SLIP-FLOW HEAT TRANSFER IN CIRCULAR MICROCHANNEL WITH VISCOUS DISSIPATION

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ABSTRACT

This paper presents an analytical solution for the Graetz problem extended to slip flow that includes rarefaction effect. The hydrodynamically developed flow is assumed to enter a circular microchannel with uniform wall temperature. The effect of velocity slip, temperature jump and viscous dissipation term are all considered. The effect of nondimensional parameters (Knudsen number, Prandtl number, Brinkman number) on local and fully developed Nusselt number is investigated. The results show that under certain conditions the viscous dissipation effect on heat transfer in microchannels is significant and should not be neglected.

Keywords: *Microchannel, Graetz Problem, Slip flow, Viscous dissipation, Heat transfer*

1. INTRODUCTION

In recent decades, progress in micro fabrication technology has lead to development in a variety of micro scale fluidic systems. Micro devices are generally referred to devices having a characteristic length scale between 1 mm and 1 μm such as microchannel heat exchangers, micropumps, micro actuators, etc. The extensive engineering applications of microdevices have provoked researchers to study on its fluid flow and heat transfer characteristics.

Modeling heat and fluid flow through such small devices is different from the macro scales counterparts. As the ratio of the mean free path to characteristic length (Knudson number, $Kn = \frac{\lambda}{L}$) increases, the continuum assumption becomes no longer valid [1,2]. For the rarefaction varying between 0.001 and 0.1 the regime is called slip-flow regime [3]. Under such circumstances the continuum modeling along with the velocity slip and temperature jump boundary conditions on the wall need to be considered.

The hydrodynamically developed and thermally developing laminar flow entering a circular tube with constant wall temperature known as Graetz problem was analytically solved by Graetz [4,5]. Many researchers developed the problem by considering different boundary conditions and different cross sections or by including the axial conduction and viscous dissipation. By increasing the applications of microchannels, their dominating boundary conditions were applied to solve Graetz problem. Barron et al. [6] extended the problem by including the effect of slip velocity. Ameel et al. [7] presented an analytical solution with constant wall heat flux in circular microchannel. Tunc and Bayazitoglu [8] solved the problem by considering slip velocity, temperature jump and viscous dissipation effect with uniform wall temperature and uniform heat flux boundary conditions. Jeong and Jeong [9] solved the energy equation with viscous dissipation and axial conduction terms in parallel plates considering both uniform temperature and uniform heat flux boundary conditions. Cetin et al. [10] presented a numerical solution by considering viscous dissipation and axial conduction with constant wall temperature in microchannel with circular cross section. Discrepancies in the fully developed Nusselt numbers can be seen in the results of Tunc and Bayazitoglu [8] (analytical solution) and Cetin et al. [10] (numerical solution). Cetin et al. [11] also solved the Graetz problem analytically with constant wall heat flux in microtubes including the effect of viscous dissipation and axial conduction.

This paper extends the Graetz problem to include the rarefaction effect and viscous dissipation term in the fluid with constant wall temperature boundary condition. The energy equation is analytically solved by the method of separation of variables. By defining appropriate nondimensional variables the energy equation becomes a well-known differential equation which is known as Kummer. The influence of viscous dissipation on Nusselt number in both cases where the fluid is being cooled or heated is thoroughly discussed.

Nomenclature			
		Nu	Nusselt number
		D	channel diameter
T	temperature		
r_0	channel radius	*Greek symbols*	
r	radius	μ	dynamic viscosity
r^*	dimensionless radius, (r/R)	γ	ratio of specific heats, (c_p/c_v)

F_t	thermal accommodation coefficient	α	thermal diffusivity, $(k/\rho c_p)$
Pr	Prandtl number, (ν/α)	λ	mean free path ; eigenvalue
x	distance along tube	θ	dimensionless temperature, $(T-T_w)/(T_0-T_w)$
x^*	dimensionless distance along tube, (x/L)		
u^*	dimensionless velocity, (u/u_m)	*Subscript*	
u	velocity in x direction	s	slip flow
k	thermal conductivity	w	wall
h_x	local heat transfer coefficient	0	at $x=0$
Nu_x	local Nusselt number	B	bulk
Kn	Knudsen number	m	mean
Re	Reynolds number	∞	infinity
Br	Brinkman number	d	Developing

2. ANALYSIS

The steady state hydrodynamically developed flow with inlet temperature T_0 enters into a microtube with radius r_0 and wall temperature T_w. The dominating energy equation may be written as:

$$u/_\alpha \frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial r^2} + 1/_r \frac{\partial T}{\partial r} + \frac{\mu}{k}\left(\frac{\partial u}{\partial r}\right)^2 \tag{1}$$

Boundary conditions are:

$$\frac{\partial T}{\partial r} = 0 \quad at \quad r = 0 \tag{2a}$$

$$T = T_0 \quad at \quad x = 0 \tag{2b}$$

$$T_s - T_w = \frac{F_t - 2}{F_t}\frac{\lambda}{Pr}\frac{2\gamma}{1+\gamma}\frac{\partial T}{\partial r}|_{r=r_0} \tag{2c}$$

where T_s is the temperature of the fluid at the wall, T_w the wall temperature, F_t is the thermal accommodation coefficient and γ is the specific heat ratio. The fully developed velocity profile can be derived from momentum equation by applying slip velocity boundary condition:

$$u = 2u_m \frac{\left(1 - \frac{r^2}{{r_0}^2} + 4Kn\right)}{(1+8Kn)} \tag{3}$$

Appropriate non-dimensional variables may be defined as:

$$r^* = \frac{r}{r_\circ B} \qquad x^* = \frac{x/_{r_\circ B^3}}{Re\,Pr} \qquad u^* = \frac{u}{u_m} \qquad \theta = \frac{T - T_w}{T_\circ - T_w} \tag{4}$$

Where

$$B = 1 + 8Kn \tag{5}$$

By applying non-dimensional variables and substituting Eq. (3) into Eq. (1), the energy equation can be written as:

$$\left(A - {r^*}^2\right)\frac{\partial \theta}{\partial x^*} = \frac{\partial^2\theta}{\partial {r^*}^2} + \frac{1}{r^*}\frac{\partial\theta}{\partial r^*} + Br\left(\frac{\partial u^*}{\partial r^*}\right)^2 \qquad A = \frac{1+4Kn}{B^2} \tag{6}$$

$$\frac{\partial\theta}{\partial r^*} = 0 \quad at \quad r^* = 0 \tag{7a}$$

$$\theta = 1 \quad at \quad x^* = 0 \tag{7b}$$

$$\theta_s = \frac{T_s - T_w}{T_\circ - T_w} = \frac{F_t - 2}{F_t}\frac{Kn}{Pr}\frac{4\gamma}{1+\gamma}\frac{1}{B}\frac{\partial\theta}{\partial r^*}|_{r^*=1/_B} \tag{7c}$$

As $x \to \infty$ then $\frac{\partial\theta}{\partial x^*} \to 0$, therefore Eq. (6) becomes

$$\frac{\partial^2\theta}{\partial {r^*}^2} + \frac{1}{r^*}\frac{\partial\theta}{\partial r^*} + Br\left(\frac{\partial u^*}{\partial r^*}\right)^2 = 0 \tag{8}$$

The fully developed temperature profile can be derived from Eq. (8) by applying Eq. (7a) and Eq. (7c) as boundary conditions:

$$\theta_\infty = -BrB^2{r^*}^4 + \frac{Br}{B^2}\left(8\frac{Kn}{Pr}\frac{2\gamma}{1+\gamma} + 1\right) \tag{9}$$

Therefore

$$\theta(x^*, r^*) = \theta_d(x^*, r^*) + \theta_\infty(r^*) \tag{10}$$

By substituting Eq. (10) into Eq. (6) we obtain:

$$\left(A - {r^*}^2\right)\frac{\partial\theta_d}{\partial x^*} = \frac{\partial^2\theta_d}{\partial {r^*}^2} + \frac{1}{r^*}\frac{\partial\theta_d}{\partial r^*} \tag{11}$$

By applying the method of separation of variables we have:

Assuming: $\theta_d(x^*, r^*) = X(x^*)R(r^*)$

$$X' + \lambda^2 X = 0 \tag{12}$$

$$R'' + \frac{1}{r^*}R' + \lambda^2\left(A - {r^*}^2\right)R = 0 \tag{13}$$

The solution for Eq. (12) is:

$$X(x^*) = Cexp(-\lambda^2 x^*) \tag{14}$$

By defining the following transformation:

$$P = \lambda {r^*}^2 \qquad W_{(P)} = e^{\frac{P}{2}} R_{(r^*)} \tag{15}$$

Eq. (13) may be written as:

$$P\frac{\partial^2 W}{\partial P^2} + (1-P)\frac{\partial W}{\partial P} + \left(\frac{\lambda A}{4} - \frac{1}{2}\right)W = 0 \tag{16}$$

Eq. (16) is known as Kummer equation and the solution is:

$$W_{(P)} = cM\left(\frac{1}{2} - \frac{\lambda A}{4}, 1, P\right) \tag{17}$$

Where $M(a, b, p)$ is written in the form of following series:

$$M(a, b, P) = 1 + \frac{a}{b}P + \frac{a(a+1)}{b(b+1)}\frac{P^2}{2!} + \cdots + \frac{a(a+1)\ldots(a+n-1)}{b(b+1)\ldots(b+n-1)}\frac{P^n}{n!} + \cdots \tag{18}$$

By substituting $\theta(r^*, x^*) = R(r^*)X(x^*)$ into Eq. (7c) we obtain:

$$R(r^*)|_{r^*={}^1/_B} = \frac{F_t - 2}{F_t}\frac{Kn}{Pr}\frac{4\gamma}{1+\gamma}\frac{1}{B}\frac{\partial R}{\partial r^*}|_{r^*={}^1/_B} \tag{19}$$

By solving Eq. (19) we can find λ_n.

Therefore the solution for the temperature distribution may be written as:

$$\theta_d(x^*, r^*) = \sum_{n=1}^{\infty} Z_n e^{-{\lambda_n}^2 x^*} R_n(r^*) \tag{20}$$

As the Eq. (13) is Sturm-Liouville equation and the eigenfunctions are orthogonal with respect to weighting function, by applying Eq. (7b) Z_n can be written as:

$$Z_n = \frac{\int_0^{1/B} R_n(r^*)\left(A - {r^*}^2\right)r^* dr^*}{\int_0^{1/B} {R_n}^2(r^*)\left(A - {r^*}^2\right)r^* dr^*} \tag{21}$$

The average temperature in the circular tube can be written as:

$$T_B = \frac{2}{u_m {r_0}^2}\int_0^{r_0} uTr\, dr \tag{22}$$

Putting the equation (22) into non-dimensional form, the dimensionless bulk temperature may be written as:

$$\theta_B = 2B^2 \int_0^{1/B} u^* \theta(x^*, r^*) r^*\, dr^* \tag{23}$$

The heat flux at the wall may be written as:

$$-k\left(\frac{\partial T}{\partial r}\right)_{r=r_0} = h_x(T_B - T_w) \tag{24}$$

by putting Eq. (24) into non-dimensional form we obtain:

$$h_x = -\frac{2k}{DB\theta_B}\left(\frac{\partial \theta}{\partial r^*}\right)_{r^*={}^1/_B} \tag{25}$$

The local Nusselt number can be written as:

$$Nu_x = -\frac{2}{B\theta_B}\left(\frac{\partial \theta}{\partial r^*}\right)_{r^*={}^1/_B} \tag{26}$$

3. RESULTS AND DISCUSSION

All the calculations have been carried out by assuming $\gamma = 1.4$ and $F_t = 1$. In Figure 1 the effect of Prandtl number on fully developed Nusselt number in different Knudsen number is presented. It is shown that by increasing Prandtl number the fully developed Nusselt number increases. When $Kn = 0$, the variation of Prandtl number does not have effect on Nusselt number. It is also evident that in lower Prandtl numbers fully developed Nusselt number changes more by variation of rarefaction.

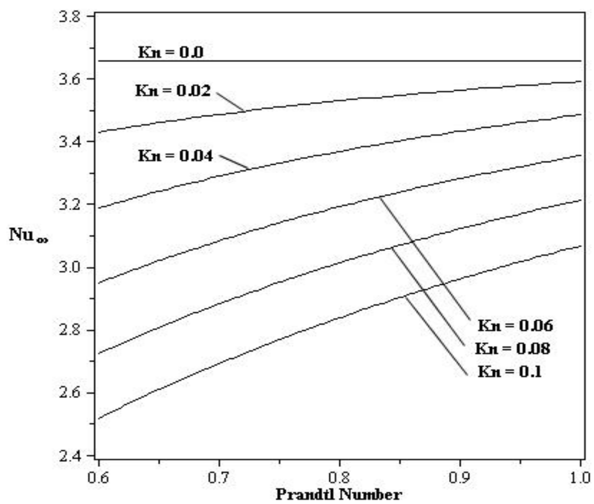

Figure 1. *Variation of fully developed Nu as a function of Pr for different Kn and Br=0*

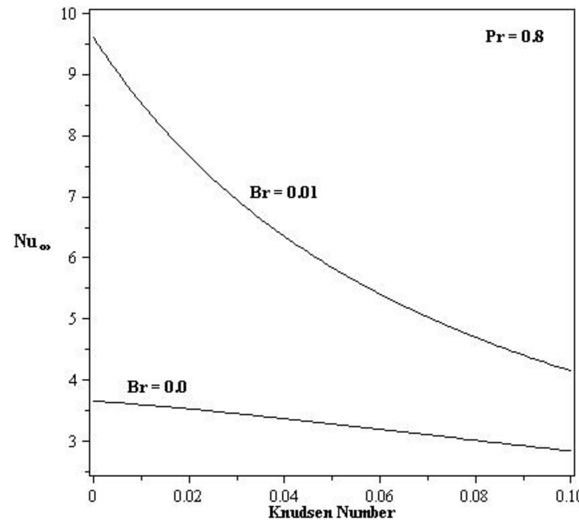

Figure 2. *Variation of fully developed Nu as a function of Kn with and without viscous dissipation (Pr=0.8)*

In Figure 2 the variation of fully developed Nusselt number by different rarefactions with and without viscous dissipation is given. It is seen that by increasing Knudsen number the fully developed Nusselt number decreases and it has more decreasing effect when viscous dissipation is taken into account.
In Figure 3 the solid lines shows the variation of Nusselt number in the length of tube at $Br = 0$. It is clear that by increasing Knudsen number the fully developed Nusselt number will decrease. By considering the influence of viscous dissipation, it can be seen that a jump occur in Nusselt profile and the fully developed Nusselt number increases. Neglecting viscous dissipation, the fully developed Nusselt number decreases from 3.656 to 3.069 by increasing Knudsen from 0 to 0.1 that is about 16% decrease in Nusselt number while in the presence of viscous dissipation the fully developed Nusselt number decreases about 50%.

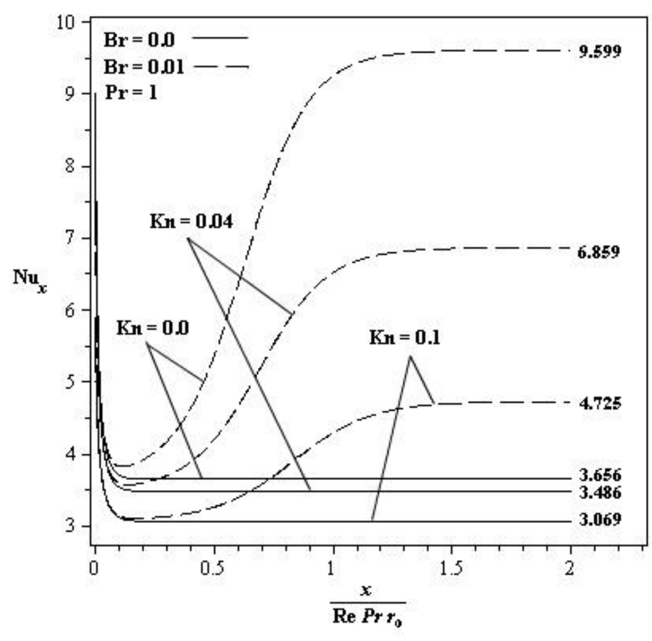

Figure 3. *Variation of the local Nu as a function of dimensionless axial coordinate for different Kn and Br (Pr=1)*

The effect of Brinkman number on Nusselt number is depicted in Figure 4. It is found that the Nusselt number in developing region has greater values for greater Brinkman numbers. It is shown that the fully developed Nusselt number increases by considering the effect of viscous dissipation. When the Brinkman number is not equal to zero which means the viscous dissipation is present, variation of Brinkman number does not have effect on fully developed Nusselt number.

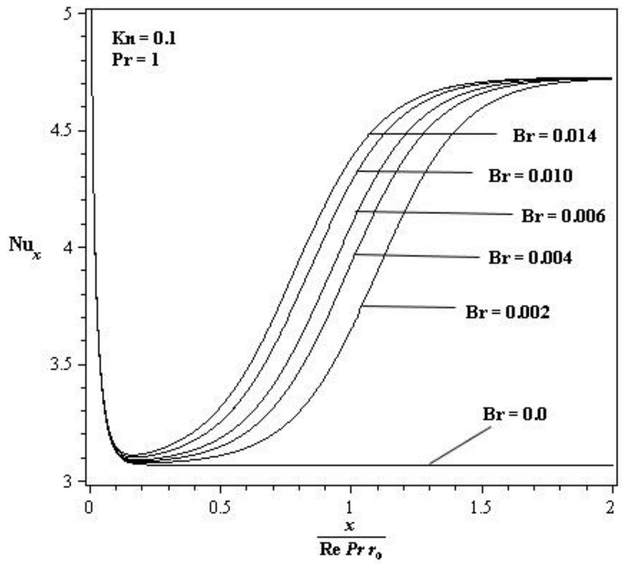

Figure 4. *Variation of the local Nu as a function of dimensionless axial coordinate for different Br (Kn=0.1, Pr=1)*

Figure 5 illustrates the variation of Nusselt with different Brinkman numbers. Fully developed Nusselt number reaches a same value for both positive and negative Brinkman numbers. When Brinkman is negative the fluid is being heated and therefore there is a location in the length of tube that the bulk temperature is equal to the wall temperature. There exists a singular point where Nusselt number goes to the infinity.

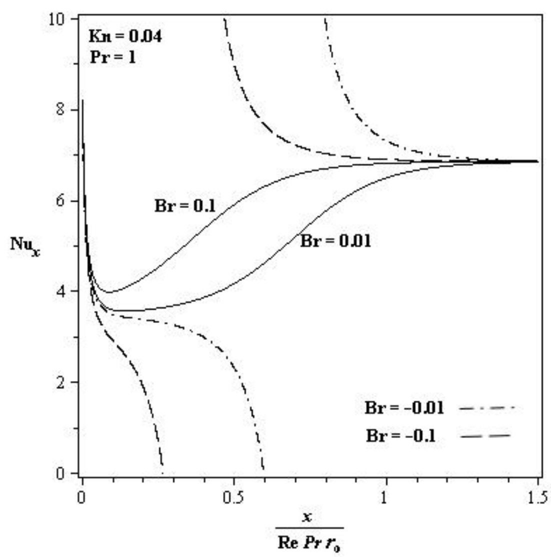

Figure 5. *Variation of the local Nu as a function of axial coordinate for positive and negative Br (Kn=0.4, Pr=1)*

Table 1 and Table 2 show the fully developed Nusselt number for different values of Knudsen, Prandtl and Brinkman.

Table 1. The fully developed Nusselt number (Br = 0)

Kn	$Pr = 0.6$	$Pr = 0.7$	$Pr = 0.8$	$Pr = 0.9$	$Pr = 1$
0.00	3.6567	3.6567	3.6567	3.6567	3.6567
0.01	3.5476	3.5770	3.5992	3.6167	3.6307
0.02	3.4310	3.4871	3.5302	3.5643	3.5919
0.03	3.3104	3.3906	3.4528	3.5024	3.5429
0.04	3.1886	3.2900	3.3695	3.4335	3.4861
0.05	3.0678	3.1875	3.2824	3.3596	3.4234
0.06	2.9494	3.0848	3.1934	3.2823	3.3565
0.07	2.8348	2.9832	3.1036	3.2031	3.2866
0.08	2.7246	2.8837	3.0142	3.1230	3.2148
0.09	2.6192	2.7871	2.9261	3.0428	3.1422
0.1	2.5188	2.6937	2.8398	2.9634	3.0693

Table 2. The fully developed Nusselt number (Br = 0.01)

Kn	$Pr = 0.6$	$Pr = 0.7$	$Pr = 0.8$	$Pr = 0.9$	$Pr = 1$
0.00	9.5999	9.5999	9.5999	9.5999	9.5999
0.01	8.1921	8.3829	8.5319	8.6515	8.7496
0.02	7.1335	7.4279	7.6652	7.8604	8.0239
0.03	6.3104	6.6607	6.9500	7.1930	7.4000
0.04	5.6531	6.0319	6.3512	6.6239	6.8595
0.05	5.1167	5.5082	5.8434	6.1338	6.3878
0.06	4.6712	5.0656	5.4080	5.7081	5.9733
0.07	4.2955	4.6870	5.0309	5.3353	5.6068
0.08	3.9746	4.3597	4.7013	5.0065	5.2806
0.09	3.6976	4.0742	4.4112	4.7144	4.9888
0.1	3.4559	3.8230	4.1538	4.4536	4.7264

4. CONCLUSIONS

In this study an analytical solution for the Graetz problem in microchannels with circular cross section is presented. Viscous dissipation effect is taken into account. It is found that viscous dissipation causes a jump in Nusselt profile and the fully developed Nusselt number increases. In the presence of viscous dissipation variation of Brinkman number does not have effect on fully developed Nusselt number. It can be concluded that under certain conditions the influence of viscous dissipation on Nusselt number is significant and should not be neglected.

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