the maps of the polar conditions in the $\zeta-plane$ as given by $\zeta=\rho e^{i\theta}$ . Using $z=c\omega(\zeta)$ in (1.4), we have

$$K\phi_1(c\omega(\zeta))-\frac{\omega(\zeta)}{\omega^{'}(\zeta)}\phi_1(c\omega(\zeta))-\psi_1(c\omega(\zeta))=f(c\omega(\zeta)). \quad (1.12)$$

The formulas (1.12) represent the first and second boundary value problems in $\zeta-plane$ .

In this paper, the complex variable method has been applied to obtain the two complex analytic potential functions, Gaursat functions, $\phi(\zeta)$, $\psi(\zeta)$ and the three stresses components $\sigma_{xx},\sigma_{yy}$, and $\sigma_{xy}$ for the first and second boundary value problems in thermoelastic of the same previous domain, for an infinite plate weakened by a curvilinear hole $C$ conformally mapped outside a unit circle $\gamma$ by the rational mapping function

$$z = c\omega(\zeta) = c\frac{\zeta + m_1\zeta^{-l} + m_2\zeta^{-2l}}{(1-n_1\zeta^{-l})(1-n_2\zeta^{-l})}, (c>0, n_1 \neq n_2, l = 1,2,..., p) \tag{1.13}$$

Here, the heat $\Theta = qy$ is following uniformly in the negative direction of $y - axis$. The increasing temperature $\Theta$ is assumed to be constant across the thickness of the plate. Also, in (1.13), the values of m's and n's are real parameters restricted such that $\omega^{'}(\zeta)$ does not vanish or become infinite outside the unit circle $\gamma$. The interesting cases when the shape of the hole is an ellipse, hypotrochoidal a crescent or a cut having the shape of a circular arc are included as special ones. Holes corresponding to certain combination of the parameters m's and n's are sketched, see Figures (1 - 8). Some applications of the first and second boundary value problems of the infinite plate with a curvilinear hole having several poles are investigated.

## 2. CONFORMAL MAPPING

Consider the conformal mapping (1.13), where the parametric equations can be obtained in the following form

$$\frac{x}{c} = \frac{1}{\prod_{c=1}^{2}[1-2n_i cosl\theta + n_i^2]}[-(n_1+n_2)m_1 + n_1 n_2 m_2 + cos\theta + L^{+} cosl\theta$$
$$-(n_1+n_2)cos(1+l)\theta + m_2 cosl\theta + n_1 n_2 cos(1+2l)\theta],$$

$$\frac{y}{c} = \frac{1}{\prod_{c=1}^{2}[1-2n_i cosl\theta + n_i^2]}[sin\theta + L^{-} sinl\theta - (n_1+n_2)sin(1+l)\theta - m_2 sin2l\theta$$
$$+ n_1 n_2 sin(1+2l)\theta].$$

where

$$L^{\pm} = \pm m_1 + n_1 n_2 m_1 \mp m_2(n_1+n_2).$$

The importance of the previous mapping comes from the following special cases

For $m_2 = 0$, we have the rational mapping function

$$z = c\frac{\zeta + m\zeta^{-l}}{(1-n_1\zeta^{-l})(1-n_2\zeta^{-l})}, \quad c>0, l=1,2,..., p \tag{2.1}$$

The conformal mapping (2.1) is studied completely by [4]

For $m_2 = n_2 = 0$, we have, see [5]

$$z = c\frac{\zeta + m\zeta^{-l}}{1-n\zeta^{-l}}, \quad |n|<1, c>0, l=1,2,..., p. \tag{2.2}$$

For $m_2 = n_1 = n_2 = 0$, we have the rational mapping function, see [1]

$$z = c\left[\zeta + m\zeta^{-l}\right] \tag{2.3}$$

The main reason of interest in this mapping is that the general shapes of the hypotrochoids are curvilinear polygons for $l = 1$, we have an elliptic hole, for $l = 2$. we have a curvilinear triangle, for $l = 3$ a curvilinear square, and hence approximate region of.

For $m_1 = m_2 = n_2 = 0$, we have the transformation

$$z = \frac{c\zeta}{1-n\zeta^{-l}}, \quad (c>0, |n| = \frac{1}{1+l}, l=1,2,..., p). \tag{2.4}$$

This mapping leads to a certain regular curvilinear polygon with $l$ round vertices which become cusps when $|n| = \frac{1}{1+l}$

For $n_1 = n_2 = 0$, we have the rational mapping function

$$z = c\left(\zeta + m_1\zeta^{-l} + m_2\zeta^{-2l}\right), \quad 0 \le m_j \le 1,\ j = 1,2, l = 1,2,...,\ p. \tag{2.5}$$

The physical interest of this map comes from the following : A circle of radius $C$ when $m_1 = m_2 = 0$.

An ellipse when $m_2 = 0, l = 1$

A square with rounded corners with diagonals parallel to the same $x$ and $y$ axises when $m_1 = 0, m_2 = about 0.1$, $l = 1$. The same square with its sides parallel to the axis's when $m_1 = 0, m_2 = about -0.1$.

An ovaloid when $m_1 = about 0.3, m_2 = about -0.05$ and $l = 1$. More information and applications on a technology for these special cases are found in England [4].

For $m_2 = n_2 = 0, l = 1$, we have the rational mapping

$$z = c\frac{\zeta + m\zeta^{-1}}{1 - n\zeta^{-1}}, \quad (c > 0, |n| < 1) \tag{2.6}$$

More information about this mapping can be found in [3].

For $n_2 = 0, l = 1$, we have the rational mapping

$$z = c\frac{\zeta + m\zeta^{-1} + m_2\zeta^{-2}}{1 - n\zeta^{-1}} \tag{2.7}$$

the physical meaning of this mapping is found in [3].

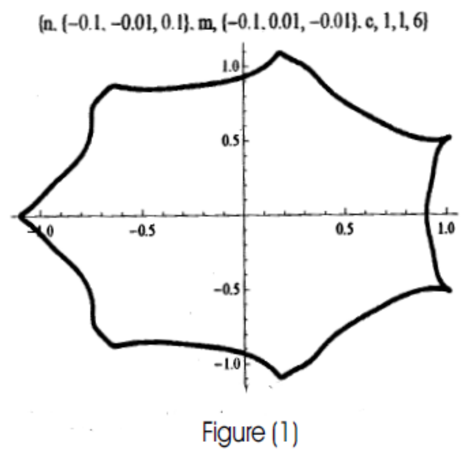


Figure 1: (n(-0.1,-0.01,0.1). m,(-0.1,0.01,-0.01).c,1,1,6)

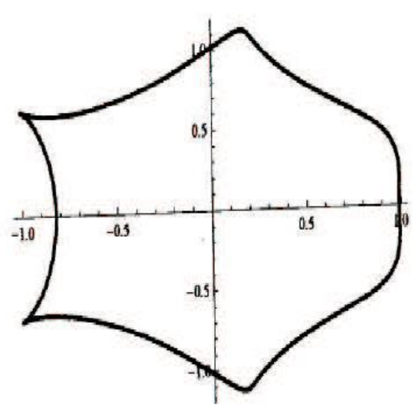


Figure 2: (n(-0.01,-0.001,0.1).m,(-0.1,0.001,-0.001).c,1,1,5)

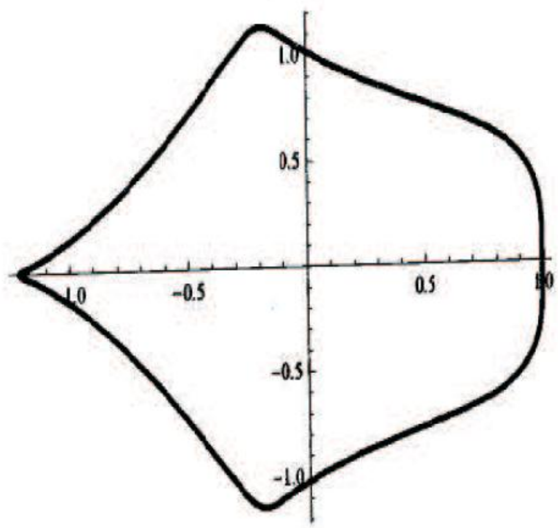


Figure 3: (n(-0.01,-0.001,0.1). m,(-0.1,0.001,-0.001).c,1,1,4)

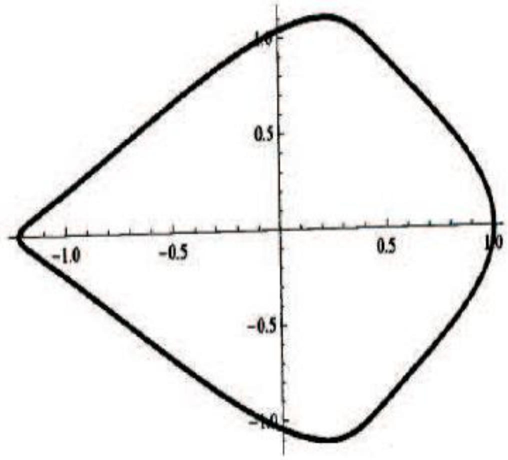


Figure 4: (n(-0.01,-0.001,0.1). m,(-0.1,0.001,-0.001).c,1,1,4)

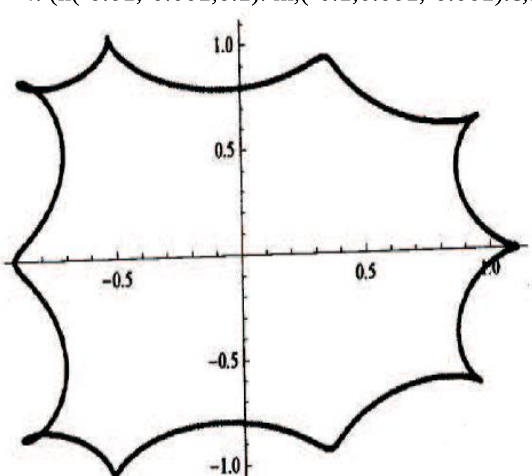


Figure 5: (n(-0.01,-0.001,0.1). m,(-0.1,0.001,-0.001).c,1,1,4)

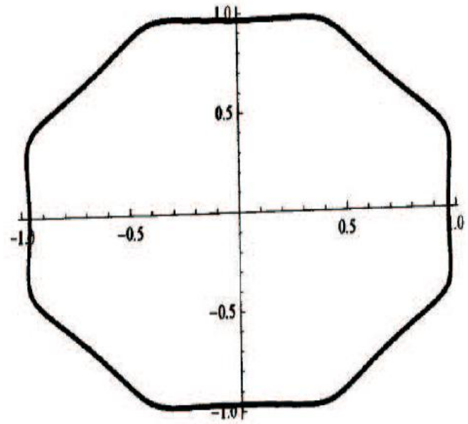


Figure 6: (n(-0.01,-0.001,0.1). m,(-0.1,0.001,-0.001).c,1,1,4)

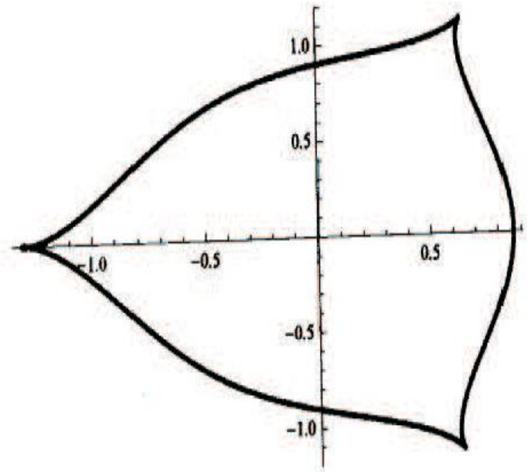


Figure 7: (n(-0.01,-0.001,0.1). m,(-0.1,0.001,-0.001).c,1,1,4)

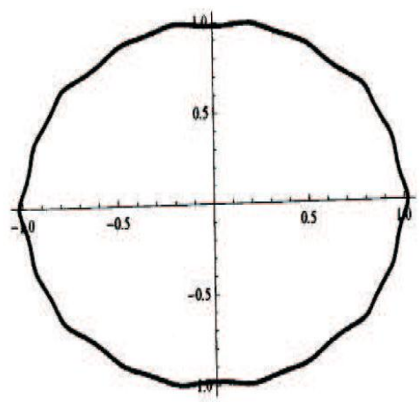


Figure 8: (n(-0.01,-0.001,0.1). m,(-0.1,0.001,-0.001).c,1,1,4)

**3. THE METHOD OF SOLUTION**

In view of the definition $z = x + iy = \rho e^{i\theta}$, and the rational mapping function of Eq.(1.13) , the solution of the boundary value problem of Eq.(1.1) is given by

$$\Theta = q\left[ Imz + \frac{r_0^2 sin^2\theta}{Imz} \right]. \tag{3.1}$$

In determining the thermoelastic potential of Eq.( 1.2), the uniform heat may be disregarded. So the formula (1.2) takes the form

$$\nabla^2\Phi = \alpha(1+\nu)\frac{r_0^2 sin^2\theta}{Imz}. \tag{3.2}$$

Using the definition of $\nabla^2\Phi$ in polar coordinates and solving (3.2), in this domain, we have

$$\Phi(z,\bar{z}) = \frac{(1+\nu)r_0^2}{4} lnz.Im(z\bar{z}) \tag{3.2}$$

Hence, the values of $\Theta$ and $\Phi$ are completely determined. Now we return to the transformation mapping. The expression $\frac{w(\zeta^{-1})}{w'(\zeta)}$ can be written in the form

$$\frac{w(\zeta^{-1})}{w'(\zeta)} = \alpha(\zeta^{-1}) + \beta(\zeta) \tag{3.4}$$

where $\alpha(\zeta) = \frac{h_1}{\zeta^l - n_1} + \frac{h_2}{\zeta^l - n_2},$

$$h_j = \frac{[n_j^{\nu+1} + m_1 n_j + m_2](1-n_j^2)(1-n_1 n_2)^2}{(n_j - n_{\pm 1})[(1-n_j^2)(1-n_1 n_2)\gamma_j^{(1)} - ln\gamma_j^{(2)}(n_1 + n_2 - 2n_j n_{j+1})]}$$

$$\gamma_j^{(1)} = 1 - lm_1 n_j^{\nu} - 2lm_2 n_j^{\nu+1}$$

$$\gamma_j^{(2)} = 1 + m_1 n_j^{\nu} + m_2 n_j^{\nu+1}, \quad (j = 1,2; \nu = 1 + \frac{1}{e}), \tag{3.5}$$

and $\beta(\zeta)$ is a regular function for $|\zeta| > 1$ . Using ( 1.10), (1.11) and (3.4), the boundary condition (1.12) can be written in the form

$$k\phi(\sigma) - \alpha(\sigma)\overline{\phi'(\sigma)} - \overline{\psi_*(\sigma)} = f_*(\sigma) \tag{3.6}$$

where $\sigma = e^{i\theta}$ denotes the value of on the boundary of the unit circle, while

$$\psi_*(\zeta) = \psi(\zeta) + \beta(\zeta)\phi'(\zeta),$$

$$f_*(\zeta) = F(\zeta) - ckl^{-1}\zeta + c\overline{\Gamma^*}\zeta^{-1} + N(\zeta)[\alpha(\zeta) + \overline{\beta(\zeta)}]$$

$$N(\zeta) = c\bar{\Gamma} - \frac{S_x - iS_y}{2\pi(1+k)}\zeta,$$

$$F(\zeta) = f(t) = f(c\omega(\zeta)). \tag{3.7}$$

Assume that the function $F(\sigma)$ with it's derivatives must satisfy Hölder condition.

Our aim is to determine the function $\phi(\zeta)$ and $\psi(\zeta)$ for the various boundary value problems, from (2.6). For this, multiplying both sides of (2.6) by $\frac{d\sigma}{2\pi i(\sigma-\zeta)}$, then integrating the integrals thus formulated by residue theorems, one has

$$K\phi(\zeta) + \frac{1}{2\pi i}\int_\gamma \frac{\alpha(\sigma)\overline{\phi'(\sigma)}}{\sigma-\zeta}d\sigma = c\overline{\Gamma^*}\zeta^{-1} + A(\zeta) + \frac{h_1}{\zeta - n_1^{\nu-1}}N(n_1^{\nu-1}), \tag{3.8}$$

where

$$\nu = 1 + \frac{1}{l},$$

and

$$A(\zeta) = \frac{1}{2\pi i}\sum_{\eta=0}^{\infty}\zeta^{-\eta-1}\int_\gamma \sigma^\eta F(\sigma)d\sigma. \tag{3.9}$$

Using (3.4) in the integral term of (3.8), we assume

$$\frac{1}{2\pi i}\int_{\gamma}\frac{\alpha(\sigma)\overline{\phi'(\sigma)}}{\sigma-\zeta}d\sigma=\frac{ch_1b_1}{n_1^{\nu-1}-\zeta}+\frac{ch_2b_2}{n_2^{\nu-1}-\zeta} \tag{3.10}$$

where b's are complex constants to be determined. Hence, we have

$$-K\phi(\zeta)=A(\zeta)-c\overline{\Gamma^*}\zeta^{-1}=\sum_{j=1}^{2}\frac{h_j(cb_j+N(n_j^{\nu-1})}{h_j^{\nu-1}-\zeta}. \tag{3.11}$$

Differentiating ( 3.11) with respect to $\zeta$ , and using the result of $\overline{\phi'(\sigma)}$ in ( 3.10), we obtain

$$cKb_j+cn_j^2\Gamma^*+d_jh_j[c\overline{b_j}+\overline{N(n_j^{\nu-1})}]=-\overline{A'(n_j)},(j=1,2.). \tag{3.12}$$

The general solution of (3.12) is

$$b_j=\frac{KE_j-h_jd_j\overline{E_j}}{c(K^2-h_j^2d_j^2)}$$

where

$$E_j=-\overline{A'(n_j)-c\Gamma^*}n_j^{2(\nu-1)}-h_jd_j\overline{N(n_j^{\nu-1})},$$

and

$$d_j=n_j^{2(\nu-1)}(1-n_1^{\nu-1}n_2^{\nu-1})^{-2}\overline{N(n_j^{\nu-1})}, \tag{3.13}$$

from the boundary condition ( 3.6), $\psi(\zeta)$ can be determined in the form

$$\psi(\zeta)=\frac{cK\overline{\Gamma}}{\zeta}-\frac{\omega(\zeta^{-1})}{\omega'(\zeta)}\phi_*(\zeta)+\frac{h_1\zeta}{1-n_1^{\omega-1}\zeta}\phi_*(n_1^{\nu-1})+\frac{h_2\zeta}{1-n_2^{\nu-1}}+B(\zeta)-B \tag{3.14}$$

where $\phi_*(\zeta)=\phi'(\zeta)+\overline{N(\zeta)},$

$$B(\zeta)=\frac{1}{2\pi i}\int_{\gamma}\frac{F(\sigma)}{\sigma-\zeta}d\sigma$$

$$B=\frac{1}{2\pi i}\int_{\gamma}\frac{\overline{F(\sigma)}}{\sigma-\zeta}d\sigma. \tag{3.15}$$

Using ( 3.1), ( 3.3), ( 3.11) and ( 3.14) in ( 1.9), after some derivatives and algebraic relations, we have

$$\begin{aligned}\sigma_{xx}&=2G\left[-\gamma(z^2+4z\bar{z}+\bar{z}^2)Imz+Re(2\phi'(\zeta)-M(\zeta,\bar{\zeta}))\right],\\ \sigma_{yy}&=2G\left[\gamma(z^2+\bar{z}^2)Imz+Re(2\phi'(\zeta)+M(\zeta,\bar{\zeta}))\right],\\ \sigma_{xy}&=2G\left[\gamma(z\bar{z}-2(Imz)^2Rez+ImM(\zeta,\bar{\zeta}))\right],\end{aligned} \tag{3.16}$$

where

$$M(\zeta,\bar{\zeta})=\left[c(\zeta)-\frac{\overline{\omega(\zeta)}}{\omega'(\zeta)}\right]\overline{\phi''(\zeta)}+\frac{S_x-is_y}{2\pi(1+k)}\frac{\overline{\omega(\zeta)}}{\omega'(\zeta)}+ck\Gamma\zeta^2$$

$$-\frac{\overline{\omega(\zeta)}}{\omega'(\zeta)}\left[\overline{\phi'(\zeta)}+N(\zeta)\right]+\overline{B'(\zeta)}+\sum_{i=1}^{2}\frac{h_j\zeta}{(\zeta-n_j^{\nu-1})^2}\phi_*(n_j^{\nu-1})$$

and

$$\gamma=\frac{(1+\nu)r_0^2}{2(z\bar{z})^2}. \tag{3.17}$$

## 4. SOME APPLICATIONS

In this section we discuss the solution of the first and second boundary value problems by assuming different values of the given functions. Then the Gaursat functions, in this case, will be obtained and directly the stress components can be calculated.

**Application 1:** Curvilinear hole for an infinite plate subjected to a uniform tensile stress and flowing heat:

For $K=-1, \Gamma^{*}=\frac{p}{4}, \Gamma^{*}=\frac{-p}{2}e^{-2i\theta}$, $0\le\theta\le 2\pi$, $S_x=S_y=f=0$ and $\Theta=qy$, we have an infinite plate stretched at infinity by the application of a uniform tensile stress of intensity $P$, making an angle $\theta$ with the $x-axis$, and a flowing heat in the negative direction of $y-axis$. The plate weakened by a curvilinear hole $C$, which is free from stresses, and the two complex functions of ( 3.12) and ( 3.14) become

$$\phi(\zeta)=\frac{cp}{2}e^{2i\theta}\zeta^{-1}+\sum_{j=1}^{2}\frac{L_j^{(1)}}{n_j^{\nu-1}-\zeta}, \tag{4.1}$$

$$\psi(\zeta)=\frac{-cp}{2}\zeta^{-1}-\frac{\omega(\zeta^{-1})}{\omega'(\eta)}\phi_*(\zeta)+\sum_{j=1}^{2}\frac{h_j\zeta}{1-n_j^{\nu-1}\zeta}\phi_*(n_j^{\omega-1}), \tag{4.2}$$

where
$$L_j^{(1)}=\frac{Ph_i}{4}\left[\frac{1-2n_j^{2(\nu-1)}cos2\theta}{1-h_jd_j}+\frac{2n_j^{2(\nu-1)}sin2\theta}{1+h_jd_j}\right]$$

and
$$\phi_*(\zeta)=\phi'(\zeta)+\frac{cp}{4}.$$

The stress components $\sigma_{xx},\sigma_{yy},\sigma_{xy}$ can be obtained directly by using ( 4.1), (4.2) in (3.16).

**Application 2:** Curvilinear hole having two poles and the edge of which is subject to a uniform pressure in the present of flowing heat :

For $K=-1, S_x=S_y=\Gamma=\Gamma^{*}=0, \Theta=qy$ and $f(t)=Pt$ where $P$ is a real constant.

The formulas (3.12)-( 3.14) become

$$\phi(\zeta)=\frac{cP(n_1^{1+\nu}+m_1n_1+m_2)}{(n_1^{\nu-1}-\zeta)(1-h_1d_1)(n_1-n_2)}+\frac{cP(n_2^{1+\nu}+m_2n_2+m_1)}{(n_2^{\nu-1}-\zeta)(1-h_2d_2)(n_2-n_1)}, \tag{4.3}$$

$$\psi(\zeta)=-cP(n_1+n_2+\zeta^{-1})-\frac{\omega(\zeta^{-1})}{\omega'(\zeta)}\phi'(\zeta)+\frac{h_1\zeta}{1-n_1^{\nu-1}\zeta}\phi^{*}(n_1^{\nu-1})$$
$$+\frac{h_2\zeta}{1-n_2^{\nu-1}\zeta}\phi^{*}(n_2^{\nu-1}). \tag{4.4}$$

Hence ( 4.3- 4.4) give the solution of the first boundary value problem when the edge of the hole is subject to a uniform pressure $P$ when the edge of hole is subject to a uniform tangential stress $T$ ,we put in ( 4.3- 4.4), $iT$ instead of $P$ .

**Application 3:**The force acts on the centre of the curvilinear hole. In this case, it will be assumed that the stresses vanish at infinity. It is easily seen that the kernel does not rotate. In general, the kernel remains in its original position. Hence one assume $\Gamma=\Gamma^{*}=f(t)=0$, $K=k$ and $\Theta=qy$, the Gaursat functions are

$$-k\phi(\zeta)=\frac{c}{2\pi(1+k)}\sum_{j=1}^{2}\frac{h_jn_j}{n_j^{\nu-1}-\zeta}\left[\frac{kh_jd_j(S_x-iS_y)}{c(k^2-h_j^2d_j^2)}\right.$$
$$\left.-(1+\frac{h_jd_j^2}{c(k^2-h_j^2d_j^2)})(S_x-iS_y)\right], \tag{4.5}$$

$$\phi_*(\zeta) = \frac{h_1\zeta}{1-n_1^{\nu-1}\zeta}\phi_*(n_1^{1-\nu}) + \frac{h_2\zeta}{1-n_2^{\nu-1}\zeta}\phi_*(n_2^{1-\nu}) - \frac{\omega(\delta^{-1})}{\omega'(\zeta)}\phi_*(\zeta) \tag{4.6}$$

where $$\phi_*(\zeta) = \phi'(\zeta) - \frac{S_x + iS_y}{2\pi(1+k)\zeta}.$$

Therefore, we have the solution of the second boundary value problem in the case when a force $S_x, S_y$ acts on the centre of the curvilinear kernel and when a heat is flowing in the negative direction of $y-axis$ .

## 5. CONCLUSION

From the above results and discussions, the following may be concluded:

[1]. In the theory of two dimensional linear elasticity one of the most useful techniques for the solution of boundary value problems for awkwardly shaped region is to transformation the region into one simpler shape.

[2]. The mapping function (1.13) maps the curvilinear hole $C$ in the $z-plane$ onto the domain outside a unit circule $\zeta - plane$ under the condition $\omega'(\zeta)$ does not vanish or become infinite outside $\gamma$ .

[3]. The physical interest of the mapping (1.13) comes from its strong special cases which discussed here. Moreover many new cases can be obtained according to the technology of the work, where the boundary value problems of the infinite plate with a curvilinear hole having finite poles are not discussed before.

[4]. The complex variables method (Cauchy method) is considered as one of the best method for solving the boundary value problems taken on closed contour $\gamma$ , and obtained the two complex potential functions $\phi(z)$ and $\psi(z)$ directly.

[5]. Here, we assumed the conformal mapping of Eq.( 1.13), which has two singular points i.e we can say that $z \to \infty$ at $\zeta = n_l, l = 1,2$ . Also at infinity we can say that the function $\Theta_{\inf}(z,\bar{z})$ is equivalent to the term $\Theta_{\inf}(z,\bar{z}) = qy$ and the thermoelasticity potential $\Phi(z,\bar{z})$ is equivalent, at infinity, to the value

$$\Phi_{\inf}(z,\bar{z}) = \frac{(1+\nu)\alpha q}{\delta} z\bar{z} Im z \tag{5.1}$$

So, the stress components, at infinity, are relative to the following equations

$$\sigma_{xx} + \sigma_{yy} + 2(1+\alpha)\Theta_{inf} = \delta GA,$$

$$\sigma_{yy} - \sigma_{xx} + 4G\left[\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial \bar{z}^2}\right]\Phi_{inf}(z,\bar{z}) = \delta GB,$$

$$\sigma_{xy} + 4iG\left[\frac{\partial^2}{\partial \bar{z}^2} - \frac{\partial^2}{\partial z^2}\right]\Phi_{inf}(z,\bar{z}) = \delta GC, \tag{5.2}$$

where the real constants $A, B$ and $C$ are related to the stress at infinity.

[6]. The complex function $\phi(z)$ is considered as the solution of the integral equation with Cauchy kernel

$$\phi'(\zeta) + \frac{1}{2\pi i}\int_\gamma M(\zeta,\sigma)\overline{\phi'(\zeta,\sigma)}d\sigma = A'(\zeta) - \lambda\omega'(\zeta) \tag{5.3}$$

Where $$\lambda = \frac{\phi'(0)}{\omega'(0)}, M(\zeta,\sigma) = \frac{\omega(\sigma) - \omega(\zeta) - (\sigma-\zeta)\omega'(\zeta)^2}{\omega'(\zeta)(\sigma-\zeta} \tag{5.4}$$

The above integral equation (5.3) with the kernel (5.4) is called Fredholm integral equation with Cauchy kernel in the closed domain $\gamma$ and complex plane.

[7]. This work can be considered as a generalization of the all previous work in the infinite plate with a curvilinear hole.

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