SINGULARITY ANALYSIS OF A 3-RRRR REDUNDANT SPHERICAL PARALLEL MANIPULATOR

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ABSTRACT
Study of parallel manipulators highlights the importance of identifying singular configurations (singularities) during the design, and of avoiding them during the operation; because in these configurations the instantaneous kinematics of the manipulator becomes indeterminate. This paper focuses on the determination of singular configurations of a kinematically redundant 3-RRRR spherical parallel manipulator while R and R denote the actuated and unactuated revolute joints respectively. Also it is shown how redundancy can help to avoid these singularities.

Keywords: *Spherical parallel manipulators; kinematic Redundancy; Singularity analysis; Angular velocity*

1. INTRODUCTION

Parallel manipulators are closed-loop mechanisms consist of a mobile platform connected to a fixed platform by several independent chains or branches. These types of manipulators have superiorities over serial ones, such as higher stiffness, improved accuracy and dynamic characteristics, higher payload to the weight and higher operating speeds. However, near singular configurations all the manipulators experience poor performances.

The mechanism configurations where the instantaneous kinematics is not determined are called singularities. Since indetermination of the instantaneous kinematics causes serious problems both to the static behavior and to the motion control of the mechanism, the research of all the singularities (singularity analysis) is a mandatory step during the design of mechanisms.

Singularity analysis of parallel manipulators has attracted many researchers in the last three decades [1-8]. Gosselin and Angeles [1] presented the concept of two Jacobian matrices which relate input velocities to output velocities, for parallel manipulator having an equal number of inputs and outputs. The singularity of each matrix corresponds to a loss or a gain of degrees of freedom (DOFs). Daniali [4] et al. have presented a generalized approach to determine singular configurations of planar parallel manipulators that have arbitrary chains and equal number of inputs and outputs. They used a velocity equation including the velocities of active and passive joints in order to determine singular configurations. They [7] also have presented a similar method to find the singular configurations of spherical 3-DOF parallel manipulators. Sefrioui [8] et al provided the expression for the singularity loci of the same manipulators with prismatic actuators. Gürsel[9] and Shirinzadeh presented a method based on the determinants of the Jacobian matrices to generate the singularity contours of a 3-DOF spherical parallel manipulator in terms of output orientation angles.

One of the ways to avoid singular configurations is Redundancy; redundancy in parallel manipulators was first discussed in [10] and [11]. According to [11] there are three types of redundancy:

1) Redundancy introduced by putting additional active joints to the limbs of the existing system.
2) Redundancy introduced by replacing the passive joints of the limbs of the existing system with the active ones.
3) Redundancy introduced by putting additional limbs.

However in some other studies such as [12, 13], redundancy has been classified into two main groups: Actuation redundancy and Kinematic redundancy.

Actuation redundancy [13-15] is defined as replacing some passive joints of a manipulator by active ones, so a manipulator with actuation redundancy has more actuators than are strictly necessary to control it for a given task. Actuation redundancy does not change the mobility or reachable workspace of a manipulator and is used to reduce the singularities within the manipulator's workspace.

Kinematic redundancy [16-18] is obtained when extra active joints and links (if needed) are added to manipulators. Fig. 1 shows a 3-RRR spherical parallel manipulator. All the joints of this manipulator are revolute and their axes pass through center of the sphere (base). The three motors A_1, A_2 and A_3 are fixed to the base. By adding one extra active prismatic joint to one limb of the 3-RRR spherical parallel manipulator, it is converted into a kinematically redundant parallel manipulator (Fig. 2). In this example, the resulting redundant parallel manipulator has 4 actuated-joint degrees of freedom (ADOFs), one more than the non redundant one. The leg with kinematic redundancy (i.e. leg 1) has an infinite number of solutions to the inverse displacement problem [17]. Thus, instead of a finite set of solutions, if there are any, there may be one or more loci of solutions. This property is very useful, because one can choose a solution that is not singular. It is worth noting that the inverse displacement problem for each of the non-redundant legs has at most two solutions.

In general, kinematically redundant parallel manipulators have the advantages of avoiding most singularities, enlarging workspace and improving dexterity. On the other hand, kinematic redundancy results in more controlling parameters than required for a set of given tasks [16].

Although for planar parallel manipulators, redundancy has been studied by many researchers, for spherical counterparts the literature is so limited e.g. [19, 20].

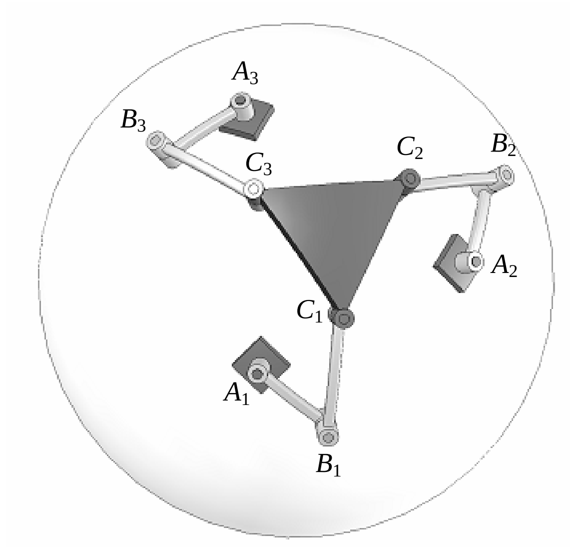

Fig. 1. 3-RRR spherical parallel manipulator.

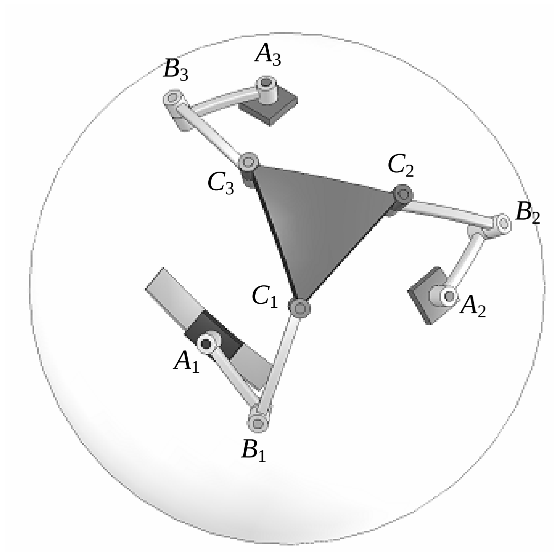

Fig. 2. A 4-ADOFs kinematically redundant spherical parallel manipulator (2-RRR+1-PRRR).

This paper focuses on the determination of singular configurations of 6-ADOFs 3-PRRR kinematically redundant spherical parallel manipulator, being originally a 3-RRR manipulator. The manipulator has three redundant actuated curved prismatic joints, as it is shown in Fig. 3. We use the term curved prismatic to denote a motion that slides on a curved path. Each prismatic joint can also be viewed as a revolute joint with its axis passing through the origin of the sphere, see Fig. 4. Therefore, each of the three moving legs can be thought of being RRRR type.

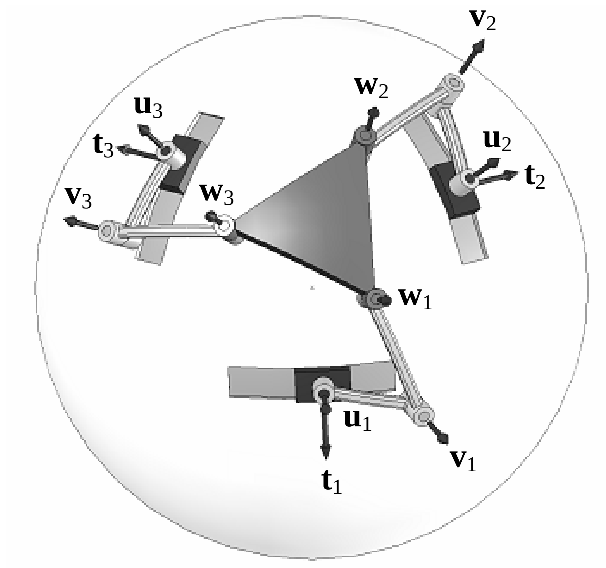

Fig. 3. The 3-RRRR kinematically redundant spherical parallel manipulator.

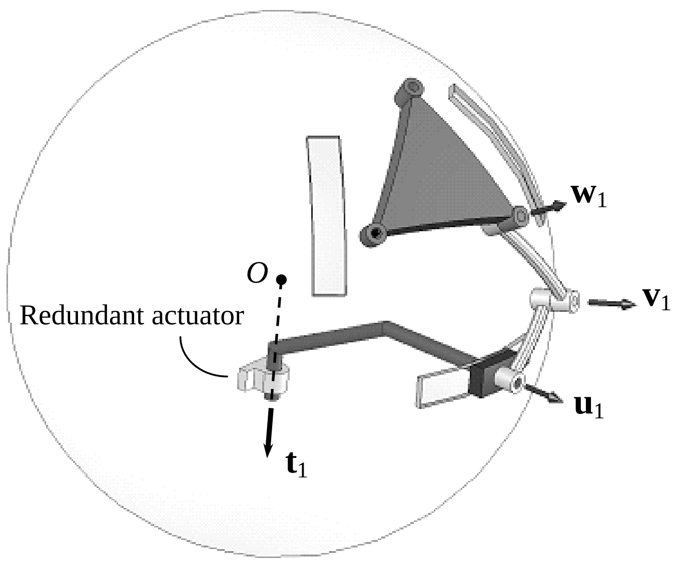

Fig. 4. One leg of the manipulator.

2. JACOBIAN MATRICES

To find singular configurations of the manipulator under study, at first its Jacobian matrices are obtained. Considering Fig. 3, angular velocity **ω** of the end-effector (EE) can be written as

$$\dot{\alpha}_i \mathbf{t}_i + \dot{\beta}_i \mathbf{u}_i + \dot{\theta}_i \mathbf{v}_i + \dot{\gamma}_i \mathbf{w}_i = \boldsymbol{\omega}, \quad i=1, 2, 3 \tag{1}$$

where $\mathbf{t}_i$ is a unit vector along the axis of redundant revolute joint, and $\mathbf{u}_i$, $\mathbf{v}_i$ and $\mathbf{w}_i$ are the unit vectors representing the direction of correspondent non redundant revolute joints of the *i*-th leg, respectively. Moreover, $\dot{\alpha}_i$, $\dot{\beta}_i$, $\dot{\theta}_i$ and $\dot{\gamma}_i$ are rates of the corresponding joint in the *i*-th leg.

We eliminate the rates of the unactuated joints, $\dot{\gamma}_i$, by dot-multiplying both sides of the foregoing equation by $\mathbf{v}_i \times \mathbf{w}_i$, thereby obtaining

$$\dot{\alpha}_i \mathbf{t}_i \cdot (\mathbf{v}_i \times \mathbf{w}_i) + \dot{\beta}_i \mathbf{u}_i \cdot (\mathbf{v}_i \times \mathbf{w}_i) + (\mathbf{w}_i \times \mathbf{v}_i) \cdot \boldsymbol{\omega} = 0, \quad i=1, 2, 3 \tag{2}$$

The above equation, for i = 1, 2, 3, are now assembled in the form

$$\mathbf{J}\dot{\boldsymbol{\theta}} + \mathbf{K}\boldsymbol{\omega} = \mathbf{0}$$

where Jacobian matrices **J** and **K** are defined as

$$\mathbf{J} = \begin{bmatrix} a_1 & b_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_2 & b_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_3 & b_3 \end{bmatrix} \tag{3}$$

and

$$\mathbf{K} = \begin{bmatrix} (\mathbf{w}_1 \times \mathbf{v}_1)^T \\ (\mathbf{w}_2 \times \mathbf{v}_2)^T \\ (\mathbf{w}_3 \times \mathbf{v}_3)^T \end{bmatrix} \tag{4}$$

in which

$$a_i = \mathbf{t}_i \cdot (\mathbf{v}_i \times \mathbf{w}_i) \text{ and } b_i = \mathbf{u}_i \cdot (\mathbf{v}_i \times \mathbf{w}_i), \quad i=1, 2, 3$$

Also $\dot{\boldsymbol{\theta}}$ is a vector of active joint rates, i.e.,

$$\dot{\boldsymbol{\theta}} = [\dot{\alpha}_1, \dot{\beta}_1, \dot{\alpha}_2, \dot{\beta}_2, \dot{\alpha}_3, \dot{\beta}_3]$$

3. SINGULARITY ANALYSIS

In this section, singular configurations of the manipulator under study are found, using the obtained Jacobian matrices.

3.1. Inverse kinematic singularities

Inverse kinematic singularities are the ones where the inverse instantaneous kinematic problem is unsolvable. This type of singularities for kinematically redundant parallel manipulators takes place when the rank of **J** is lower than the degrees of freedom of the end effector (number of rows of **J**), in other words, when the determinant of $\mathbf{JJ}^T$ is equal to zero [11].

$$\left|\mathbf{JJ}^T\right| = \begin{vmatrix} a_1^2 + b_1^2 & 0 & 0 \\ 0 & a_2^2 + b_2^2 & 0 \\ 0 & 0 & a_3^2 + b_3^2 \end{vmatrix} = 0 \tag{5}$$

In which |*| denotes the determinant. Equation (5) is correct when

$$a_i^2 + b_i^2 = 0, \quad i=1 \text{ or } 2 \text{ or } 3 \tag{6}$$

So this type of singularities occurs when four vectors $\mathbf{t}_i$, $\mathbf{u}_i$, $\mathbf{v}_i$ and $\mathbf{w}_i$ for i=1 or 2 or 3 are coplanar which means that at least one of the legs are fully extended or folded and direction of the correspondent curved prismatic joint is spherically perpendicular to the links with revolute joints. At these configurations, the motion of actuators of the legs with singular poses does not produce any motion of the EE and the EE cannot move in the direction of the extended or folded legs. Thus the manipulator loses one or more degrees of freedom. Figure 5 shows the manipulator in a configuration in which legs one and three produce this type of singularities.

Note that, as long as the manipulator is not at the workspace boundary, the inverse kinematic singularities of the manipulator are avoidable as it is possible to choose at least a set of solutions which is free of inverse singularities. For instance, for the given pose of the end effector in Fig. 5, the poses of legs one and three can be changed to the poses which are not at the inverse singularity.

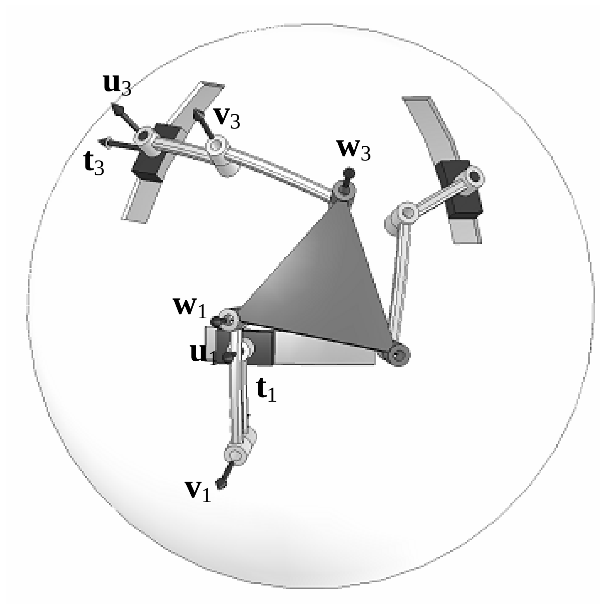

Fig. 5. An inverse kinematic singularity of spherical 3-RRRR manipulator in which legs one and three are in branch-singular poses.

3.2. Direct kinematic singularities

Direct kinematic singularities are the ones where the forward instantaneous kinematic problem is unsolvable. This type of singularities occurs when the determinant of **K** vanishes which, in turn, occurs when the rows or columns of **K** are linearly dependent.

Matrix **K**, introduced in Eq. (4), can be written as

$$\mathbf{K} = \begin{bmatrix} \mathbf{n}_1^T \\ \mathbf{n}_2^T \\ \mathbf{n}_3^T \end{bmatrix} \tag{7}$$

In which

$$\mathbf{n}_i = \mathbf{v}_i \times \mathbf{w}_i,\ i=1, 2, 3$$

It can be readily shown that

$$|\mathbf{K}| = \mathbf{n}_1 \cdot (\mathbf{n}_2 \times \mathbf{n}_3) \tag{8}$$

Determinant of **K** vanishes if and only if

$$\mathbf{n}_1 \cdot (\mathbf{n}_2 \times \mathbf{n}_3) = 0 \tag{9}$$

The above condition is satisfied when the three vectors $\mathbf{n}_1$, $\mathbf{n}_2$ and $\mathbf{n}_3$ are coplanar. Therefore, direct kinematic singularities of the manipulator take place when three planes passing through distal links intersect at a common line. An example of this type of singularities is depicted in Fig. 6. At this configuration, the EE can rotate freely in the direction of intersection line, even if all actuators are locked. Moreover, the EE cannot balance torques along this direction, so the manipulator gains some additional degrees of freedom.

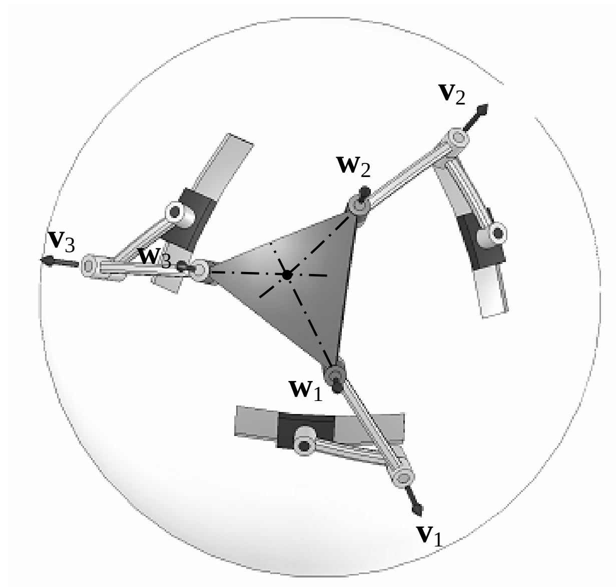

Fig. 6. A direct kinematic singularity of spherical 3-RRRR manipulator in which planes of the distal links intersect at a common line.

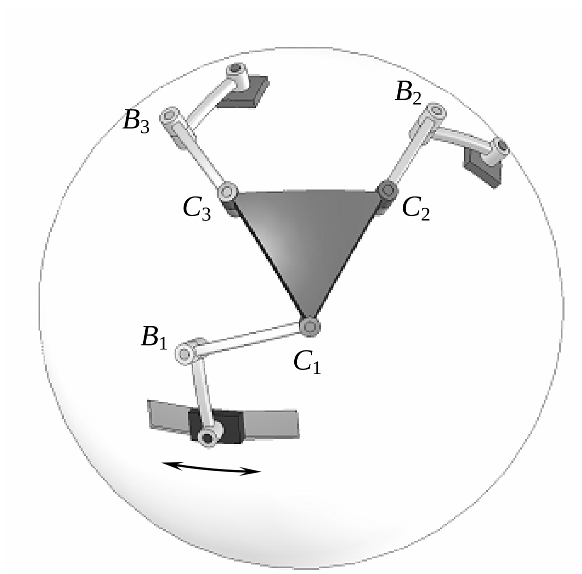

Fig. 7. A direct kinematic singularity of a 4-ADOFs redundant spherical parallel manipulator with four revolute active joints.

When an actuated revolute (or curved prismatic) joint is added to the 3-RRR, it is still possible that the determinant of **K** becomes zero. Such a configuration is shown in Fig 7. For the pose shown, the manipulator is in a singular configuration for any pose of the RRRR redundant leg (i.e., leg 1). That is, all three planes passing through distal links always intersect at a common line (the line passing through points C_1 and center of the sphere). Adding another redundant revolute joint to one of the other two legs would help avoid singular configurations by allowing changing the direction of the corresponding distal link. In turn, this would prevent intersection at a common line of planes passing through distal links. Adding three redundant revolute joints not only allows avoiding singularities but also increases the workspace and gives a symmetrical architecture to the manipulator which is desirable.

3.3. Combined singularities

The combined singularities correspond to singularities both of the inverse and of the direct kinematic problems. These singularities occur when both $\mathbf{JJ}^T$ and **K** are rank deficient. That is to say, this type of singularities is a combination of the two aforementioned types of singularities, Fig. 8.

So, at the configurations correspondent to third type of singularities, the motion of actuators of at least one leg do not produce any motion of the EE. Therefore, the manipulator loses one or more DOF. As well, the EE can rotate freely in a direction even if all the actuators are locked and some torques applied to it in that direction cannot be balanced by the actuators. Thus, the manipulator gains some uncontrollable DOFs.

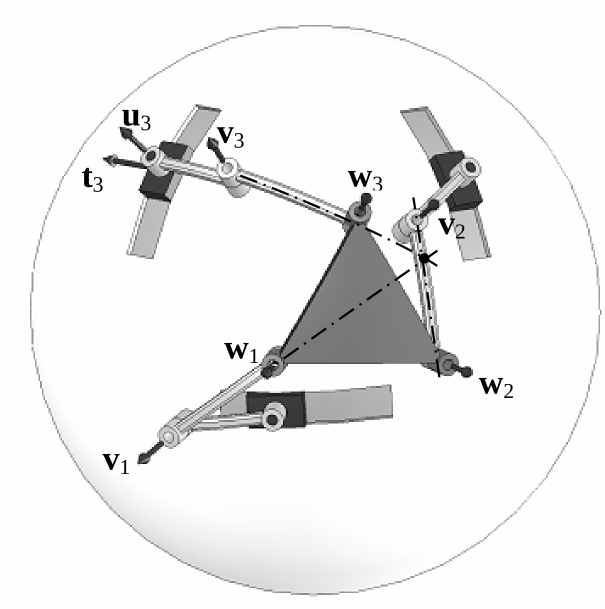

Fig. 8. A combined singularity of the spherical 3-RRRR manipulator where the two first singularities occur simultaneously.

4. CONCLUSIONS

Singularities of A 6-ADOFs 3-RRRR kinematically redundant spherical parallel manipulator has been identified in this paper. The concise analytical expressions showing the singular configurations of the manipulator were obtained using the kinematic solutions. Most of the singularities are avoidable by considering poses that are singular-free. That is, of course, as long as the pose is not on the outer boundary of the workspace which is caused by the physical restriction of the manipulator and it is inevitable. The presented method can be used for singularity analysis of all types of kinematically redundant spherical parallel manipulators.

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