TWO DIMENSIONAL UNSTEADY MOTION OF MICROPOLAR FLUID IN THE HALF-PLANE WHEN THE VELOCITY ARE GIVEN ON THE BOUNDARY

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ABSTRACT
The object of this work is to investigate the unsteady two dimensional motion of micropolar fluid within the half-plane (-∞< x <∞, y>0 | t>0) due to the sudden motion of its horizontal boundary. Using the technique of Laplace-Fourier transform, numerical results of velocities, pressure, microrotation, stream function, stresses and moments are obtained and illustrated graphically. The classical problem of viscous fluid is included as special case and compared numerically with its analytical solution.
Keywords: *Micropolar Fluid, Laplace-Fourier Transform, Mathcad Program*

1. INTRODUCTION

The theory of micropolar fluids was originally formulated by Eringen [1] . This theory describes some physical systems of fluids which can provide couple stresses, body couples, exhibit microrotational effects and microrotational inertia. Moreover it has supported satisfactory model for describing the flow properties of liquid crystals, fluid with certain additives and animal blood. The study of micropolar fluids has been considered and attracted the attention of many authors [2]-[8].

In a previous work [9], Belonosov and Elsirafy used the technique of Laplace –Fourier transform to obtain exact solution of Navier-Stockes homogeneous equations for the slow motion of viscous incompressible fluid within the half-plane initially at rest while the velocities are given on the boundary. The aim of this manuscript is to solve the boundary problem of micropolar fluid within the half-plane (-∞ < x < ∞ , y >0 | t > 0), initially at rest , due to a sudden motion of its horizontal boundary by using the same technique of Laplace-Fourier transform. Numerical results of velocities, pressure, microrotation, stream function, stresses and moments are obtained and illustrated graphically. The solution of the Newtonian problem of viscous fluid is included as special case and compared with the corresponding analytical solution.

2. FORMULATION OF THE PROBLEM

In the absence of both external forces and body couples the linearized equations of motion of an incompressible micropolar fluid [1] are

$$\nabla.\vec{q} = 0$$
$$-(\mu+k)\nabla\wedge(\nabla\wedge\vec{q}) + k\ \nabla\wedge\vec{\nu} - \nabla p = \rho\frac{\partial\vec{q}}{\partial t} \tag{1}$$
$$(\alpha+\beta+\gamma)\nabla(\nabla.\vec{\nu}) - \gamma\nabla\wedge(\nabla\wedge\vec{\nu}) + k\nabla\wedge\vec{q} - 2k\vec{\nu} = \rho J\frac{\partial\vec{\nu}}{\partial t}$$

where $\alpha,\beta,\gamma,\mu,k$ are viscosity coefficients ,J is guration parameter,$\vec{q}$ is the velocity vector and $\vec{\nu}$ is the microrotation vector.

We consider the case of two-dimensional unsteady motion of a micropolar incompressible fluid in the half-plane (-∞ < x < ∞, y> 0 |t> 0).In this case

$$\vec{q} = (u, v, 0) \quad ,\vec{\nu} = (0,0,\nu)$$

The system (1) is reduced to

$$Re\left\{\frac{\partial w}{\partial z}\right\} = 0,$$
$$\rho\frac{\partial w}{\partial t} = (\mu+k)\nabla^2 w - 2\frac{\partial}{\partial\bar{z}}(p+ik\nu) \tag{2}$$
$$\rho J\frac{\partial\nu}{\partial t} = \gamma\nabla^2\nu - 2k\nu + 2kIm\left\{\frac{\partial w}{\partial z}\right\}$$

where , $z = x + i\,y, \qquad w = u + iv.$

Now we need to solve the system (2) subject to the boundary conditions

$$u(x,0,t) = u_0(x,t)\ , v(x,0,t) = 0,\ \ \nu(x,0,t) = 0, \tag{3}$$

$w(x,\infty,t) = \nu(x,\infty,t) = 0.$
with the initial conditions

$$w(x,y,0) = 0, \qquad \nu(x,y,0) = 0. \tag{4}$$

For sudden motion on the boundary initially at rest , we can take

$$u_0(x,t) = V_0 H(t) H(a - |x|), \qquad a > 0 \tag{5}$$

where $H(z)$ is the Heaviside function of z and V_0 is a constant.
The stresses and couples are given by

$$\widehat{yz} = \widehat{zy} = \widehat{xz} = \widehat{zx} = 0 \ , \ \widehat{zz} = -p$$
$$\widehat{xx} + \widehat{yy} = -2p \ , \ \widehat{xx} - \widehat{yy} = 2(2\mu + k)\frac{\partial u}{\partial x},$$
$$\widehat{yx} + \widehat{xy} = (2\mu + k)\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right), \qquad \widehat{yx} - \widehat{xy} = k\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} + 2\nu\right) \tag{6}$$
$$\frac{m_{yz}}{\gamma} = \frac{m_{zy}}{\beta} = \frac{\partial \nu}{\partial y}, \qquad \frac{m_{xz}}{\gamma} = \frac{m_{zx}}{\beta} = \frac{\partial \nu}{\partial x},$$
$$m_{xx} = m_{yy} = m_{zz} = m_{xy} = m_{yx} = 0$$

We now introduce the following non-dimensional variables

$$u^* = \frac{u}{V_0},\ v^* = \frac{v}{V_0},\ y^* = \frac{\rho V_0}{\mu + k} y,\ x^* = \frac{\rho V_0}{\mu + k} x, \quad t^* = \frac{\rho V_0^2}{\mu + k} t \tag{7}$$
$$\nu^* = \frac{k}{\rho V_0^2}\nu,\ p^* = \frac{p}{\rho V_0^2},\ u_0^* = \frac{u_0}{V_0},\ \widehat{ij}^* = \frac{\mu + k}{\mu \rho V_0^2}\widehat{ij},\ m_{ij}^* = \frac{k(\mu + k)}{\gamma \rho^2 V_0^3} m_{ij},\ i,j = x,y,z$$

Thus the system (2) and (6) can be written as (dropping asterisks for convenience)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \nabla^2 u - \frac{\partial p}{\partial x} + \frac{\partial \nu}{\partial y} = \frac{\partial u}{\partial t}$$
$$\nabla^2 v - \frac{\partial p}{\partial y} - \frac{\partial \nu}{\partial x} = \frac{\partial v}{\partial t}, \qquad \nabla^2 \nu + f\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - g\nu = h\frac{\partial \nu}{\partial t}, \tag{8}$$
$$\widehat{yz} = \widehat{zy} = \widehat{xz} = \widehat{zx} = 0 \ , \ \widehat{zz} = -\mathrm{k}_0 p$$
$$\widehat{xx} + \widehat{yy} = -2\mathrm{k}_0 p \ , \ \widehat{xx} - \widehat{yy} = 2(1 + \mathrm{k}_0)\ \frac{\partial u}{\partial x},$$
$$\widehat{yx} = \frac{\partial v}{\partial x} + k_0\left(\frac{\partial u}{\partial y} + \nu\right), \qquad \widehat{xy} = \frac{\partial u}{\partial y} + k_0\left(\frac{\partial v}{\partial x} - \nu\right) \tag{9}$$
$$m_{yz} = \frac{\gamma}{\beta} m_{zy} = \frac{\partial \nu}{\partial y}, \qquad m_{xz} = \frac{\gamma}{\beta} m_{zx} = \frac{\partial \nu}{\partial x},$$
$$m_{xx} = m_{yy} = m_{zz} = m_{xy} = m_{yx} = 0$$

where

$$f = \frac{k^2(\mu + k)}{\gamma V_0^2 \rho^2},\ g = \frac{2k(\mu + k)^2}{\gamma V_0^2 \rho^2},\ h = \frac{J}{\gamma}(\mu + k),\ \eta = \frac{g}{L} = \frac{2k_0}{1 + k_0},\ k_0 = 1 + \frac{k}{\mu} \ \text{and} \ L = g - f. \tag{10}$$

3. METHOD OF SOLUTION

It is convenient to write the velocities $u(x,y,t)$ and $v(x,y,t)$ in terms of two real functions $\phi(x,y,t)$ and $\psi(x,y,t)$ in the form

$$u = \frac{\partial \psi}{\partial y} + \frac{\partial \phi}{\partial x}, \qquad v = \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x} \tag{11}$$

Inserting (11) in the system (8), we have

$$\nabla^2 \phi = 0, \qquad p = -\frac{\partial \phi}{\partial t}, \qquad \nabla^2 \psi - \frac{\partial \psi}{\partial t} + \nu = 0,$$
$$\nabla^2 \nu - f\nabla^2 \psi - g\nu = h\frac{\partial \nu}{\partial t}. \tag{12}$$

while the boundary conditions (3) become

$$\frac{\partial \psi}{\partial y} + \frac{\partial \phi}{\partial x}\Big|_{y=0} = u_0(x,t),\ \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x}\Big|_{y=0} = 0, \qquad \nu(x,0,t) = 0 \tag{13}$$

and we can assume that each of $\phi(x,y,t), \psi(x,y,t)$ *and* $\nu(x,y,t)$ tends to zero as $y \to \infty$.
also

$$\phi(x,y,0) = \psi(x,y,0) = \nu(x,y,0) = 0. \tag{14}$$

Thus in the upper half –plane y>0 , we need to solve the system (12)with conditions (13)-(14).

Taking Laplace transform with parameter s of the system (11)-(12) and using the initial conditions (14),we obtain

$$\bar{u} = \frac{\partial \bar{\psi}}{\partial y} + \frac{\partial \bar{\phi}}{\partial x}, \quad \bar{v} = \frac{\partial \bar{\phi}}{\partial y} - \frac{\partial \bar{\psi}}{\partial x}, \quad \bar{p} = -s\bar{\phi}$$
$$\nabla^2 \bar{\phi}(x,y,s) = 0, \qquad \nabla^2 \bar{\psi} - s\bar{\psi} + \bar{\nu} = 0,$$

$$\nabla^2\bar{v} - f\nabla^2\bar{\psi} - (g + hs)\bar{v} = 0 , \tag{15}$$

where

$$\bar{F}(x, y, s) = \int_0^\infty e^{-st} F(x, y, t)dt, \tag{16}$$

$$F(x, y, t) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} e^{st}\bar{F}(x, y, s)ds, \qquad \sigma = Res > 0 \tag{17}$$

We now introduce the exponential Fourier transform (denoted by an asterisk) with respect to the coordinate x, defined by

$$\bar{F}^*(q, y, s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-iqx}\, \bar{F}(x, y, s)dx \tag{18}$$

with its corresponding inversion formula

$$\bar{F}(x, y, s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{iqx}\bar{F}^*(q, y, s)dq. \tag{19}$$

Taking the Fourier transform of the system (15),we obtain

$$(D^2 - q^2)\bar{\phi}^*(q, y, s) = 0, \qquad D^2\bar{\psi}^* - (q^2 + s)\bar{\psi}^* + \bar{v}^* = 0,$$

$$D^2\bar{v}^* - (q^2 + hs + g)\bar{v}^* - f(D^2 - q^2)\bar{\psi}^* = 0 \tag{20}$$

where $D = \frac{d}{dy}$

with conditions

$$D\bar{\psi}^* + iq\bar{\phi}^*|_{y=0} = \bar{u}_o^*(q, s) , \qquad D\bar{\phi}^* - iq\bar{\psi}^*|_{y=0} = 0 , \qquad \bar{v}^*(q, 0, s) = 0,$$

$$\bar{\phi}^*(q, \infty, s) = \bar{\psi}^*(q, \infty, s) = \bar{v}^*(q, \infty, s) = 0 \tag{21}$$

and

$$\bar{u}_0^*(q, s) = \sqrt{\frac{2}{\pi}}\frac{sinaq}{qs} , \tag{22}$$

H(a-|x|)=1, -a$\leq x \leq$ a , $H(\text{a} - |x|) = 0 , |x| > a$.

Now we may take as a trail solution

$$\bar{\psi}^* = Ae^{-ry} , \qquad \bar{v}^* = Be^{-ry} \qquad \text{and} \qquad \bar{\phi}^* = Ce^{-|q|y}$$

By substituting in (20) we get

$$(r^2 - q^2 - s)A + B = 0,$$

$$-f(r^2 - q^2)A + (r^2 - q^2 - g - hs)B = 0.$$

Then the condition for a nonzero solution is

$$(r^2 - q^2)^2 - [(h + 1)s + g - f](r^2 - q^2) + s(hs + g) = 0 . \tag{23}$$

We confine our study to a special class of fluid in which the microinertia coefficient J is given by

$$J = \frac{2\gamma}{2\mu+k} \qquad \text{or} \qquad h = \eta = \frac{2(1+n)}{2+n} . \quad n = \frac{k}{\mu} \geq 0 \tag{24}$$

Then the roots of (23) become

$$r_1^2 = q^2 + s + \zeta(\eta - 1) \qquad ,r_2^2 = q^2 + \eta s . \tag{25}$$

where,

$$\zeta = \frac{(1+n)(2+n)^2}{\Lambda} \qquad \text{and} \qquad \Lambda = \frac{\gamma\rho^2 V_0^2}{\mu^3} . \tag{26}$$

Thus after some calculations we obtain the solution of the system (20) with conditions (21)and (22)in the form

$$\bar{\phi}^*(q, s) = i\sqrt{\frac{2}{\pi}}\frac{.e^{-|q|y}(\zeta - s)}{|q|s\Omega(|q|, s)}sinaq ,$$

$$\bar{\psi}^*(q, s) = \sqrt{\frac{2}{\pi}}\frac{se^{-r_1y} - \zeta e^{-r_2y}}{qs\Omega(|q|, s)}sinaq , \tag{27}$$

$$\bar{v}^*(q, s) = \sqrt{\frac{2}{\pi}}\frac{\zeta(\eta - 1)(e^{-r_2y} - e^{-r_1y})}{q\Omega(|q|, s)}sinaq.$$

where

$$\Omega(q, s) = \zeta(r_2 - q) - s(r_1 - q) . \tag{28}$$

Using the inversion formula (19), (14) and (27)-(28) we get

$$\bar{u}(x, y, s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{iqx}\, U(q, y, s)dq$$

$$\bar{v}(x, y, s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{iqx}\, V(q, y, s)dq , \tag{29}$$

$$\bar{v}(x, y, s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{iqx}\ \ W(q, y, s)dq,$$

$$\bar{p}(x, y, s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{iqx}\, P(q, y, s)dq,$$

Where $U(q,y,s) = \frac{2sinaq}{qs\Omega(|q|,s)}\left[-sr_1 e^{-r_1 y} + \zeta r_2 e^{-r_2 y} + (s-\zeta)|q|e^{-|q|y}\right]$,

$V(q,y.s) = \frac{2sinaq}{is\Omega(|q|,s)}\left[se^{-r_1 y} - \zeta e^{-r_2 y} + (\zeta - s)e^{-|q|y}\right]$

(30) $W(q,y,s) = \frac{2\zeta(\eta-1)sinq}{q\Omega(|q|,s)}(e^{-r_2 y} - e^{-r_1 y})$,

$P(q,y,s) = \frac{2sinaq(\zeta - s)}{i|q|\Omega(|q|,s)}e^{-|q|y}$.

Noting that $U(q,y,s)$and $W(q,y,s)$are even functions in q, while $V(q,y,s)$ and $P(q,y,s)$ are odd functions in q. Using the inverse Laplace transform (17),we have

$$u(x,y,t) = \frac{e^{\sigma t}}{2\pi}\int_{-\infty}^{\infty} e^{i\xi t}\,\bar{u}(x,y,\sigma + i\xi)d\xi, \tag{31}$$

Similar relations could be written for the functions $v,\bar{v}$;$v,\bar{v}$; p ,$\bar{p}$.

Finally the total velocity $c(x,y,t)$ and the stream function $\psi(x,y,t)$ can be calculated from

$$c(x,y,t) = \sqrt{u(x,y,t)^2 + v(x,y,t)^2},$$

$$u(x,y,t) = \frac{\partial\psi(x,y,t)}{\partial y}, \quad v(x,y,t) = -\frac{\partial\psi(x,y,t)}{\partial x}, \tag{32}$$

$$\psi(x,y,t) = \frac{e^{\sigma t}}{2\pi}\int_{-\infty}^{\infty} e^{i\xi t}\,\bar{\psi}(x,y,\sigma + i\xi)d\xi, \tag{33}$$

$$\overline{\Psi}(x,y,s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{iqx}\,\mathrm{E}(q,y,s)dq,$$

$$\mathrm{E}(q,y,s) = -\frac{2\sin aq}{qs\Omega(|q|,s)}\left[\zeta e^{-r_2 y} - se^{-r_1 y} + (s-\zeta)e^{-|q|y}\right].$$

Since the function $\Omega(q,s)$ can be take the form

$$\frac{1}{\Omega(q,s)} = \frac{\mathrm{f}^2\Omega(-q,s)}{\Delta(q,s)}Q(q,s),$$

where

$$\mathrm{Q(q\,,s)}=(\zeta r_2 + sr_1)^2 - q^2(\zeta - s)^2,$$

$$\Delta(q,s) = s^2(s-\zeta)^2[s^2 f^2 + 2fgLs + L^2(g^2 + 4fq^2)],$$

then $s = \zeta$ is the only root of the function $\Omega(q,s)$ lies in the right half-plane $Re\ s > 0$, we can take $\sigma > \zeta$ and the relation (32)and its similar can be put in the form

$$w(x,y,t) = \frac{e^{\sigma t}}{2\pi}\sum_{j=-\infty}^{\infty} e^{i\Delta\xi tj}\,\bar{w}(x,y,\sigma + i\Delta\xi j)\Delta\xi. \tag{34}$$

4. NUMERICAL CALCULATIONS

For numerical calculations, the formula (34) can be approximated in the form

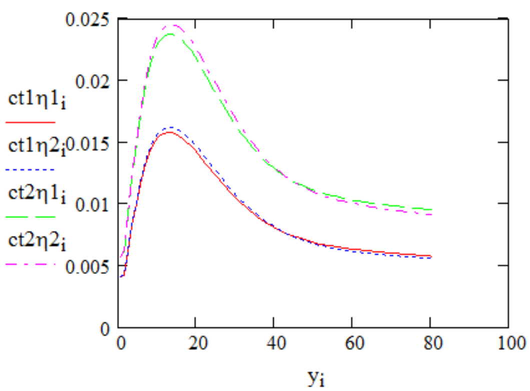
$$w_N(x,y,t) = \frac{e^{\sigma t}}{T}\left[\frac{1}{2}\bar{w}(x,y,\sigma) + Re\left\{\sum_{j=1}^{N} e^{\frac{i\pi tj}{T}}\bar{w}(x,y,\sigma + \frac{i\pi j}{T})\right\}\right], \tag{35}$$

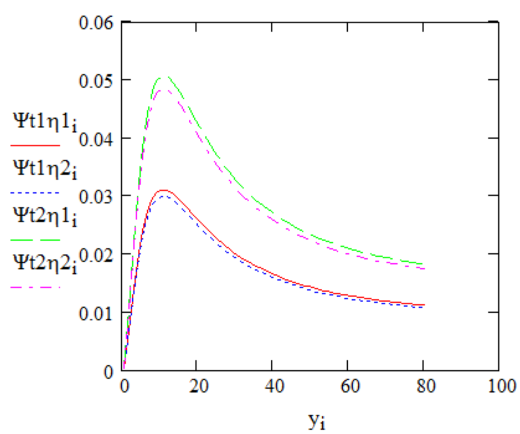
where,

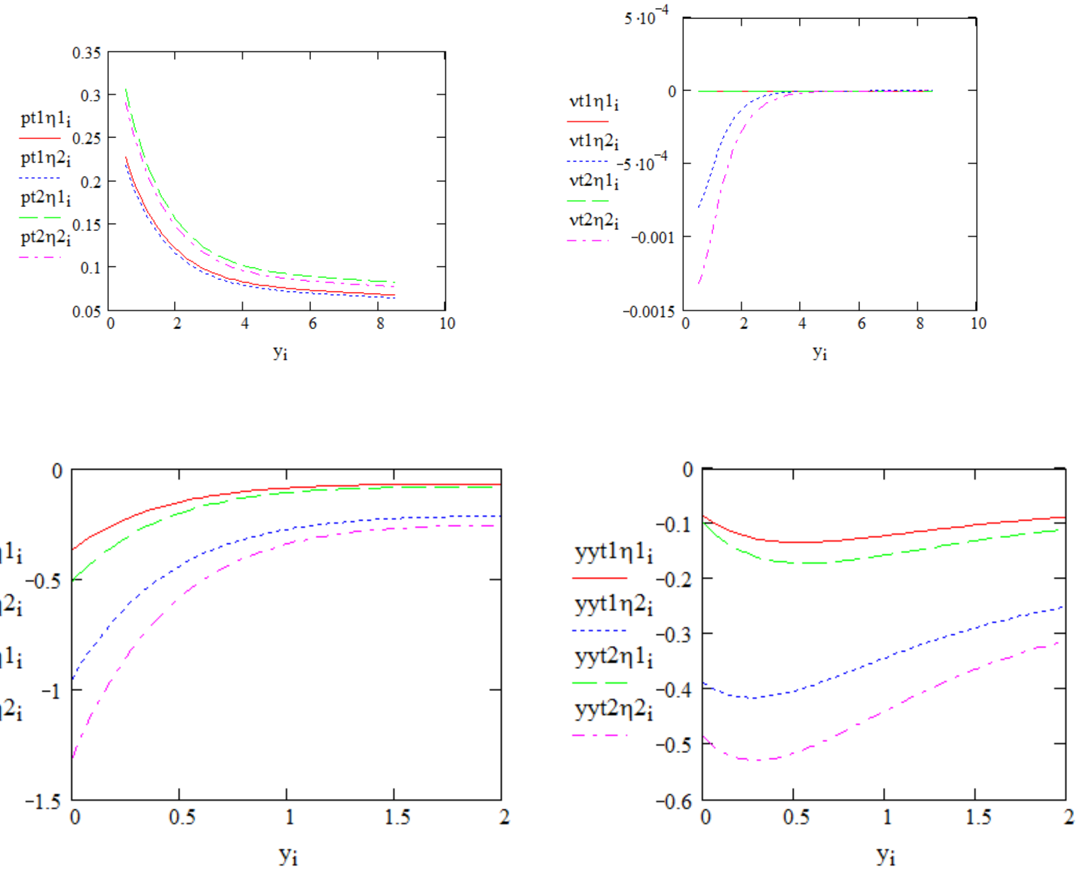
$$T = \pi/\Delta\xi, \qquad 0 \le t \le 2T$$

and we choose N to be a sufficiently large integer, such that

$$\frac{e^{\sigma t}}{T}Re\left\{e^{\frac{i\pi Nt}{T}}\bar{w}(x,y,\sigma + i\frac{i\pi N}{T})\right\} < \epsilon,$$



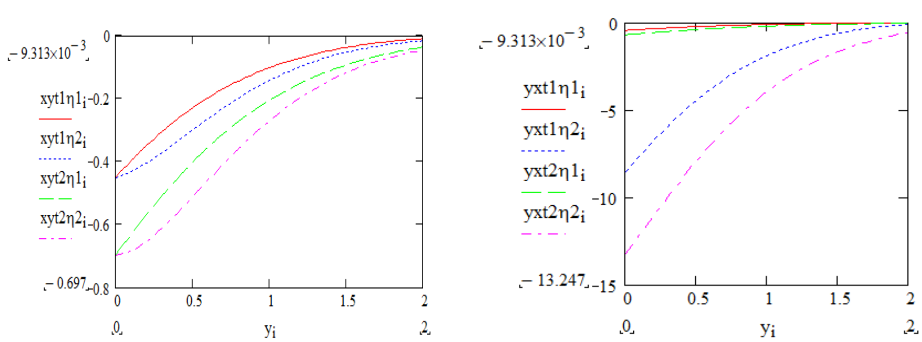


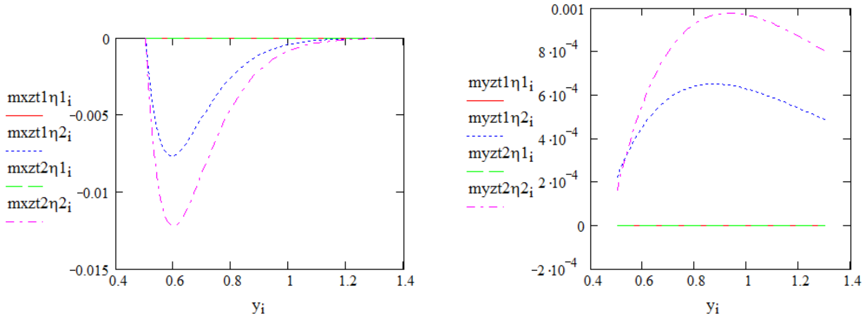


Distribution of absolute velocity c, stream function ψ , pressure p, stresses $\widehat{xx}$, $\widehat{yy}$ at $t_1 = 1, t_2 = 1.5$ for $\eta_1 = 1$ (Newtonian fluid) and $\eta_2 = 1.9$ (micropolar fluid)

`` Figure 1"

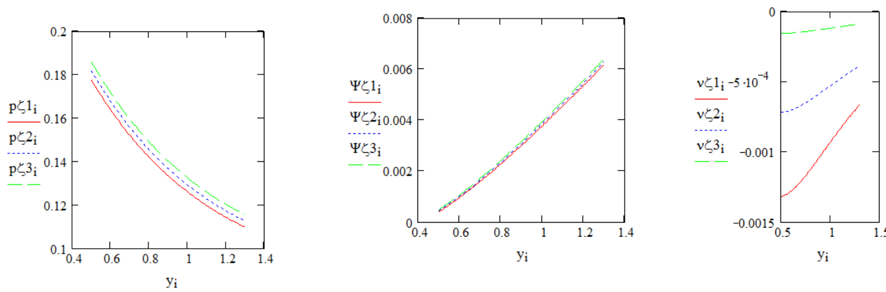
where $\epsilon > 0$ is chosen to be small enough according to the degree of accuracy required. The micropolar incompressible fluid can be chosen with the parameter $1 \le \eta(n) < 2$ and $\zeta(n, \Lambda) = 1$. The computations of the total velocity $c(x, y, t)$,the stream function $\psi(x, y, t)$, the pressure$p(x, y, t)$ microrotation $\nu(x, y, t)$, the components of stresses $\widehat{xy}$, $\widehat{yx}$,$\widehat{xx}, \widehat{yy}$ and couples m_{yz}, m_{xz} were performed as a function of y for the values $\zeta(n, \Lambda) = 1$, $x = 10, a = 1$,`` Figure 1"-``Figure 2". These figures give a comparison of the behaviour of Newtonian fluid($\eta 1 = 1$) and the corresponding micropolar fluid ($\eta 2 = 1.9$) at two different values of $t(t1 = 1, t2 = 1.5)$. ``Figures3" show the pressure, the stream function and the microrotation at $t = 1.5, \eta = 1.9$ for different values of ζ ($\zeta 1 = 1, \zeta 2 = .5, \zeta 3 = .1$). The variation of the horizontal velocity , the vertical velocity, the stream function , the pressure and the microrotation are conveniently represented in the form of contour plots ``Figure 4-a"- ``Figure 8-a" and $3 - D$ surface plot plots``Figure 4-b"- ``Figure 8-b" at $t = 1$ and $\eta = 1.5, \zeta = 1$. All figures were computed using the Mathcad program on an IBM Pc. Each contour plot and 3-D surface plot was needed 3200 mesh points on the average.





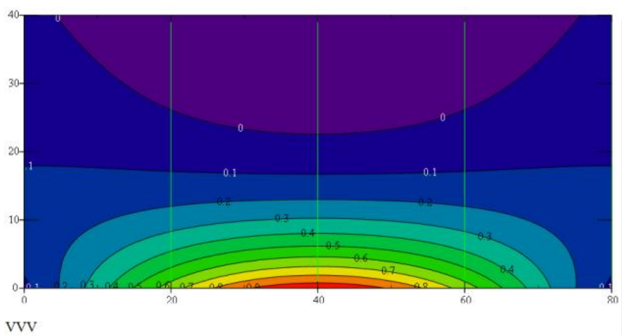
Distribution of shear stresses $\widehat{xy}$, $\widehat{yx}$, couples , m_{xz}, m_{yz} at $t_1 = 1, t_2 = 1.5$ for $\eta_1 = 1$ (Newtonian fluid) and $\eta_2 = 1.9$ (micropolar fluid)

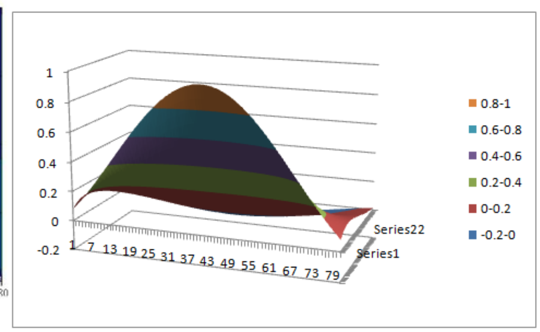
`` Figure 2"



Distribution of the pressure p , the stream function ψ and the microrotation ν at $t = 1.5, \eta = 1.9$ for different values of ζ ($\zeta 1 = 1, \zeta 2 = .5, \zeta 3 = .1$).

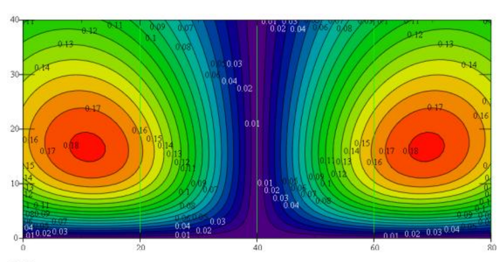
`` Figure 3"

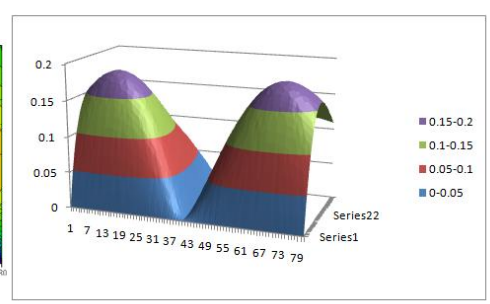




horizontal velocity u(x,y,1)

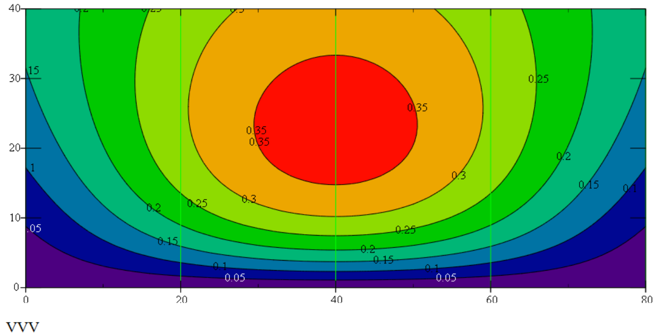
``Figure 4-a"** **`` Figure 4-b"

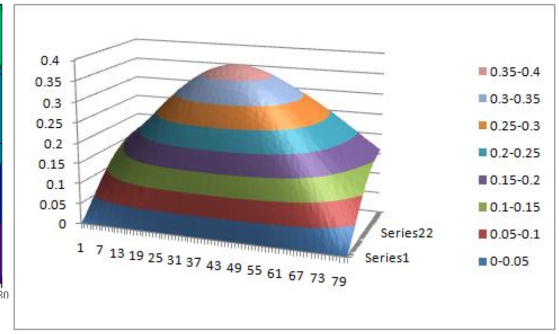




Vertical velocity v(x,y,1)

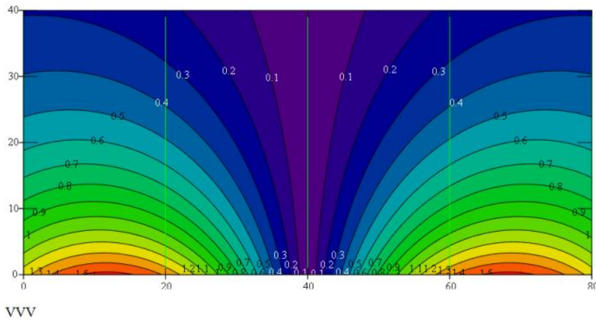
``Figure5-a"** **``Figure5-b"

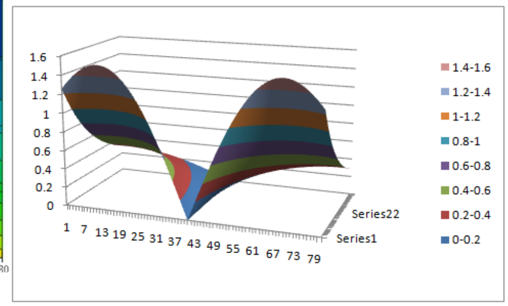




Stream function $\psi(x, y, 1)$

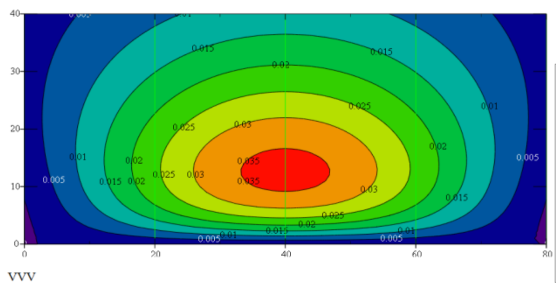
``Figure 6-a"** **``Figure 6-b"

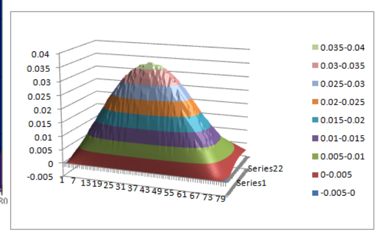




Pressure $p(x, y, 1)$

``Figure7-a"** **``Figure7-b"





Microrotation $\nu(x, y, 1)$

``Figure 8-a"** **``Figure 8-b"

5. COMPARISON WITH EXACT SOLUTION

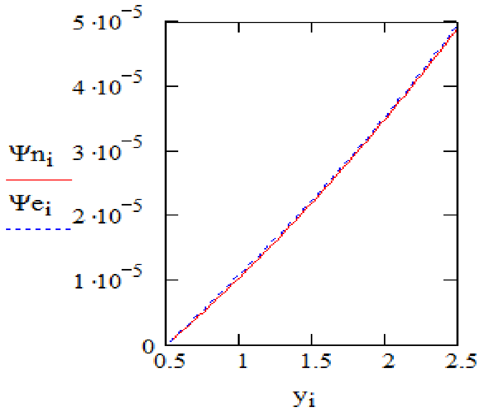
It is worthy of mention that the exact solution of the system (8) and (32) for the special case $k = 0 \;\; or \, \eta = 1$ which correspond to Newtonian fluid, is given by [8]

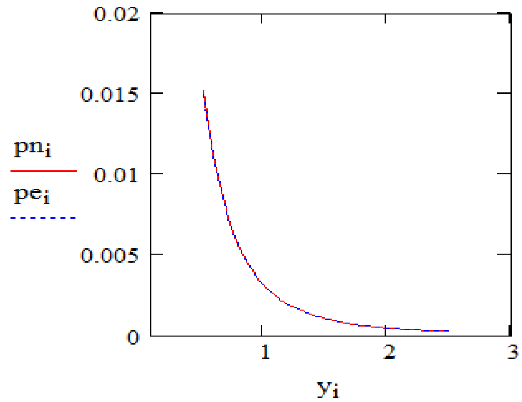
$$\psi(x, y, t) = \int_0^t \int_{-a}^a \Im m\{G(x-\xi, y, \tau)\} d\xi d\tau \; , \; p(x, y, t) = \frac{1}{\pi} \mathcal{R}e \left\{ \int_{-a}^a \left[\frac{\iota}{(z-\xi)^2} + \frac{\iota e^{-Z^2} erfc\,(-\iota Z)}{(z-\xi)^2} + \frac{1}{2\sqrt{\pi t} Z} \right] d\xi \right\}$$

,where

$$G(x-\xi, y, t) = \frac{1}{4\pi t} \left\{ \frac{2}{\sqrt{\pi} Z} \left(e^{-Y^2} - 1 \right) - \frac{\iota \Phi(Y)}{Z^2} + \iota \left(\frac{1}{Z^2} - \frac{2\iota Y}{Z} \right) e^{-|Z|^2} erfc(-\iota X) - \frac{\iota}{Z^2} e^{-Z^2} erfc(-\iota Z) \right\},$$

$$Z = X + \iota Y = \frac{z-\xi}{2\sqrt{t}} \;\; , \; 1 - erfc(Z) = \Phi(Z) = \frac{2}{\sqrt{\pi}} \int_0^Z e^{-\lambda^2} d\lambda = \frac{2}{\sqrt{\pi}} \sum_{j=0}^{\infty} \frac{(-1)^j Z^{2j+1}}{j!(2j+1)} \; , \; |Z| < \infty \, .$$





The comparison of the numerical solution $\psi_n(1.85, y, 2.5\ )$, $p_n(.29, y, 2.5)$and exact solution $\psi_e(1.85, y, 2.5)$, $p_e(.29, y, 2.5)$ respectively at $\zeta = .242\ \ , \eta = 1$ (Newtonian fluid)

``Figure 9

``Figure 9" shows the comparison of the numerical, exact solutions of the stream functions ψ_n, ψ_e for $x = 1.85$, and the numerical, exact solutions of the pressures $p_n p_e$ for x=.29 at $\zeta = .242\ \ , \eta = 1, t = 2.5$.

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