AN INVENTORY MODEL FOR WEIBULL DETERIORATING ITEMS WITH PERMISSIBLE DELAY IN PAYMENTS UNDER INFLATION

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ABSTRACT

This paper derives an optimal inventory replenishment policy for two parameters Weibull deteriorating items with a permissible delay in payment under inflation over finite planning horizon. The theoretical results reflected in this paper is also studied through numerical examples and sensitivity analysis.

KEY WORDS: *Inventory, Inflation, Weibull deterioration, Delay in payment.*

1. INTRODUCTION

The deterioration of goods is a realistic phenomenon in many inventory systems. The controlling and regulating of deteriorating items is a measure problem in any inventory system. Certain products like food stuff, pharmaceuticals, chemicals, volatile liquid, blood and etcetera deteriorate during their normal storage period. Hence while developing an optimal inventory policy for such products, the loss of inventory due to deterioration cannot be ignored. The researchers have continuously modified the deteriorating inventory models so as to become more practicable and realistic. The analysis of deteriorating inventory model is initiated by Ghare and Schrader [6] with a constant rate of decay. It has been realized that failure and life expectancy of many items can be expressed interms of Weibull distribution function. Convert and Philip [5] extended Ghare and Schrader's model for variable rate of deterioration by assuming two parameters Weibull distribution function. The models for these type products have been developed by Phillip [14], Mishra [11], Chakrabarty, Giri and Choudhury [4]. Hence realistic model always treats that deteriorate rate is a time varying function.

In all of the models mention above, the inflation and time value of money does not have significant role to play in the inventory policy, on the other hand, it is realized that most of countries have been suffering from high inflation which leads decline of time value of money. Thereby dependent countries suffer from purchasing of power, oil etc. Hence ignorance of inflation and time value of money while developing an inventory model is disregarded. Buzacott [2] was the pioneer researcher of inventory model with inflation. Several researchers have extended their idea to different situations considering various inflation rates and time value of money. The valuable models in this direction are the models of Mishra [12], Bierman and Thomas [1] Moon and Giri [13] Gor and Shah [7], Chandra and Bahner [3], KuoLung Huo [9].

In the traditional inventory EOQ model, the purchaser pays for his items as soon as it is received. However in real competitive business world, the supplier may allow a credit period to encourage the customers. Delay in payment to the supplier is an alternative way of price discount. Hence paying later in directly reduces the purchase cost which attracts the customers to enhance their ordering quantity. The reference of valuable models in this regards are Goyal [8], Lio [10]. Most of inventory models taking constant rate of deterioration with delay in payment under inflation have been developed by researchers. It has been empirically observed that failure and life expectancy of many items can be expressed in terms of Weibull distribution function. Hence consideration of Weibull distribution function for rate of deterioration in our model is more reasonable and practicable. The present model is discussed and solved analytically. The conditions for concavity of optimality are established and also studied through numerical examples applying sensitivity analysis.

2. NOTATION AND ASSUMPTIONS

The proposed mathematical model of inventory replenishment policy is developed under the following notations and assumptions:

H = Length of finite planning horizon
R = Demand rate
p = Selling price per unit at $t = 0$

$p(t) = pe^{rt} =$ Instantaneous selling price per unit
C = Purchasing price per unit at $t = 0$
$C(t) = Ce^{rt} =$ Instantaneous ordering cost per order
$I_C =$ Interest charged in stock per annum by the supplier
$I_e =$ Interest earned per annum by the retailer
$T_1 =$ Allowable delay period during settlement of the account
$Q =$ Optimum ordering quantity
T = Optimum length of cycle time.
$\lambda(t) = \alpha\,\beta t^{\beta-1} =$ Time proportional deterioration $0 < \alpha < 1, \quad \beta > 1$
$r =$ Rate of inflation
$B = e^{rH} - 1$
$I(t) =$ Instantaneous level of inventory
$TC(T) =$ Total relevant cost over [0,H]
$NP(T) =$ Total relevant profit over finite planning horizon

The components of total inventory cost are

i. Cost of placing order (OC)
ii. Cost of deterioration (CD)
iii. Inventory holding cost excluding interest charged (IHC)
iv. Interest charged (IC) for unsold items at the initial time or after permissible delay period T_1
v. Interest earned IE from sales revenue during the permissible delay period $[0, T_1]$.

Hence

$$TC = OC + CD + IHC + IC - IE$$

Gross revenue $= R(P - C)\, e^{rT}$.

3. ASSUMPTIONS

i. Inflation is constant
ii. Demand rate is constant
iii. Shortages are not allowed
iv. Lead time is zero
v. During the permissible delay period the sales revenue generated is deposited in an interest bearing account. At the end of trade credit period, the customer pays off all units ordered and begins paying for the interest charged on the items in stock.
vi. There is no repair or replacement of deteriorated units during the given cycle.

4. MATHEMATICAL FORMULATION

Let the length of planning horizon $H = nT$ where n is the number of replenishments occur during the period H. The falling of inventory level happens due to the combined effect of demand and deterioration in interval [0,T]. Hence the differential equation governing the instantaneous level of inventory $I(t)$ is given by

$$\frac{dI}{dt} + \alpha\beta t^{\beta-1}\, I = -R \quad 0 < t < T \tag{1}$$

With the boundary conditions

$$I(0) = Q \text{ and } I(T) = 0$$

The solution boundary condition is given by

$$I(t) = \frac{\mathrm{Re}^{-\alpha t^{\beta}}}{\beta+1}\left[(\beta+1)(T-t) + \alpha\left(T^{\beta+1} - t^{\beta+1}\right)\right] \tag{2}$$

The initial ordering quantity can be obtained from equation (2) after putting $t = 0$.
Hence

$$Q = I(0) = \frac{R}{\beta+1}\left[(\beta+1)\,T + \alpha T^{\beta+1}\right] \tag{3}$$

Since I(t) is a periodic function with period *T* hence we have

$$I(kT+t) = I(t) \quad k \in z$$

$$\Rightarrow I(kT+t) = \frac{R}{\beta+1}\left[(\beta+1)(T-t) + \alpha\left(T^{\beta+1} - t^{\beta+1}\right) - \alpha(\beta+1)\left(Tt^{\beta} - t^{\beta+1}\right)\right] \tag{4}$$

After neglecting higher powers of α as $\alpha < 1$. The components of total cost of the inventory are given as follows

(a). Ordering cost OC :

Since $A(t) = Ae^{rt}$,

Therefore

$$OC = A(O) + A(T) + A(2T) + A(3T) + \cdots + A[(n-1)\,T]$$

$$= A\left(e^{rH} - 1\right) / \left(e^{rT} - 1\right) = AB\left(\frac{1}{rT} + 1 + \frac{rT}{4}\right) e^{-rT} \tag{5}$$

where

$$B = e^{rH} - 1$$

(b). Cost of Deteriorated Units CD:

The number of deteriorated units is

$$D = I(0) - RT = \frac{\alpha RT^{\beta+1}}{\beta+1} \tag{6}$$

The cost total deteriorated units is

$$CD = \frac{\alpha RBCT^{\beta+1}}{(\beta+1)\left(e^{rT}-1\right)} \tag{7}$$

(c). Inventory holding cost IHC is given by:

$$IHC = h\sum_{k=0}^{n-1} c(kT) \int_0^T I(kT+t)\,dt$$

$$= \frac{hRBC}{(\beta+1)\,(e^{rT}-1)}\left[\left(\frac{\beta+1}{2}\right)T^2 + \frac{\alpha\beta T^{\beta+1}}{\beta+2}\right] \tag{8}$$

(d). Depending upon the length of the cycle time T and customer's choice, the following two possible cases are taken into account while computing the interest charged and interest earned.

CASE-I

INTEREST CHARGED IC IN [0,H]

Since, optimal cycle length T is greater than the permissible delay time T_1, the interest charged during the period $[T_1, T]$ is given by

$$IC_1 = I_C \sum_{k=0}^{n--1} C(kT) \int_{T_1}^{T} I(kT+t)\,dt$$

$$= \frac{k_1}{\left(e^{rT}-1\right)}\left[(\beta+1)(T-T_1)^2 + \frac{2\alpha\beta}{\beta+2}\left(T^{\beta+2} - T_1^{\beta+2}\right) - 2\alpha T_1\, T\left(T^{\beta} - T_1^{\beta}\right)\right] \tag{9}$$

Where $k_1 = \dfrac{I_c RBC}{2(\beta+1)}$ Interest earned during [0,H] is given by

$$IE_1 = I_e \sum_{k=0}^{n-1} P(kT) \int_0^{T_1} Rt\ dt = \frac{RPBI_e\ T_1^2}{2(e^{rT}-1)} \tag{10}$$

AVERAGE COST

The total average cost per unit during the time [0,H] is

$$TC_1(T) = \frac{1}{T}\left[OC + CD + IHC + IC_1 - IE_1\right] \tag{11}$$

NET PROFIT

The net profit during the interval [0,H] is given by

$$NP_1(T) = R(P-C)e^{rT} - TC_1(T) \tag{12}$$

OPTIMALITY CONDITIONS

Differentiating equation (11) and (12) w.r.t. T once and twice, we have

$$\frac{\partial(OC/T)}{\partial T} = -AB\left[\frac{2}{rT^3} + \frac{3}{2T^2} + \frac{r}{2T} + \frac{r^2}{4}\right]e^{-rT} \tag{13}$$

$$\frac{\partial^2(OC/T)}{\partial T^2} = AB\left[\frac{6}{rT^4} + \frac{5}{T^3} + \frac{2r}{T^2} + \frac{r^2}{2T}\right]e^{-rT} > 0 \tag{14}$$

$$\frac{\partial(CD/T)}{\partial T} = \frac{\alpha RBC}{\beta+1}\left[\left(\frac{\beta-1}{r}\right)T^{\beta-2} + \left(\frac{\beta-2}{2}\right)T^{\beta-1} + \frac{r(\beta-1)T^{\beta}}{4} - \frac{r^2T^{\beta+1}}{4}\right]e^{-rT} \tag{15}$$

$$\frac{\partial(CD/T)}{\partial T} = \frac{\alpha RBC}{\beta+1}\left[\left(\frac{(\beta-1)(\beta-2)\,T^{\beta-3}}{r} + \frac{(\beta-4)(\beta-1)\,T^{\beta-2}}{2}\right) + \frac{r}{4}\left(\beta^2 - 3\beta + 4\right)T^{\beta-1} - \frac{r^2\,\beta T^{\beta}}{2}\right]e^{-rT} > 0 \tag{16}$$

$$\frac{\partial(IHC/T)}{\partial T} = \frac{2hRBC}{8r(\beta+1)(\beta+2)}\left[-(\beta+1)(\beta+2)\,r + 4\alpha\beta^2T^{\beta-1} + 2\alpha r\,\beta(\beta-1)T^{\beta}\right] \tag{17}$$

$$\frac{\partial^2(IHC/T)}{\partial T^2} = \frac{2hRBC}{8r(\beta+1)(\beta+2)}\left[2\alpha\beta^2(\beta-1)\,T^{\beta-2} + 2\alpha r\,\beta^2\,(\beta-1)\,T^{\beta-1}\right] > 0 \tag{18}$$

$$\frac{\partial(IC_1/T)}{\partial T} = \frac{2RBI_C}{8(\beta+1)\,rT^3}\Big[\left(4T + 2rT^2 + r^2T^3\right)\left\{(T-T_1)(\beta+1+\alpha\beta T^{\beta}) - \alpha T_1\left(T^{\beta} - T_1^{\beta}\right)\right\} - \left(4 + 3rT + r^2T^2\right)\Big\{(\beta+1)\left(T-T_1\right)^2 + \frac{2\alpha\beta}{\beta+2}\left(T^{\beta+2} - T_1^{\beta+2}\right) - 2\alpha T_1\,T\left(T^{\beta} - T_1^{\beta}\right)\Big\}\Big]e^{-rT} \tag{19}$$

Let $IC_1/T = kXY$.

Then

$$\frac{\partial^2 (IC_1/T)}{\partial T^2} = k_1\left[2X'Y' + XY'' + X''Y\right]$$

Where

$$X = \left(\frac{1}{rT^2} - \frac{1}{2T} + \frac{3r}{4} + \frac{r^2 T}{4}\right) > 0$$

$$X' = \left[-\frac{2}{rT^3} + \frac{1}{2T^2} + \frac{r^2}{4}\right] < 0$$

$$X'' = \frac{6}{rT^4} - \frac{1}{T^3} = \frac{6 - rT}{rT^4} > 0,$$

$$Y = (\beta+1)(T - T_1)^2 + \frac{2\alpha\beta}{\beta+2}\left(T^{\beta+2} - T_1^{\beta+2}\right)$$
$$-2\alpha T_1\, T\left(T^{\beta} - T_1^{\beta}\right) > 0$$

$$Y' = 2(\beta+1)\,(T - T_1) + 2\alpha\beta T^{\beta+1} - 2\alpha T_1\left[(\beta+1)\,T^{\beta} - T_1^{\beta}\right] > 0,$$

$$Y'' = 2(\beta+1) + 2\alpha\,\beta^2(\beta+1)\,T^{\beta-1}\,(T - T_1) > 0,$$

Where

$$k_1 = IcRBC/2(\beta+1).$$

Therefore

$$\frac{\partial^2 (IC_1/T)}{\partial T^2} > 0 \tag{20}$$

$$\frac{\partial (IE_1/T)}{\partial T} = -k_2\left[\frac{2}{rT^3} + \frac{3}{2T^2} + \frac{r}{2T} + \frac{r^2}{4}\right] e^{-rT} \tag{21}$$

Where

$$k_2 = RPBI_e T_1^2/2.$$

$$\frac{\partial^2 (IE_1/T)}{\partial T^2} = k_2\left[\frac{6}{rT^4} + \frac{5}{2T^3} + \frac{2r}{T^2} + \frac{r^2}{2T}\right] e^{-rT} > 0$$

$$\frac{\partial^2 (OC/T)}{\partial T^2} - \frac{\partial^2 (IE_1/T)}{\partial T^2} = B\left(\frac{2A - RPI_e\, T_1^2}{2}\right)\left[\frac{6}{rT^4} + \frac{5}{T^3} + \frac{2r}{T^2} + \frac{r^2}{2T}\right] e^{-rT} > 0 \tag{22}$$

$$\frac{\partial NP(T)}{\partial T} = r(p - C)\,\mathrm{Re}^{rT} - \frac{\partial TC_1(T)}{\partial T} = 0 \tag{23}$$

Equation(23) represents condition of optimality. Let the solutions of equation (23) be ‘ γ ’.
Therefore

$$\frac{\partial^2 NP_1(T)}{\partial T^2} < 0,\ \alpha t\, T = \gamma \tag{24}$$

In equation (24) represents sufficient condition for maximum net profit.

INTEREST CHARGED

In this case the retailer pays the procurement cost to the supplier prior to expiration of the delay period T_1 provided by the supplier. Hence the interest charged (IC_2) in this case is zero.

CASE-II

INTEREST EARNED

For $T < T_1$, the interest earned during [0,H] is given by

$$IE_2 = RI_e \sum_{k=0}^{n-1} p(kT)\left[\int_0^T t dt + T(T_1 - T)\right]$$

$$= \frac{RI_e PB}{2} \frac{\left(2T_1 T - T^2\right)}{\left(e^{rT} - 1\right)}$$

$$\frac{\partial(IE_2/T)}{\partial T} = \frac{I_e RPB}{2}\left[\frac{-2T_1}{rT^2} - \frac{rT_1}{2} + \frac{1}{2} - \frac{2T_1}{T} - \frac{r^2 T_1 T}{2} + \frac{r^2 T^2}{4}\right] \quad (25)$$

$$\frac{\partial^2\left(IE_2/T\right)}{\partial T^2} = \frac{I_e RPB}{2}\left[\frac{4T_1}{rT^3} + \frac{2T_1}{T^2} - \frac{r^2 T_1}{2} + \frac{r^2 T}{2}\right] > 0 \quad (26)$$

From the in equations (14) and (26) we have

$$\frac{\partial^2 (OC/T)}{\partial T^2} - \frac{\partial^2 (IE_2/T)}{\partial T^2} > 0.$$

The net profit during the interval [0,H] is given by

$$NP_2(T) = R(P - C)e^{rT} - TC_2(T) \quad (27)$$

where

$$TC_2(T) = \frac{1}{T}\left(OC + CD + IHC + IC_2 - IE_2\right).$$

Differentiating equation (27) w.r.t *T* we have

$$\frac{\partial NP_2(T)}{\partial T} = R(P - C)\, re^{rT} - \frac{\partial TC_2(T)}{\partial T} = 0 \quad (28)$$

Equation (28) represents necessary condition of optimally. Let the solution of equation (28) be θ.

Therefore,

$$\left.\frac{\partial^2 NP_2(T)}{\partial T^2}\right| < 0 \quad T = \theta \quad (29)$$

In equation (29) represents sufficient condition of maximum.

Table-1: Optimal Solution for different values of parameters associated with model

r/T_1		0.0411	0.82192	0.123288
0.01	T	0.6915	0.5616	0.4438
	Q	697.668	564.836	445.3598
	P(t)	30.2082	30.169	30.1334
	NP	8064.6036	9232.4558	8527.3425
0.02	T	0.75967	0.60133	0.46266
	Q	767.9251	605.3298	464.4346
	P(t)	30,4593	30.363	30.2789
	NP	8134.8832	8322.7992	8568.7477
0.03	T	0.8705	0.66032	0.4868
	Q	883.0913	665.666	488.8777
	P(t)	30.7938	30.6002	30.4413
	NP	8215.4698	8383.1926	8612.6461

NUMERICAL EXAMPLES

Let A = 100, R = 1000, C = 20, P = 30, r = 0.02, α = 0.06, β =2.1, H = 1 year, h = 0.02, I_e = 0.12, I_c =0.15, $T_1 = \frac{30}{365}$.

SENSITIVITY ANALYSIS

1. Under constant Weibull deteriorating parameters α, β and the allowable credit period T_1, the increase in the rate of inflation enhances the optimum cycle time, optimum procurement quantity, selling price and net profit.
2. Keeping the rate of inflation *r* and parameter α, β parameters as constants, the increase in the allowable credit period reduces the optimum cycle time, optimum procurement quantity, selling price but enhances the net profit due to substantially low interest charged.
3. The rate of inflation, the allowable credit period and β remaining constant, increase in α causes decrease in optimum cycle time, optimum procurement quantity, selling price and net profit.
4. The rate of inflation, the allowable credit period and α remaining unaltered.

Table-2: Optimal Solution for different values of parameters associated with model

α / β		2.0	2.1	2.2	2.3
0.05	T	0.64049	0.63708	0.6351	0.6347
	Q	644.8691	641.0666	638.7552	638.0801
	P(t)	30.3668	30.3847	30.3835	30.3832
	NP	8342.7019	8359.3992	8374.6696	8388.6431
0.06	T	0.60328	0.60133	0.60063	0.60125
	Q	607.6712	605.3298	604.2989	604.6425
	P(t)	30.3642	30.363	30.3626	30.3629
	NP	8299.2815	8319.4901	8337.9394	8354.6141
0.07	T	0.57493	0.57396	0.57418	0.57532
	Q	579.3642	577.9989	577.8859	578.742
	P(t)	30.3469	30.3463	30.3464	30.3472
	NP	8259.3709	8281.8656	8303.3301	8322.7413
0.08	T	0.55218	0.5519	0.55272	0.55432
	Q	556.6696	555.9878	556.4693	557.8405
	P(t)	30.3331	30.3329	30.3334	30.3344
	NP	8222.3909	8246.0267	8273.1756	8292.5462

The increase in β causes the enhancement of the net profit.

5. r,T and α are very sensitive in this model and β is moderately sensitive.

6. CONCLUSION

The quality and quantity of goods decrease in course of time due to deterioration. Hence consideration of Weibull distribution time varying deteriorating function in place of constant deterioration taken in various models of this type carries a significant meaning for perishable, volatile and failure of any kind of item. In this model we have considered two opposite characteristic elements such as inflation and deterioration which suggest the market that the presence of inflation impacts larger in optimum cycle length, whereas deterioration forces smaller in optimum cycle length. In addition to the above, the providing of delay in payment by the supplier suggests the retailer to take the advantage of repeating of ht optimum cycle period within substantially reducible time in turn of which interest charged by the supplier becomes nominal. Consequently the model becomes more profitable and very useful in business community dealing with perishable products, electronic components, domestic goods volatile substances and other products.

This model can further be enriched by incorporating shortages demand as a function of time.

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