RELIABILITY AND AVAILABILITY ANALYSIS OF PARALLEL REDUNDANT COMPLEX SYSTEM WITH TWO TYPES OF FAILURE WITH CRITICAL HUMAN ERROR UNDER PREEMPTIVE-REPEAT REPAIR DISCIPLINE

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ABSTRACT

This paper presents the analytical study for point wise availability of a complex system consisting of two sub systems A and B. Subsystem A consists of two identical units arranged in parallel redundancy (1-out-of-2:G). Subsystem B consists of only one unit with two types of failure, viz. partial and catastrophic.

Key words: *Laplace Transforms, Supplementary Variables Techniques, Repair Disciplines.*

1. INTRODUCTION

Joseph and Tang [1] have calculated reliability with degradation data, whereas Guo and Haitao [2] have considered a new stochastic model under general repairs but no consideration has been given to failure due to critical Human Errors.

In view of above, the authors have evaluated point wise availability of a complex system consisting of two sub systems A and B incorporating the concept of critical Human Errors. The failure and repair times follow exponential and general time distributions respectively.

By using Supplementary Variable Technique, Laplace transforms of various state probabilities have been obtained. Numerical examples have also been added along with graphical illustration to high light the important results. The state transition diagram showing in figure-1.

2. ASSUMPTIONS

1. Initially the complex system is good.
2. Subsystems *A* & *B* are in series (1-out-of-2:F).
3. Subsystem *A* has 2 units in parallel (1-out-of-2:G).Both units are s-independent.
4. Each unit of subsystem *A* has same constant failure rate.
5. Each unit of subsystem *A* has 2 states good & failed.
6. Subsystem *B* is a single unit with two types of failure partial & catastrophic.
7. Any of the two subsystems *A* & *B* can also fail due to critical Human Error and consequently the complex system will automatically fail.
8. Partial failure brings subsystem *B* to degraded state and hence the complex system. Catastrophic failure can occur in this state too.
9. The catastrophic failure brings subsystem *B* to failed state and hence the complex system.
10. Subsystem *B* has 3 states: good, degraded & failed.
11. The failure & repair times for both subsystems follow exponential & general distributions respectively.
12. The complex system has only one repair facility & repair is like new. Repair never damages anything.
13. Failed *A*-units are repaired only when the complex system is in the failed state.
14. The repair of partial failure is opportunistic i.e. it would be undertaken along with the repair of catastrophic failure & subsystem *A.*
15. Under Preemptive-repeat repair discipline, priority is given to the repair of subsystem A over the repair of partial failure in subsystem B, and after completion of repair of subsystem A the repair of partial failure restarts and is considered as a fresh failure i.e., the repair already carried out on the partial failure goes waste.

NOTATIONS

λ_a	:	Failure rate of both the s-independent units of subsystem *A*.
λ_p, λ_c	:	Failure rates of subsystems *B* for partial & catastrophic failures respectively.

f	:	Constant Human Error failure rate.
η	:	Constant repair rate of failure due to Human Error.
$\psi_i(r)\overline{S_i}(r)$ $i=a, r=x$ $i=p, r=y$ $i=c, r=z$	:	Transition rates and probability density functions, repair of *A* is completed in elapsed repair time *x*, whereas repair of partial & catastrophic failures in *B* is completed in elapsed repair time *y* and *z* respectively.
$P_i(t)$	:	Probability that complex system is in good state at time t for i=0,1 & probability of being in failed state for i=8.
$P_4(x,t) \equiv$	:	The probability that at time *t* complex system is in failed state and elapsed repair time lies in the interval $(x, x+\Delta)$
$P_i(y,t) \equiv$	:	The probability that at time *t* the complex system is in degraded state and elapsed repair time lies in the interval $(y, y+\Delta)$ for i=2,3,5.
$P_j(z,t) \equiv$	:	Probability that at time *t*, the complex system is in failed state and is under repair of subsystem *A*, *x* time has elapsed in the repair of subsystem *A* & *y* time has already elapsed in the repair of partial failure of subsystem *B*.
$P_5(x,t) =$	:	Probability that at time t, the complex system is in failed state and is under repair of subsystem A, x time has elapsed in the repair of subsystem A and the partial failure, which has been in repair is awaiting repair a fresh.
$M_a, M_p, M_c =$	:	Mean time to repair of subsystem *A*, partial failure & catastrophic failure of subsystem *B* respectively and are defines as
$M_a =$	:	$\int_0^\infty x S_a(x)dx$
$M_p =$	:	$\int_0^\infty y S_p(y)dy$
$M_c =$	:	$\int_0^\infty z S_c(z)dz$
Where	:	$S_a(x), S_p(y) and S_c(z) = \psi_a(x)e^{-\int_0^\infty \psi_a(x)dx}, \psi_p(y)e^{-\int_0^\infty \psi_p(y)dy} and\, \psi_c(z)e^{-\int_0^\infty \psi_c(z)dz}$ Respectively

3. STATE TRANSITION DIAGRAM

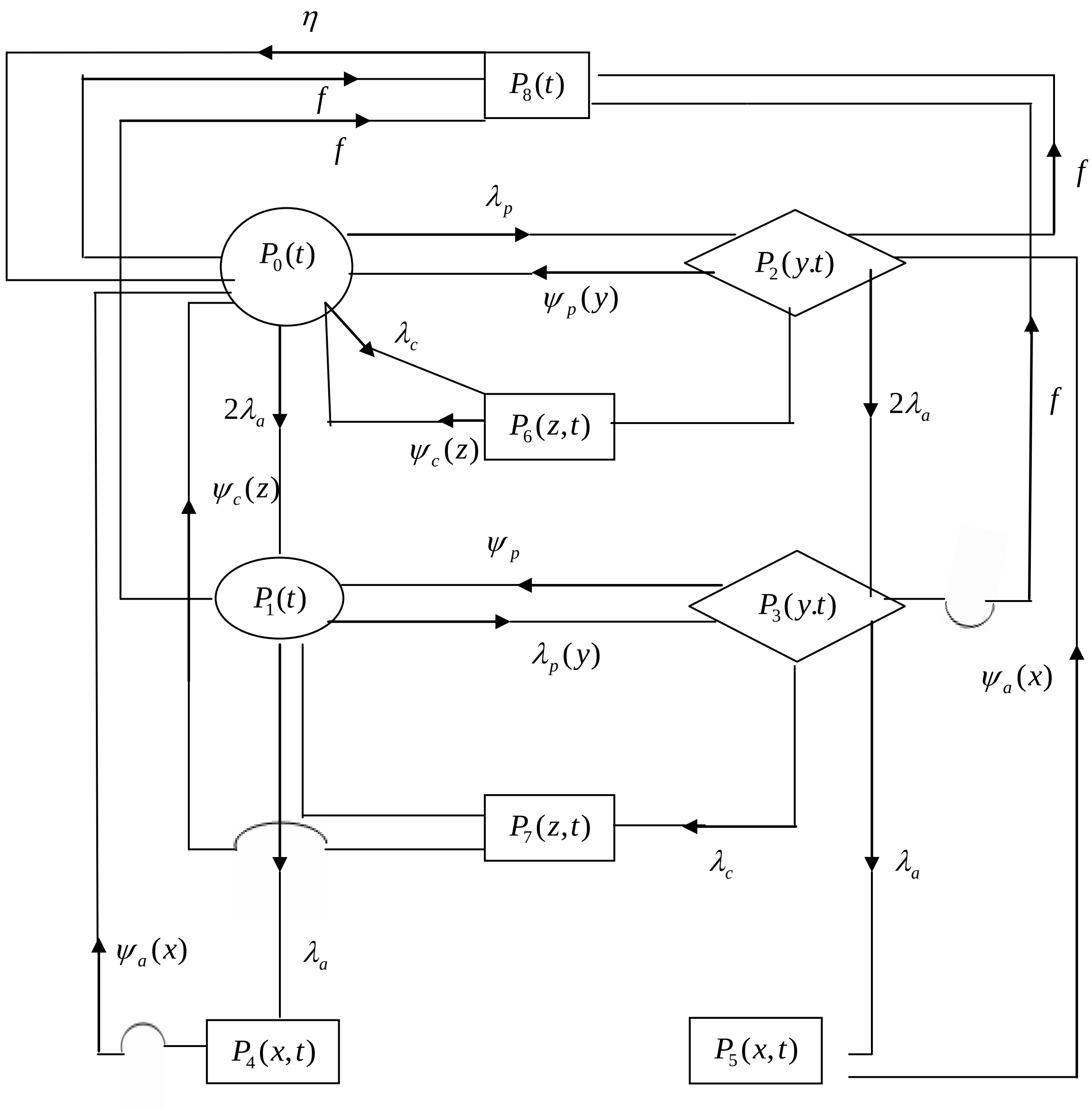
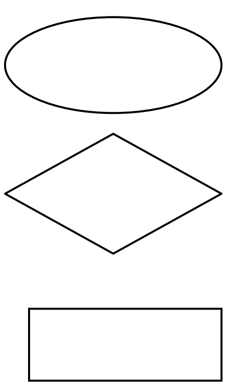


Fig:- 1.3.1



Operable State

Degraded State

Failed State

4. FORMULATION OF MATHEMATICAL MODEL

By elementary probability & continuity arguments, the differences-differential equations for stochastic process are:

$$\left(\frac{\partial}{\partial t}+2\lambda_a+\lambda_p+\lambda_c+f\right)P_0(t)=\int_0^\infty P_2(y,t)\psi_p(y)dy+\int_0^\infty P_4(x,t)\psi_a(x)dx$$

$$+\int_0^\infty P_6(z,t)\psi_c(z)dz+\int_0^\infty P_7(z,t)\psi_c(z)dz+\eta P_8(t) \qquad \ldots(1.3.1)$$

$$\left(\frac{\partial}{\partial t}+\lambda_a+\lambda_c+\lambda_p+f\right)P_1(t)=2\lambda_a P_0(t)+\int_0^\infty P_3(y,t)\psi_p(y)dy \qquad \ldots(1.3.2)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+2\lambda_a+\lambda_c+f+\psi_p(y)\right)P_2(y,t)=0 \qquad \ldots(1.3.3)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda_a+\lambda_c+f+\psi_p(y)\right)P_3(y,t)=2\lambda_a P_2(y,t) \qquad \ldots(1.3.4)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\psi_a(x)\right)P_4(x,t)=0 \qquad \ldots(1.3.5)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\psi_a(x)\right)P_5(x,t)=0 \qquad \ldots(1.3.6)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\psi_c(z)\right)P_6(z,t)=0 \qquad \ldots(1.3.7)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\psi_c(z)\right)P_7(z,t)=0 \qquad \ldots(1.3.8)$$

$$\left(\frac{\partial}{\partial t}+\eta\right)P_8(t)=f\left[p_0(t)+P_1(t)+\int_0^\infty P_2(y,t)dy+\int_0^\infty P_3(y,t)dy\right] \qquad \ldots(1.3.9)$$

These equations are to be solved under the following boundary & initial conditions:

BOUNDARY CONDITIONS

$$P_2(0,t)=\lambda_p P_0(t)+\int_0^\infty P_5(x,t)\psi_a(s)dx \qquad \ldots(1.3.10)$$

$$P_3(0,t)=\lambda_p P_1(t) \qquad \ldots(1.3.11)$$

$$P_4(0,t)=\lambda_a P_1(t) \qquad \ldots(1.3.12)$$

$$P_5(0,t)=\lambda_a P_3(t) \qquad \ldots(1.3.13)$$

$$P_6(0,t)=\lambda_c P_0(t)+\lambda_c P_2(t) \qquad \ldots(1.3.14)$$

$$P_7(0,t)=\lambda_c P_1(t)+\lambda_c P_3(t) \qquad \ldots(1.3.15)$$

INITIAL CONDITIONS

$$P_i(0)=\begin{cases}1, & \text{where } i=0\\ 0, & \text{otherwise}\end{cases} \qquad \ldots(1.3.16)$$

Taking Laplace Transforms from eq^n (1.3.1) to (1.3.15) one may obtain

$$\left(s+2\lambda_a+\lambda_c+\lambda_p+f\right)\overset{*}{P}_0(s)=1+\int_0^\infty \overset{*}{P}_2(y,s)\psi_P(y)dy+\int_0^\infty \overset{*}{P}_4(x,s)\psi_a(x)dx$$

$$+\int_0^\infty \overset{*}{P}_6(z,s)\psi_c(z)dz+\int_0^\infty \overset{*}{P}_7(z,s)\psi_c(z)dz+\eta\overset{*}{P}_8(s) \qquad \text{…(1.3.17)}$$

$$\left(s+\lambda_a+\lambda_c+\lambda_p+f\right)\overset{*}{P}_1(s)=2\lambda_a\overset{*}{P}_0(s)+\int_0^\infty \overset{*}{P}_3(y,s)\psi_P(y)dy \qquad \text{…(1.3.18)}$$

$$\left\{\frac{\partial}{\partial y}+s+2\lambda_a+\lambda_c+f+\psi_P(y)\right\}\overset{*}{P}_2(y,s)=0 \qquad \text{…(1.3.19)}$$

$$\left\{\frac{\partial}{\partial y}+s+\lambda_a+\lambda_c+f+\psi_P(y)\right\}\overset{*}{P}_3(y,s)=2\lambda_a\overset{*}{P}_2(y,s) \qquad \text{…(1.3.20)}$$

$$\left\{s+\frac{\partial}{\partial x}+\psi_a(x)\right\}\overset{*}{P}_4(x,s)=0 \qquad \text{…(1.3.21)}$$

$$\left\{s+\frac{\partial}{\partial x}+\psi_a(x)\right\}\overset{*}{P}_5(x,s)=0 \qquad \text{…(1.3.22)}$$

$$\left\{s+\frac{\partial}{\partial z}+\psi_c(z)\right\}\overset{*}{P}_6(z,s)=0 \qquad \text{…(1.3.23)}$$

$$\left\{s+\frac{\partial}{\partial z}+\psi_c(z)\right\}\overset{*}{P}_7(z,s)=0 \qquad \text{…(1.3.24)}$$

$$\left(s+\eta\right)\overset{*}{P}_8(s)=f\left[\overset{*}{P}_0(s)+\overset{*}{P}_1(s)+\int_0^\infty \overset{*}{P}_2(y,s)dy+\int_0^\infty \overset{*}{P}_3(y,s)dy\right] \qquad \text{…(1.3.25)}$$

$$\overset{*}{P}_2(0,s)=\lambda_p\overset{*}{P}_0(s)+\int_0^\infty P_5\left(x,s\right)\psi_a\left(s\right)dx \qquad \text{…(1.3.26)}$$

$$\overset{*}{P}_3(0,s)=\lambda_p\overset{*}{P}_1(s) \qquad \text{…(1.3.27)}$$

$$\overset{*}{P}_4(0,s)=\lambda_a\overset{*}{P}_1(s) \qquad \text{…(1.3.28)}$$

$$\overset{*}{P}_5(0,s)=\lambda_a\overset{*}{P}_3(s) \qquad \text{…(1.3.29)}$$

$$\overset{*}{P}_6(0,s)=\lambda_c\overset{*}{P}_0(s)+\lambda_c\overset{*}{P}_2(s) \qquad \text{…(1.3.30)}$$

$$\overset{*}{P}_7(0,s)=\lambda_c\overset{*}{P}_1(s)+\lambda_c\overset{*}{P}_3(s) \qquad \text{…(1.3.31)}$$

Solving equations (1.3.17) to (1.3.25) & making use of equations (1.3.26) to (1.3.31) one may obtain

$$\overset{*}{P}_0(s)=\frac{A(s)}{G(s)} \qquad \text{…(1.3.32)}$$

$$\overset{*}{P}_1(s)=\frac{B(s)}{G(s)} \qquad \text{…(1.3.33)}$$

$$\overset{*}{P}_2(s)=\left[\left\{\lambda_p+\lambda_a\alpha\left(s\right)\right\}\frac{A\left(s\right)}{G\left(s\right)}+\lambda_a\beta\left(s\right)_a\left(s\right)\frac{B\left(s\right)}{G\left(s\right)}\right]r_p\left(s+2\lambda_a+\lambda_c+f\right) \qquad \text{…(1.3.34)}$$

$$\overset{*}{P}_3(s)=\alpha(s)\frac{A(s)}{G(s)}+\beta(s)\frac{B(s)}{G(s)} \quad \text{...(1.3.35)}$$

$$\overset{*}{P}_4(s)=\lambda_a r_a(s)\frac{B(s)}{G(s)} \quad \text{...(1.3.36)}$$

$$\overset{*}{P}_5(s)=\lambda_a r_a(s)\left[\alpha(s)\frac{A(s)}{G(s)}+\beta(s)\frac{B(s)}{G(s)}\right] \quad \text{...(1.3.37)}$$

$$\overset{*}{P}_6(s)=\lambda_c r_c(s)\left[\left\{1+\left(\lambda_p+\lambda_a\alpha(s)S_a(s)\right)r_p(s+2\lambda_a+\lambda_c+f)\right\}\frac{A(s)}{G(s)}\right.$$
$$\left.+\left\{\lambda_a S_a(s)\beta(s)r_p(s+2\lambda_a+\lambda_c+f)\right\}\frac{B(s)}{G(s)}\right] \quad \text{...(1.3.38)}$$

$$\overset{*}{P}_7(s)=\lambda_c r_c(s)\left[\alpha(s)\frac{A(s)}{G(s)}+\left\{1+\beta(s)\right\}\frac{B(s)}{G(s)}\right] \quad \text{...(1.3.39)}$$

$$\overset{*}{P}_8(s)=C(s)\frac{A(s)}{G(s)}+D(s)\frac{B(s)}{G(s)} \quad \text{...(1.3.40)}$$

Where,

$$\alpha(s)=\frac{2\lambda_p\left\{r_p(s+\lambda_a+\lambda_c+f)-r_p(s+2\lambda_a+\lambda_c+f)\right\}}{1-2\lambda_a S_a(s)\left\{r_p(s+\lambda_a+\lambda_c+f)-r_p(s+2\lambda_a+\lambda_c+f)\right\}}$$

$$\beta(s)=\frac{\lambda_p r_p(s+\lambda_a+\lambda_c+f)}{1-2\lambda_a S_a(s)\left\{r_p(s+\lambda_a+\lambda_c+f)\right\}-r_p(s+2\lambda_a+\lambda_c+f)}$$

$$\gamma(s)=\left[2\lambda_p+2\lambda_a S_a(s)\alpha(s)\right]\left[S_p(s+\lambda_a+\lambda_c+f)-S_p(s+2\lambda_a+\lambda_c+f)\right]$$

$$\delta(s)=\left[\lambda_p S_p(s+\lambda_a+\lambda_c+f)+2\lambda_a S_a(s)\beta(s)\left\{S_p(s+\lambda_a+\lambda_c+f)-S_p(s+2\lambda_a+\lambda_c+f)\right\}\right]$$

$$A(s)=s+\lambda_a+\lambda_c+f+\lambda_p-\delta(s)$$

$$B(s)=2\lambda_a+\gamma(s)$$

$$C(s)=\left[1+\alpha(s)+\left\{\lambda_a S_a(s)\alpha(s)+\lambda_p\right\}r_p(s+2\lambda_a+\lambda_c+f)\right]$$

$$D(s)=\left[1+\beta(s)+\lambda_a r_p(s+2\lambda_a+\lambda_c+f)S_a(s)\beta(s)\right]$$

$$E(s)=s+2\lambda_a+\lambda_p+\lambda_c+f-\left\{\lambda_p+\lambda_a\alpha(s)S_a(s)\right\}S_p(s+2\lambda_a+\lambda_c+f)-\lambda_c S_c(s)$$
$$-\lambda_c S_c(s)r_p(s+2\lambda_a+\lambda_c+f)\left\{\lambda_p+\lambda_a\alpha(s)S_a(s)\right\}-\lambda_c\alpha(s)S_c(s)-\frac{f}{(s+\eta)}C(s)$$

$$F(s)=\lambda_a\beta(s)S_a(s)S_p(s+2\lambda_a+\lambda_c+f)+\lambda_a S_a(s)+\lambda_c S_c(s)\beta(s)$$
$$+\lambda_c S_c(s)\lambda_a S_a(s)\beta(s)r_p(s+2\lambda_a+\lambda_c+f)+\frac{f}{(s+\eta)}D(s)$$

$$G(s)=A(s)E(s)-B(s)F(s)$$

EVALUATION OF UP AND DOWN STATE PROBABILITIES:

The probabilities of the complex system being in up state is given by

$$\dot{P}_{up}(s)=\dot{P}_0(s)+\dot{P}_1(s)+\dot{P}_2(s)+\dot{P}_3(s)$$
$$=\left[1+\alpha(s)+\left\{\lambda_p+\lambda_a\alpha(s)S_a(s)\right\}r_p(s+2f_a+f_c+f)\right]\frac{A(s)}{G(s)}$$
$$+\left[1+\beta(s)+\lambda_a\beta(s)S_a(s)r_p(s+2\lambda_a+\lambda_c+f)\right]\frac{B(s)}{G(s)} \qquad \ldots(1.3.41)$$

The probability of the complex system being in down state is given by,

$$\dot{P}_{down}(s)=\dot{P}_4(s)+\dot{P}_5(s)+\dot{P}_6(s)+\dot{P}_7(s)$$
$$=\Big[\lambda_a r_a(s)\alpha(s)+\lambda_c r_c(s)\{1+\lambda_p r_p(s+2\lambda_a+\lambda_c+f)+\alpha(s)$$
$$+\lambda_a S_a(s)\alpha(s)r_p(s+2\lambda_a+\lambda_c+f)\}+C(s)\Big]\frac{A(s)}{G(s)}$$
$$+\Big[\lambda_a r_a(s)+\lambda_a r_a(s)\beta(s)+\lambda_c r_c(s)\{\lambda_a S_a(s)\beta(s)r_p(s+2\lambda_a+\lambda_c+f)$$
$$+1+\beta(s)\}+D(s)\Big]\frac{B(s)}{G(s)} \qquad \ldots(1.3.42)$$

It is interesting to note that

$$\dot{P}_{up}(s)+\dot{P}_{down}(s)=\frac{1}{s} \qquad \ldots(1.3.43)$$

STEADY STATE BEHAVIOUR OF THE SYSTEM

Using Abel's Lemma in Laplace transform

$$\lim_{t\to\infty}G(t)=\lim_{s\to 0}sG(s)=G(say)$$
$$P_{up}=\left[1+\lambda_p r_p(2\lambda_a+\lambda_c+f)+\lambda_a\alpha(0)r_p(2\lambda_a+\lambda_c+f)+\alpha(0)\right]P_0$$
$$+\left[1+\lambda_p\beta(0)r_p(2\lambda_a+\lambda_c+f)+\beta(0)\right]P_1 \qquad \ldots(1.3.44)$$
$$P_{down}=\left[\lambda_a M_a\alpha+\lambda_c M_c+\lambda_p r_p\ \lambda_a+\lambda_c+f+\lambda_a\alpha(0)r_p(2\lambda_a+\lambda_c+f)+\alpha(0)+C(0)\right]P_0$$
$$+\left[\lambda_a M_a\beta(0)+\lambda_c M_c\{\lambda_a\beta(0)r_p(2\lambda_a+\lambda_c+f)+1+\beta(0)\}+\lambda_a M_a+D(0)\right]P_1 \qquad \ldots(1.3.45)$$

$$Where\ P_0 = \frac{A(0)}{G(0)}$$

$$P_1 = \frac{B(0)}{G'(0)}$$

$$G'(0) = \left[\frac{d}{ds}G(s)\right]_{s=0}$$

5. NUMERICAL COMPUTATION:

To study the effect of practical catastrophic & human error failure on the availability of the complex system in steady state, some computation are made using equation (1.3.44) as under:

CASE I. PARTIAL FAILURE:

To study the effect of partial failure λ_p on system availability, taking λ_a = .0025, λ_c = .0085, f = .009, Ψ =1, η = 0.8, Ψ_p = .69, Ψ_a = .0045 varying of partial failure from 0.0105 to 0.0185 one may obtain Table 1.3.1

S.No.	λ_p	P_{up}
1	0.0105	0.952624
2	0.0115	0.9521015
3	0.0125	0.951588
4	0.0135	0.9511015
5	0.0145	0.950589
6	0.0155	0.950064
7	0.0165	0.949582
8	0.0175	0.9490595
9	0.0185	0.948274

Table 1.3.1

CASE II. CATASTROPHIC FAILURE:

The effect of catastrophic failure on system availability can be studies by

Fixing $\lambda_a = 0.0025, \lambda_p = .009, f = 0.008, \psi_a = 0.0045, \psi_p = 0.86, \psi_c = 1, \eta = 1$ and varying of λ_c from 0.15 to 0.85 one may obtain Table 1.3.2

S.No.	λ_c	P_{up}
1	0.15	0.87973395
2	0.25	0.80477045
3	0.35	0.7452345
4	0.45	0.6934895
5	0.55	0.647996
6	0.65	0.608363
7	0.75	0.573671
8	0.85	0.542436

Table 1.3.2

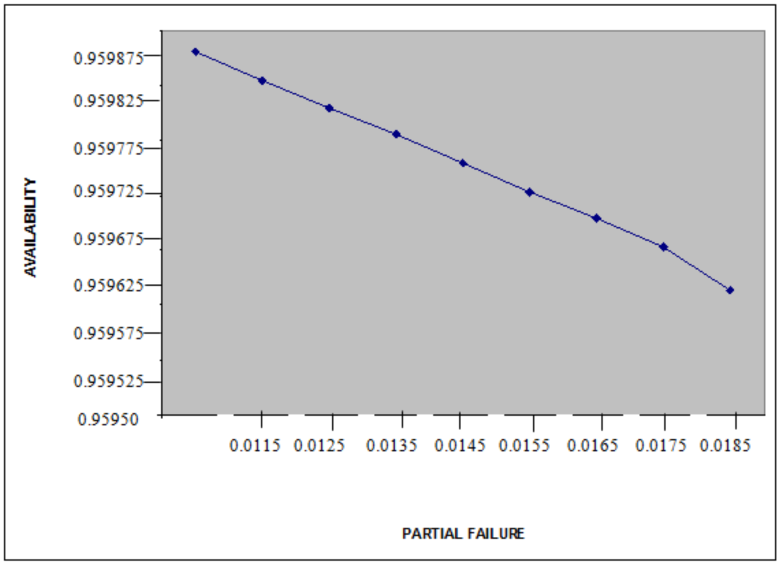
CASE III. CATASTROPHIC FAILURE:

To study the effect of Human Error failure on system availability

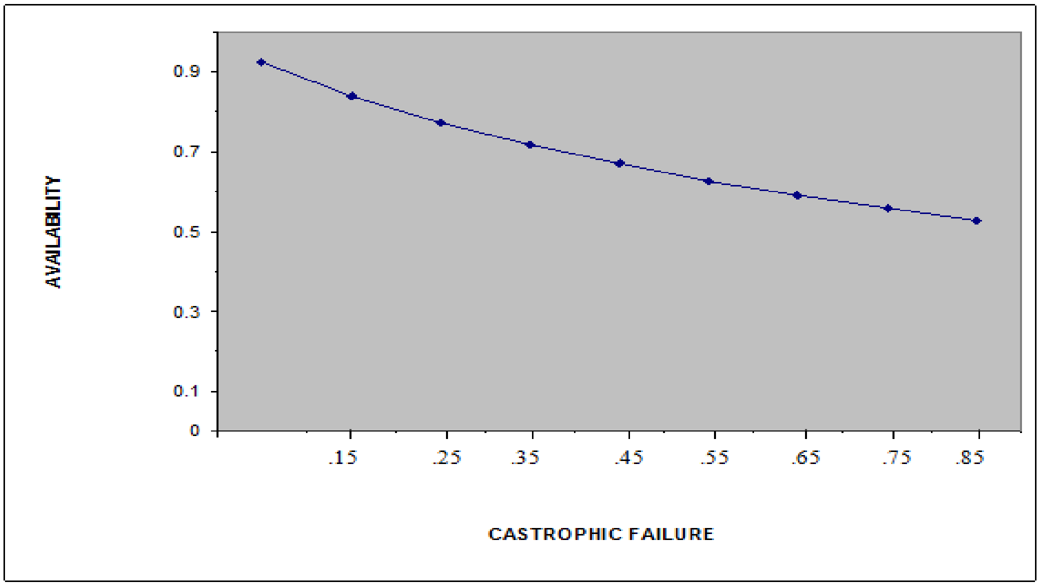
Fixing $\lambda_a = 0.0025, \lambda_p = .009, f = 0.008, \psi_a = 0.0045, \psi_p = 0.86, \psi_c = 1, \eta = 1$ and varying of f from 0.15 to 0.85 one may obtain Table 1.3.3

S.No.	λ_c	P_{up}
1	0.15	0.851358
2	0.25	0.788829
3	0.35	0.7333405
4	0.45	0.6826695
5	0.55	0.6370605
6	0.65	0.6000505
7	0.75	0.568178
8	0.85	0.5377275

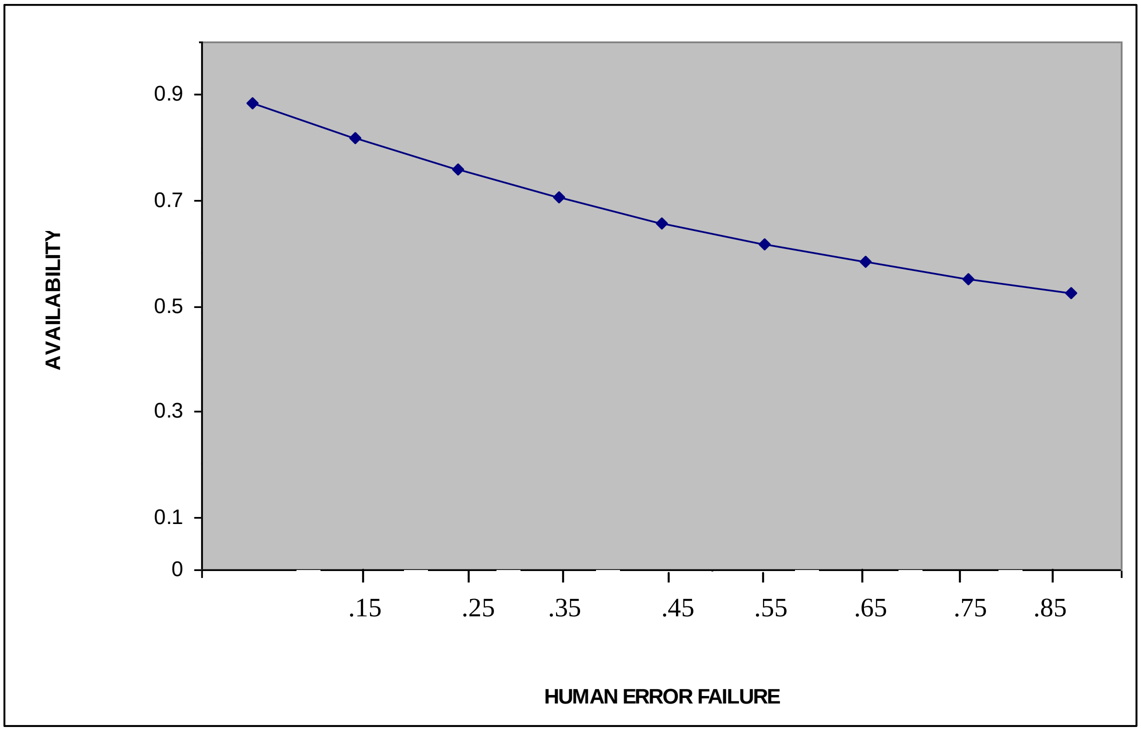
Table 1.3.3



Grap 1.3.1
Availability V/S Partial Failure



Graph 1.3.2
Availability V/S Catastrophic Failure



Graph 1.3.3
Availability V/S Human Error Failure

6. INTERPRETATION OF THE RESULTS

Case I: A critical examination of the Table 1.3.1 and Graph, "Availability V/S Partial Failure Rate" reveals the fact that the availability of the system decreased with increased in partial failure rate.
Case II: An observation of Table 1.3.2 and Graph, "Availability V/S Catastrophic failure rate" indicates that the availability of the system decreases rapidly with the increase of catastrophic failure rate & this decrease becomes small as the failure rate approaches towards 1.
Case III: An observation of the Table 1.3.3 and Graph, "Availability V/S Human Error failure" discloses the fact that the availability of the system decreases rapidly with increase in Human Error failure & this decrease becomes negligible as the failure rate approaches towards 1.

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