# PEAK TO AVERAGE POWER RATIO REDUCTION USING BANDWIDTH EFFICIENCY INCREASING METHOD IN OFDM SYSTEM

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**ABSTRACT**

An efficient scheme to reduce peak to average power ratio (PAPR) of OFDM signal is presented, which introduces bandwidth efficiency increasing method to OFDM. In other words the spacing of the sub-carriers in this new method is twice as dense when compared to conventional OFDM system. In this new method the frequency separation of the sub-carriers is halved compared to the conventional OFDM. It is evident from simulation results that the new technique has significantly reduced the PAPR for high data rate systems. This improvement in the systems performances have been maintained even when different combination modulation scheme and number of subcarrier were used. Using statistical hypotheses testing with confidence level 95%, we have sufficient evidence that the proposed method is better than the conventional OFDM system.

***Keywords*** **:** *OFDM, Peak to Average Power Ratio, Hypotheses Testing*

## 1. INTRODUCTION

OFDM is considered as a very promising candidate for future mobile communication systems. OFDM uses the Inverse Fast Fourier Transform (IFFT) operation to generate a large number of sub-channels that are orthogonal. A cyclic prefix is added in the time domain that simplifies equalization and also eliminates inter-block interference (IBI). OFDM is a widely used communication technique in broadband access applications requiring high data rates. It is already used in different WLAN standards (HIPERLAN 2, IEEE 802.11a), ADSL and digital video broadcasting (DVB).Even though OFDM has a number of advantages it has a potential drawback of high Peak to Average Power Ratio (PAPR) [1]. This high peak to average power ratio causes nonlinearities in the transmitted signal and also degrades the power efficiency of the system. In order to reduce the PAPR problem many researcher have made efforts and a large variety of different PAPR reduction approaches are proposed. An OFDM signal is basically a bundle of narrowband carriers transmitted in parallel at different frequencies from the same source. In fact, this modulation scheme is often termed “multicarrier” as opposed to conventional “single carrier” schemes. Each individual carrier, commonly called a subcarrier, transmits information by modulating the phase and possible the amplitude of the subcarrier over the symbol duration. That is, each subcarrier uses either phase-shift keying (PSK) or quadrature-amplitude-modulation (QAM) to convey information just as conventional single carrier systems. However, OFDM or multi-carrier systems use a large number of low symbol rate subcarriers. The spacing between these subcarriers is selected to be the inverse of the symbol duration so that each subcarrier is orthogonal or non-interfering. This is the smallest frequency spacing that can be used without creating interference. At first glance it might appear that OFDM systems must modulate and demodulate each subcarrier individually. Fortunately, the well-known Fast Fourier transform (FFT) provides designers with a highly efficient method for modulating and demodulating these parallel subcarriers as a group rather than individually [2].

## 2. ORTHOGONAL FREQUENCY DIVISON MULTIPLEXING

The OFDM concept is based on spreading the data to be transmitted over a large number of carriers, each being modulated at a low rate. The carriers are made orthogonal to each other by appropriately choosing the frequency spacing between them. In contrast to conventional Frequency Division Multiplexing, the spectral overlapping among sub-carriers are allowed in OFDM since orthogonality will ensure the subcarrier separation at the receiver, providing better spectral efficiency and the use of steep bandpass filter was eliminated [10]. OFDM transmission system offers possibilities for alleviating many of the problems encountered with single carrier systems. It has the advantage of spreading out a frequency selective fade over many symbols [4]. This effectively randomizes burst errors caused by fading or impulse interference so that instead of several adjacent symbols being completely destroyed, many symbols are only slightly distorted.  This allows successful reconstruction of majority of them even without forward error correction. Because of dividing an entire signal bandwidth into many narrow subbands, the frequency response over individual subbands is relatively flat due to subband are smaller than coherence bandwidth of the channel. Thus, equalization is potentially simpler than in a single carrier system and even equalization may be avoided altogether if differential encoding is implemented [7,8,9]. The orthogonality of subchannles in OFDM can be maintained and individual subchannels can be completely separated by the FFT at the receiver when there are no intersymbol interference (ISI) and intercarrier interference (ICI) introduced by the transmission channel distortion. Since the spectra of an OFDM signal is not strictly band limited, linear distortions such as multipath propagation causes each subchannel to spread energy into the adjacent channels and consequently cause ISI [6].  One way to prevent ISI is to create a cyclically extended guard interval, where each OFDM symbol is preceded by a periodic extension of the signal itself. When the guard interval is longer than the channel impulse response or multipath delay, the ISI can be eliminated.

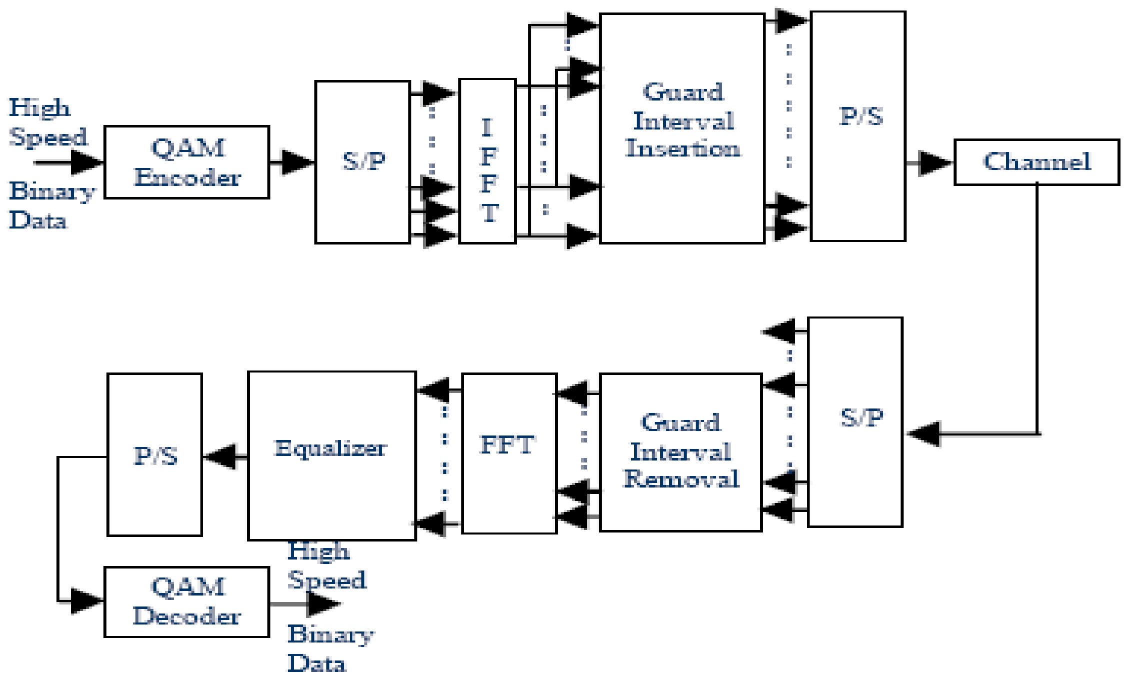


Figure 1 : General OFDM transceiver.

## 3. THE DISTRIBUTION OF PEAK TO AVERAGE POWER RATIO

An OFDM symbol consists of $N$ sub-carriers by the frequency spacing of $\Delta f$ . The total band-with $B$ will be divided into $N$ equally spaced sub-carriers with all sub-carriers are orthogonal to each other within a time interval of length $T = \frac{1}{\Delta f}$. Each sub-carrier can be modulated independently with complex modulation symbol $X_{m,n}$, where m is a time index and n is a sub-carrier index.

The m-*th* OFDM block period can be described as :

$$x_m(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{m,n} g_n(t - mT) \qquad (1)$$

Where, $g_n(t)$ is defined through (2).

$$g_n(t) = \begin{cases} \exp\left(j2\pi n\Delta f t\right), 0 \le t \le T \\ 0, else \end{cases} \qquad (2)$$

Where $g_n(t)$ is a rectangular pulse applied to each subcarrier [4]. The total continuous time signal $x(t)$ consisting of all the OFDM block is given by (3).

$$x(t) = \frac{1}{\sqrt{N}} \sum_{m=0}^{\infty} \sum_{n=0}^{N-1} X_{m,n} g_n\left(t - mT\right) \qquad (3)$$

Consider a single OFDM symbol $(m = 0)$ without loss of generality. This can be shown because there is no overlap between different OFDM symbols. Since $m = 0$, $X_{m,n}$ can be replaced by $X_n$. Then, the OFDM signal can be described as follows,

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi n\Delta f t} \qquad (4)$$

If the bandwith of the OFDM signal is $B = N \times \Delta f$ and the signal $x(t)$ is sampled by the sampling time of $\Delta t = \frac{1}{B} = \frac{1}{N\Delta f}$, then the OFDM signal is in discrete time form and can be written as shown in (5).

$$x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{\frac{j2\pi kn}{N}}, k = 0,2,..N-1 \ (5)$$

For an OFDM signal with *N* subcarriers, the PAPR can be defined as

$$PAPR = \frac{\max\left|x(t)\right|^2}{E\left|x(t)\right|^2} \qquad (6)$$

In particular, a baseband OFDM signal with N subchannels has

$$PAPR_{\max} = 10\log_{10} N \qquad (7)$$

From the central limit theorem, it follows that for large values of N (*N*>64), the real and imaginary values of $x(t)$ become Gaussian distributed [5]. Therefore the amplitude of the OFDM signal has a Rayleigh distribution, with a cumulative distribution given by

$$F(z) = 1 - e^{-z} \tag{8}$$

The probability that the PAPR is below some threshold level can be written as

$$P(PAPR \leq z) = \left(1 - e^{-z}\right)^N \tag{9}$$

In fact, the complementary cumulative distribution function (CCDF) of PAPR of an OFDM is usually used, and can be expressed as

$$P(PAPR > z) = 1 - \left(1 - e^{-z}\right)^N \tag{10}$$

## 4. THE PROPOSED METHOD

OFDM stands for Orthogonal Frequency Division Multiplexing. Its basic concept is the division of the available bandwidth into a number of overlapping sub-carriers, orthogonal to each other. The *N* low rate data streams modulate the *N* orthogonal sub-carriers. In order for the sub-carriers to be orthogonal, their frequency separation must be $\frac{1}{T}\,Hz$ where *T* is the duration of the signalling interval in each sub-carrier. The orthogonality of the sub-carriers will ensure that the signal can be recovered at the receiver without any intercarrier interference by using correlation techniques [12]. The complex envelope representation of an OFDM signal is given by:

$$S_{tx}(t) = \sum_{k=-\infty}^{\infty} \sum_{n=0}^{N-1} a_{n,k}\, g_n(t - kt) \tag{11}$$

$$g_n(t) = \begin{cases} \frac{1}{\sqrt{T - T_{CP}}} e^{j\frac{2\pi n t}{T - T_{CP}}} & , t \in [0, T] \\ 0 & , t \notin [0, T] \end{cases} \tag{12}$$

The complex envelope representation of the new method signal is expressed as:

$$S_{tx}(t) = \sum_{k=-\infty}^{\infty} \sum_{n=0}^{N-1} a_{n,k}\, g_n(t - kt) \tag{13}$$

$$g_n(t) = \begin{cases} \frac{1}{\sqrt{T - T_{CP}}} e^{j\frac{2\pi n t}{2(T - T_{CP})}} & , t \in [0, T] \\ 0 & , t \notin [0, T] \end{cases} \tag{14}$$

The system proposed, uses the partial symmetry of the samples out of the IFFT. In every OFDM symbol, the first $\frac{N}{2}+1$ samples are transmitted with the rest discarded. At the receiver the remaining samples are reconstructed using the partial symmetry of the Fourier transform as explained in [12]. The new system constructed here uses this partial symmetry of the Fourier samples with the only difference that at the receiver instead of reconstructing the remaining samples, zeros are inserted. In this model the QPSK modulated data is converted from frequency to time-domain using a 64-sample IFFT. The new signal is created by discarding the last $\frac{N+2}{2}$ IFFT samples [12]. In this way equation (13) and (14) are satisfied. An additional guard interval is then inserted into the OFDM frame using the cyclic prefix property.

## 5. SIMULATION AND RESULTS

The modelling of the systems is performed in MATLAB simulation software. The new system can either be modelled in a *"continuous"* or *"discrete"* time model respectively [12]. In this paper the new system is constructed using FFTs as shown in Fig. 2. In our simulation, we assume and OFDM system with different number of subcarriers and different modulation scheme. Fig. 3 and Fig. 4 show the CCDF of both conventional OFDM system and those after using the new method. As could be seen, the curves also show that maximum reduction in PAPR value slightly more than 5 dB was achieved by the proposed method.

Table 1 : PAPR performance for QPSK

| MODULATION | N | PAPR(ORIGINAL) | PAPR(NEW METHOD) | DIFFERENCE |
|---|---|---|---|---|
| QPSK | 256 | 9.843 | 3.118 | 6.725 |
| QPSK | 512 | 10.1 | 3.127 | 6.973 |
| QPSK | 1024 | 10.36 | 3.134 | 7.226 |

Table 1 shows the result of the simulation part of QPSK mapped data. In this part, we tried to increase the system complexity by using different number of subcarrier. This table shows that our proposed work still capable of reducing the PAPR value in a significant manner. A huge reduction in PAPR values when we use N=1024 compare with others number of subcarrier.

Table 2 : PAPR performance for 16-QAM

| MODULATION | N | PAPR(ORIGINAL) | PAPR(NEW METHOD) | DIFFERENCE |
|---|---|---|---|---|
| 16-QAM | 256 | 9.252 | 3.671 | 5.581 |
| 16-QAM | 512 | 9.533 | 3.714 | 5.819 |
| 16-QAM | 1024 | 9.795 | 3.762 | 6.033 |

In Table 2, we changed the modulation criteria from phase modulation to amplitude modulation; it shows the result of the simulation part of 16 QAM mapped data. This is another proof of the compatibility of the new technique with different modulation schemes. It shows that the huge reduction in PAPR value still appears in N=1024. It is slightly lower than QPSK by 1.19 dB.

The reason is that each symbol of QPSK convey 2 bits but that of 16 QAM is 4 bits/symbol. Therefor, 16 QAM can carry more traffic than QPSK that cause changes in mean value of the transmit signal.

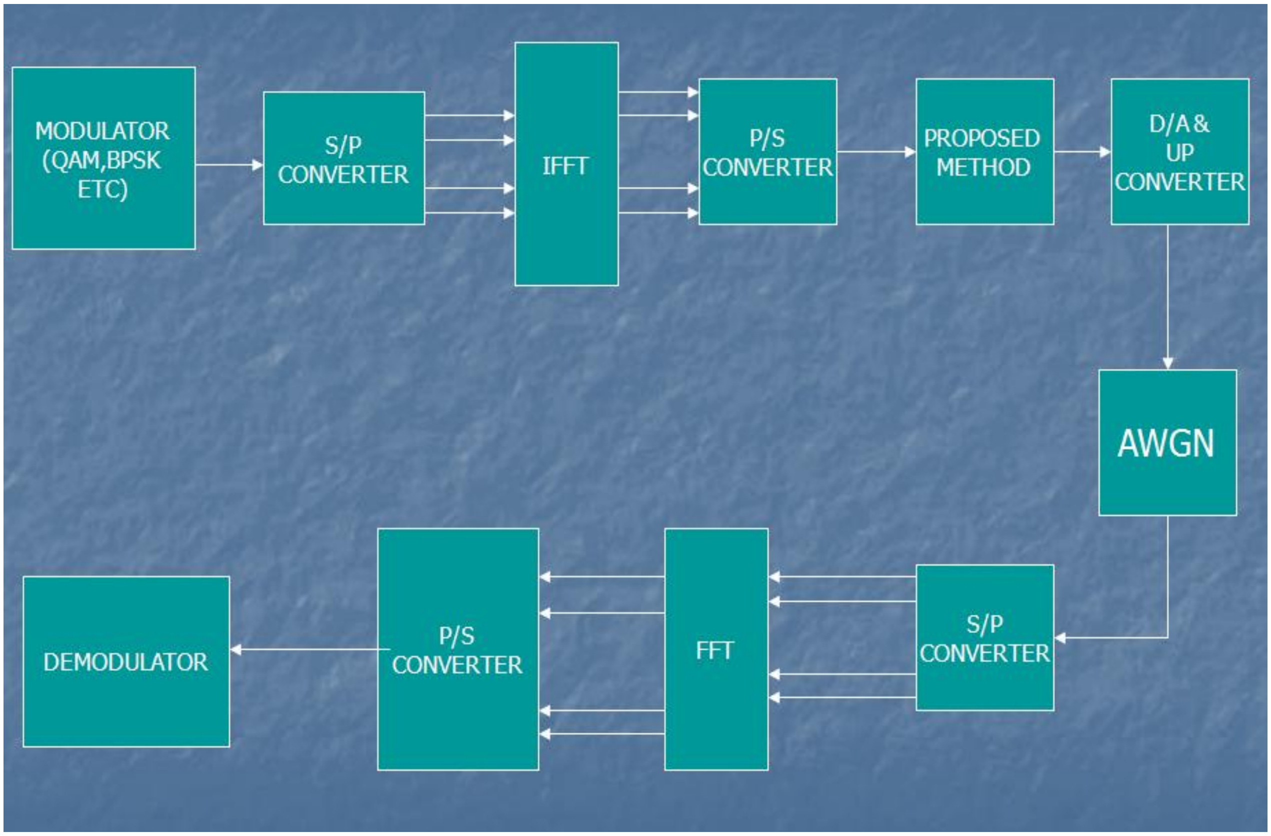


Figure 2: New system model.

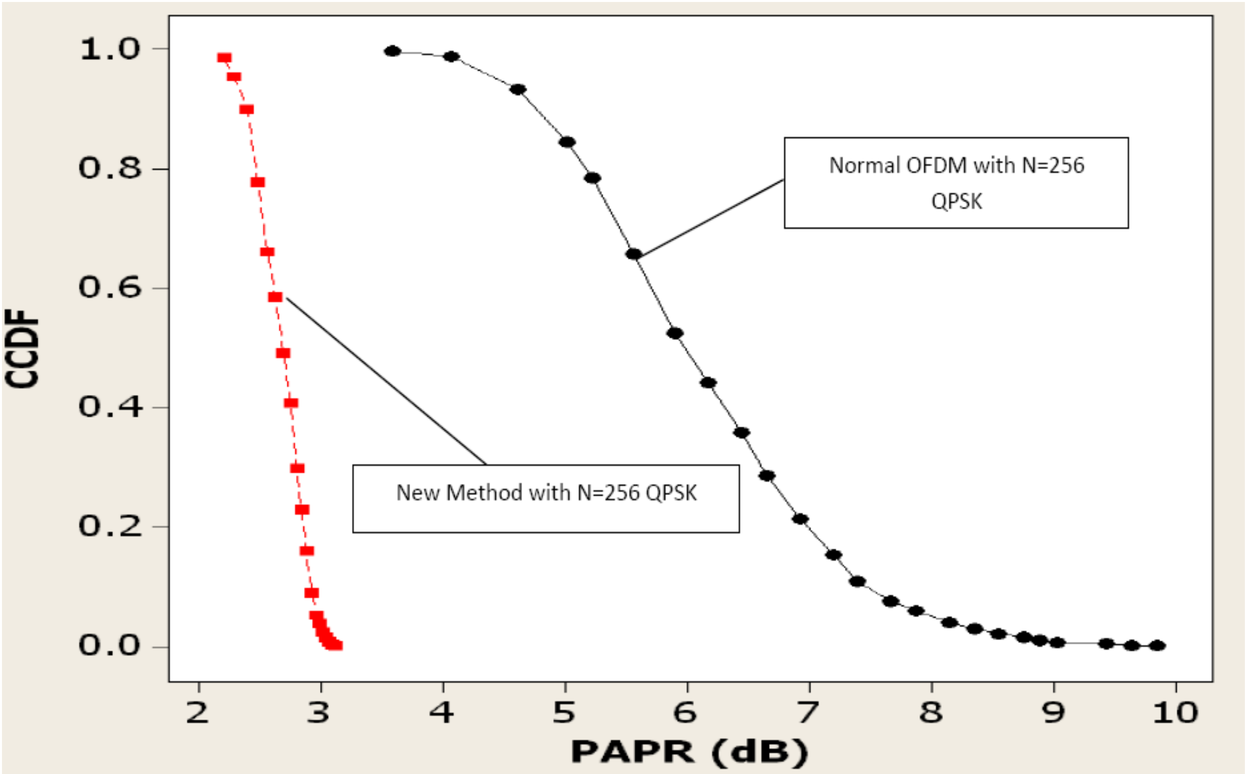

Figure 3: CCDF comparison of conventional OFDM and new method with QPSK modulation

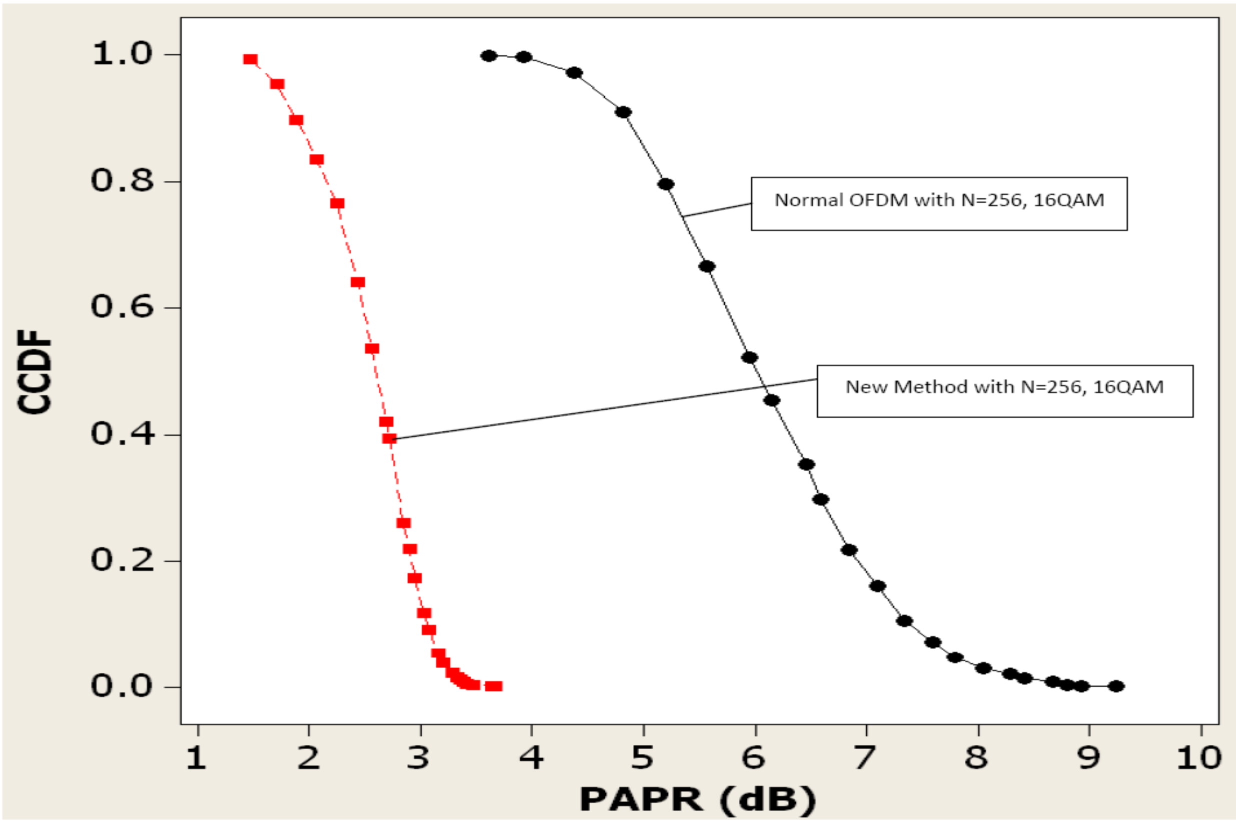


Figure 4 : CCDF comparison of conventional OFDM and new method with 16 QAM modulation

To prove our findings, we have to perform the hypotheses testing using the method called inference of paired samples. A paired difference experiment is one in which the sample data consist of matched observations randomly selected from a population of paired observations. The parameter of interest is the mean $\mu_D$ of the population differences. In this study the population differences is between the original OFDM and the new method. The *t-test* can be used to make an inference on $\mu_D$. To test whether the original OFDM ( $\mu_1$ ) mean score exceeds the new method ( $\mu_2$ ) mean score, the null and alternative hypotheses are

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_a : \mu_1 - \mu_2 > 0 \qquad (15)$$

Table 3 : T-Test For Mean Differences

| Variable | N | Mean | Standard Deviation | T Statistics | P-value |
|---|---|---|---|---|---|
| $\mu_D$ | 18 | 4.41 | 1.44 | 12.97 | 0.00 |

From Table 3, since the observed significance level or *p-value* is 0.00 and *t-test* equal to 12.97 we reject the null hypotheses. We have sufficient evidence to conclude that the mean difference is greater than 0 or the mean for original OFDM exceeds the mean of the new method. From the result we can conclude that the new method will reduce the PAPR value up to 7 dB.

## 6. CONCLUSSIONS

A new PAPR reduction method for the OFDM system by using bandwidth efficiency increasing method has been proposed and investigated. In other words the spacing of the sub-carriers in this new method is twice as dense when compared to OFDM. In this new method the frequency separation of the sub-carriers is halved compared to the conventional OFDM. It is evident from simulation results that the new technique has significantly reduced the PAPR for high data rate systems. This improvement in the systems performances have been maintained even when different combination modulation scheme and number of subcarrier were used. Using statistical hypotheses testing with confidence level 95%, we have sufficient evidence that the proposed method is better than the conventional OFDM system.

## REFERENCES

[1] R. V. Nee and R. Prasad, "OFDM for Wireless Multimedia Communications," Artech House, 2000.

[2] " Broadband Radio Access Network (BRAN); High Performance Radio Local Area Network (HIPERLAN) Type 2; Requirements and Architecture for Wireless Broadband Access," TR101031, 1999.

[3] S.H. Han and J. H. Lee, "An Overview of Peak-To-Average Power Ratio Reduction Techniques For Multicarrier Transmission," IEEE Wireless Comm., Vol. 12, No. 2, April 2005, pp. 56-65.

[4] Rohling H., "Broadband OFDM Radio Transmission For Multimedia Applications," IEEE Proceedings, Vol. 87, pp. 1778-1788, 1999.

[5] May, T. and Rohling, H., "Reducing the Peak To Average Power Ratio Of OFDM and Radio Transmission System," *IEEE Vehicular Technology Conference*, vol. 3, 1998.

[6] Wu, Y. and Zou, W. Y., "Orthogonal Frequency Division Multiplexing: A Multicarrier Modulation Scheme," *IEEE Transactions on consumer Electronics*, Vol. 41, No. 3, pp. 392-399,1995.

[7] Tiku, M. L. " Robust Statistics For Testing Equality Of Means and Variaces," *Communications In Statistics, Theory and Methods*, pp. 2543-2558, 1982.

[8] De Couasnon T., Monnier R., Rault J.B. (1994) OFDM for digital TV broadcasting, Signal processing (ELSEVIER) 39 (1994) 1-32.

[9] Lee D., Cheun K., "A new symbol timing recovery algorithm for OFDM systems", *IEEE Transactions on Consumer Electronics*, Vol. 43, No. 3, pp 766-775, August 1997.

[10] G. Fay, "Wireless Data Networking," International Journal of Network Management, 8 March 1992, pp. 8-17.

[11] Thibault L. and Le M.T., "Performance Evaluation of COFDM for Digital Audio Broadcasting, Part I: Parametric Study", *IEEE Transactions on Broadcasting*, Vol. 43, No. 1, pages 64-75, March 1997.

[12] D. Karampatsis, M.R.D. Rodrigues and I. Darwazeh, "Implications of linear phase dispersion on OFDM and Fast-OFDM systems," London Communications Symposium UCL, pp.117-120, London, UK, Sept. 2002