STOCHASTIC BEHAVIOUR OF COMMUNICATION SUBSYSTEM OF COMMUNICATION SATELLITE

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ABSTRACT

The authors in this paper have discussed the stochastic behavior of communication satellite for evaluation of some important reliability parameters. One Standby control unit has been taken to improve system's performance. This Standby unit can take place through a perfect switching device. Supplementary variable technique has been used to convert a Non-Markovian process in to Markovian one. Steady-State behavior of the system has obtained. All the transition State probabilities, in case repairs follows exponential time distribution, have also computed to improve practical utility of the model.

Keywords: *Supplementary variables, Perfect Switching, Standby redundancy, Availability and cost function.*

1. INTRODUCTION

The heart of a satellite is the communication subsystem. It consists of multiple transponders, which as explained earlier receive and amplify up-link signals, translate them in frequency and amplify again for retransmission as down-link signals. In the single conversion transponder (fig.1) only single frequency translation process takes place from the received signal to the transmitted signal.

The author divided the system into five subsystems, namely A (Antenna), R (Receiver), F_t (frequency translator), T (transmitter) and C (two control units in standby). Laplace transform and supplementary variable technique have been used to solve and formulate of mathematical model. All failures follow exponential time distribution where as all repairs follow general time distribution. Laplace transforms of various state probabilities have been obtained. A numerical example together with its graphical illustration has appended in last to highlight important results of this study.

2. ASSUMPTIONS

The following assumptions have been associated with this model:

1. Initially, the system works with its full efficiency.
2. Repair facilities are always available and after repair units work like new one.
3. Switching device used to online standby control unit, is perfect.
4. Failures are statistically independent. Nothing can fail from a failed state.
5. All the failures and repairs follow exponential and general time distribution, respectively.

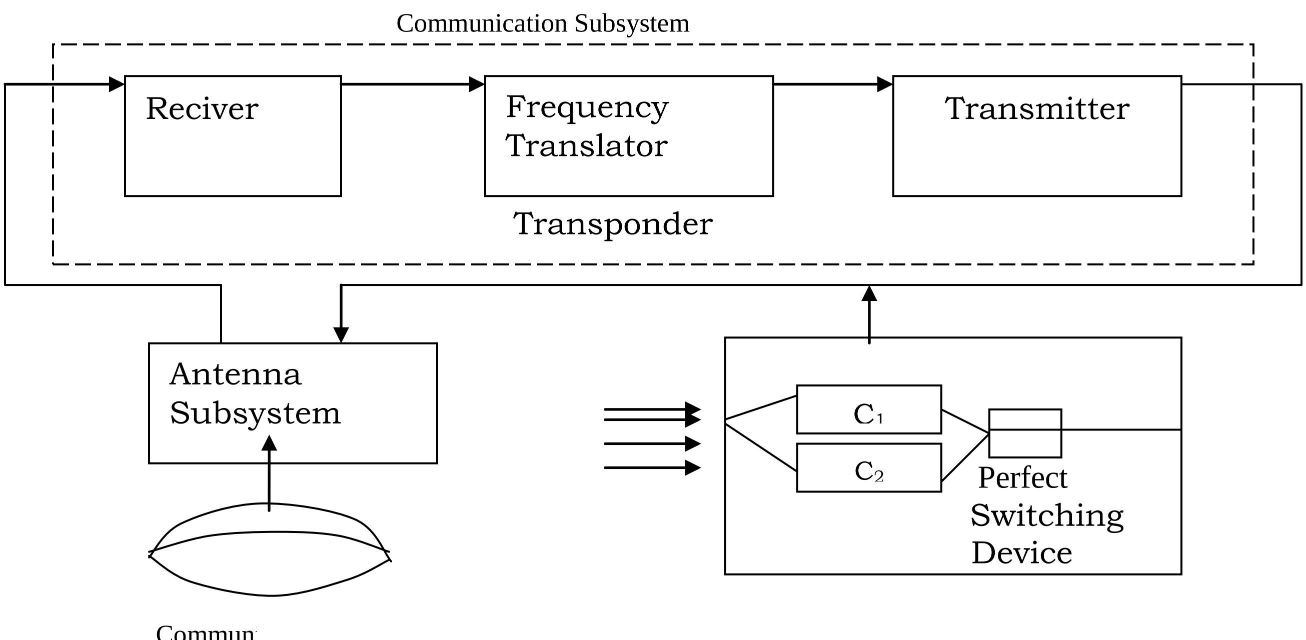

Fig-1: General block diagram of a satellite

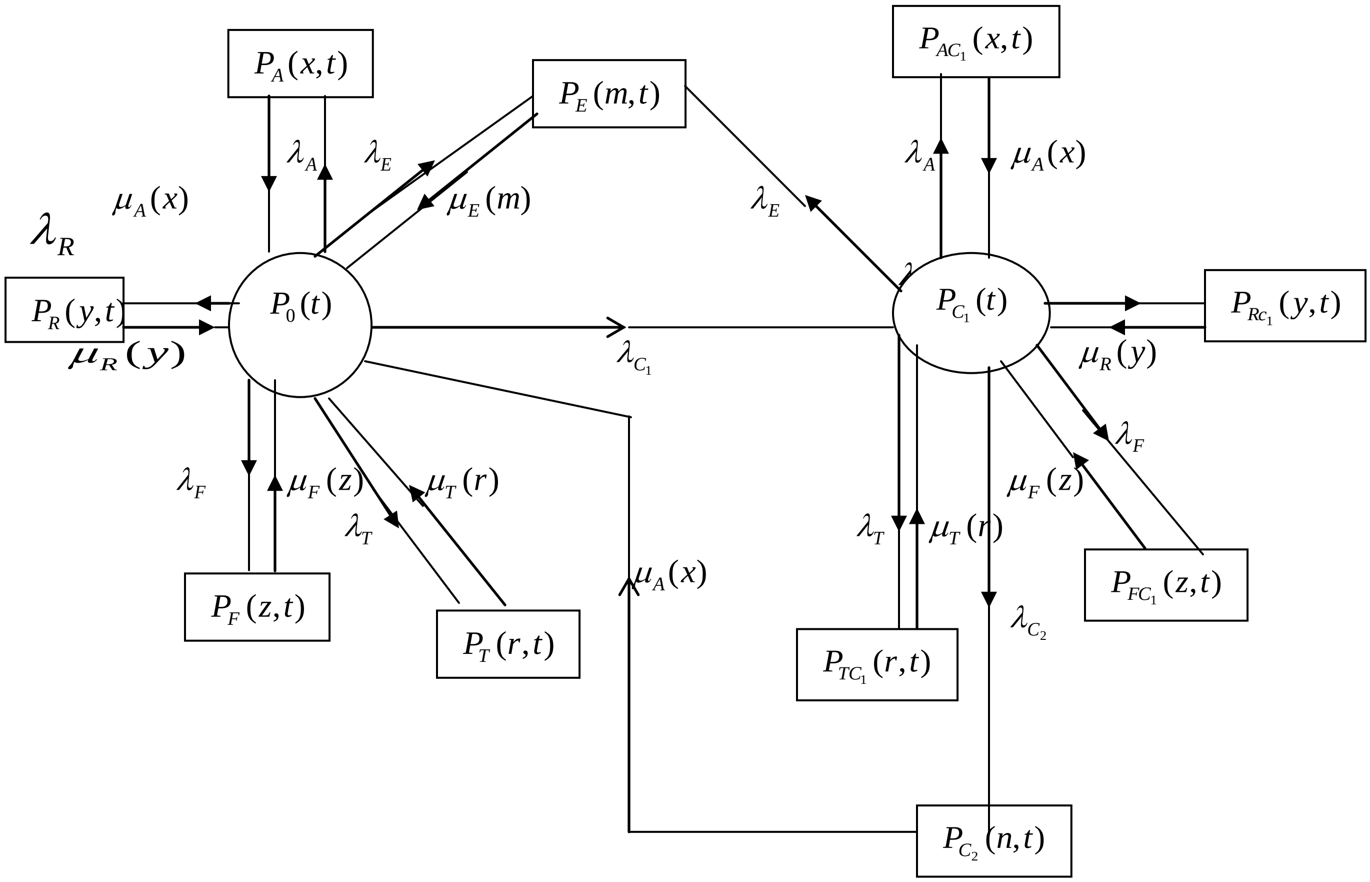

Fig-2: State-Transition Diagram

States: Good Failed

NOMANCLATURE

Following notations have been used throughout this study:

$P_0(t)$: The probability that at time‘t’, the system is in good state of full efficiency.

$P_i(j,t)\,\Delta$: The probability that at time‘t’, the system is in failed state due to failure of i th subsystem and elapsed repair time lies in the interval $(j, j+\Delta)$.

where, $$\mathrm{i = A, R, F, T, E, AC_1, RC_1, FC_1, TC_1, C_2 \text{ and }} \\ \mathrm{j = x, y, z, r, m, x, y, z, r, n, \quad respectively.}$$

$\mu_i(j)\Delta$: The first order probability that ith subsystem will be repaired in the time interval $(j, j+\Delta)$ conditioned that it was not repaired up to the time j.

$\lambda_A / \lambda_R / \lambda_F / \lambda_T / \lambda_E$	:	Failure rate of Antenna, receiver, frequency translator, transmitter and whole system due to environmental reasons.
$\lambda_{C_1} / \lambda_{C_2}$	:	Failure rate of control unit I and II.
$\overline{F}(s)$	:	Laplace transform of function F(t)
M_i	:	$-\left\{\frac{d}{ds}\left(\overline{S}_i(s)\right)\right\}_{s=0}$ = mean time to repair i^{th} unit.
$S_i(t)$	:	$\mu_i(j)\exp\left\{-\int \mu_i(j)dj\right\}$, $\forall$ i for j

3. FORMULATION OF MATHEMATICAL MODEL:

Using continuity argument, we obtain the following set of difference-differential equations, which are discrete in space and continuous in time, governing the behaviour of the model under consideration:

$$\left(\frac{\partial}{\partial t} + \lambda_A + \lambda_R + \lambda_F + \lambda_T + \lambda_E + \lambda_{C_1}\right)P_0(t) = \int_0^\infty \mu_A(x)P_A(x,t)dx + \int_0^\infty \mu_R(y)P_R(y,t)dy$$

$$+\int_0^\infty \mu_F(z)P_F(z,t)dz + \int_0^\infty \mu_T(r)P_T(r,t)dr + \int_0^\infty \mu_E(m)P_E(m,t)dm \quad \ldots(1)$$

$$+\int_0^\infty \mu_{C_2}(n)P_{C_2}(n,t)dn$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_A(x)\right]P_A(x,t) = 0 \quad \ldots(2)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \mu_R(y)\right]P_R(y,t) = 0 \quad \ldots(3)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \mu_F(z)\right]P_F(z,t) = 0 \quad \ldots(4)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial r} + \mu_T(r)\right]P_T(r,t) = 0 \quad \ldots(5)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial m} + \mu_E(m)\right]P_E(m,t) = 0 \quad \ldots(6)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial n}+\mu_{C_2}(n)\right]P_{C_2}(n,t)=0 \quad \ldots(7)$$

$$\left(\frac{\partial}{\partial t}+\lambda_E+\lambda_A+\lambda_R+\lambda_F+\lambda_T+\lambda_{C_2}\right)P_{C_1}(t)=\int_0^\infty \mu_A(x)P_{AC_1}(x,t)dx+\int_0^\infty \mu_R(y)P_{RC_1}(y,t)dy$$

$$+\int_0^\infty \mu_F(z)P_{FC_1}(z,t)+\int_0^\infty \mu_T(r)P_{TC_1}(r,t)dr+\lambda_{C_1}P_0(t) \quad \ldots(8)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_A(x)\right]P_{AC_1}(x,t)=0 \quad \ldots(9)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_R(y)\right]P_{RC_1}(y,t)=0 \quad \ldots(10)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\mu_F(z)\right]P_{FC_1}(z,t)=0 \quad \ldots(11)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial r}+\mu_T(r)\right]P_{TC_1}(r,t)=0 \quad \ldots(12)$$

Boundary conditions are:

$$P_A(0,t)=\lambda_A P_0(t) \quad \ldots(13)$$

$$P_R(0,t)=\lambda_R P_0(t) \quad \ldots(14)$$

$$P_F(0,t)=\lambda_F P_0(t) \quad \ldots(15)$$

$$P_T(0,t)=\lambda_T P_0(t) \quad \ldots(16)$$

$$P_E(0,t)=\lambda_E P_0(t)+\lambda_E P_{C_1}(t) \quad \ldots(17)$$

$$P_{AC_1}(0,t)=\lambda_A P_{C_1}(t) \quad \ldots(18)$$

$$P_{RC_1}(0,t)=\lambda_R P_{C_1}(t) \quad \ldots(19)$$

$$P_{FC_1}(0,t)=\lambda_F P_{C_1}(t) \quad \ldots(20)$$

$$P_{TC_1}(0,t)=\lambda_T P_{C_1}(t) \quad \ldots(21)$$

$$P_{C_2}(0,t)=\lambda_{C_2} P_{C_1}(t) \quad \ldots(22)$$

Initial conditions:

$P_0(0)=1$ and all other state probabilities at $t=0$ are zero. …(23)

4. SOLUTION OF THE MODEL:

Taking Laplace transforms of equations (1) through (22) by making use of initial conditions (23) and then on solving them one by one we obtain the following Laplace transforms of different state probabilities:

$$\overline{P}_0(s)=\frac{1}{A(s)} \quad \text{…(24)}$$

$$\overline{P}_A(s)=\frac{1}{A(s)}\lambda_A D_A(s) \quad \text{…(25)}$$

$$\overline{P}_R(s)=\frac{1}{A(s)}.\lambda_R D_R(s) \quad \text{…(26)}$$

$$\overline{P}_F(s)=\frac{1}{A(s)}.\lambda_F D_F(s) \quad \text{…(27)}$$

$$\overline{P}_T(s)=\frac{1}{A(s)}.\lambda_T D_T(s) \quad \text{…(28)}$$

$$\overline{P}_{C_1}(s)=\frac{1}{A(s)}.B(s) \quad \text{…(29)}$$

$$\overline{P}_E(s)=\frac{1}{A(s)}.[1+B(s)]\lambda_E D_E(m) \quad \text{…(30)}$$

$$\overline{P}_{C_2}(s)=\frac{1}{A(s)}.B(s).\lambda_{C_2} D_{C_2}(s) \quad \text{…(31)}$$

$$\overline{P}_{AC_1}(s)=\frac{1}{A(s)}.B(s).\lambda_A D_A(s) \quad \text{…(32)}$$

$$\overline{P}_{RC_1}(s)=\frac{1}{A(s)}.B(s).\lambda_R D_R(s) \quad \text{…(33)}$$

$$\overline{P}_{FC_1}(s)=\frac{1}{A(s)}.B(s).\lambda_F D_F(s) \quad \text{…(34)}$$

$$\overline{P}_{TC_1}(s)=\frac{1}{A(s)}.B(s).\lambda_T.D_T(s), \qquad \ldots(35)$$

where

$$B(s)=\frac{\lambda_{C_1}}{s+\lambda_A+\lambda_R+\lambda_F+\lambda_T+\lambda_E+\lambda_{C_2}-\lambda_A\overline{S}_A(s)-\lambda_R\overline{S}_R(s)-\lambda_F\overline{S}_F(s)-\lambda_T\overline{S}_T(s)} \qquad \ldots(36)$$

$$A(s)=s+\lambda_A+\lambda_R+\lambda_F+\lambda_T+\lambda_{C_1}-\lambda_A\overline{S}_A(s)-\lambda_R\overline{S}_R(s)-\lambda_F\overline{S}_F(s)-\lambda_T\overline{S}_T(s)$$
$$-\lambda_E\overline{S}_E(s)-\lambda_E B(s)\overline{S}_E(s)-\lambda_{C_2}B(s)\overline{S}_{C_2}(s) \qquad \ldots(37)$$

and $D_i(j)=\frac{1-\overline{S}_i(j)}{j}$, for all i and j …(38)

5. ERGODIC BEHAVIOUR OF THE SYSTEM:

By making use of Abel's Lemma, *Viz*, $\lim_{s\to o} s\overline{P}(s)=\lim_{t\to\infty}P(t)=P(say)$, provided the limit on right exists, we have the following time- independent state probabilities from equations (24) through (35):

$$P_0=\frac{1}{A'(0)} \qquad \ldots(39)$$

$$P_A=\frac{1}{A'(0)}\lambda_A M_A \qquad \ldots(40)$$

$$P_R=\frac{1}{A'(0)}.\lambda_R M_R \qquad \ldots(41)$$

$$P_F=\frac{1}{A'(0)}.\lambda_F M_F \qquad \ldots(42)$$

$$P_T=\frac{1}{A'(0)}.\lambda_T M_T \qquad \ldots(43)$$

$$P_{C_1}=\frac{1}{A'(0)}.B(0)=\frac{1}{A'(0)}.B \text{ (say)} \qquad \ldots(44)$$

$$P_E=\frac{1}{A'(0)}[1+B]\lambda_E M_E \qquad \ldots(45)$$

$$P_{C_2} = \frac{1}{A'(0)}.B\lambda_{C_2} M_{C_2}$$ …(46)

$$P_{AC_1} = \frac{1}{A'(0)}.B\lambda_A M_A$$ …(47)

$$P_{RC_1} = \frac{1}{A'(0)}.B\lambda_R M_R$$ …(48)

$$P_{FC_1} = \frac{1}{A'(0)}.B\lambda_F M_F$$ …(49)

$$P_{TC_1} = \frac{1}{A'(0)}.B\lambda_T M_T ,$$ …(50)

where $B(0) = \frac{\lambda_{C_1}}{\lambda_{C_2} + E} = B$ …(51)

$$M_i = -\overline{S}'_i(0) \;\; for\ all \;\; \text{i}$$ …(52)

$$A'(0) = \left[\frac{d}{ds}(A(s))\right]_{s=0}$$ …(53)

6. SOME PARTICULAR CASES:

(a) ***when repairs follow exponential time distribution***

Setting $\overline{S}_i(j) = \frac{\mu_i}{j + \mu_i}$, for all *i* and *j* in equations (24) through (35), one may obtain the following transition state probabilities, in this case:

$$\overline{P}_0(s) = \frac{1}{A_1(s)}$$ …(54)

$$\overline{P}_A(s) = \frac{\lambda_A}{A_1(s)}.\frac{1}{s + \mu_A}$$ …(55)

$$\overline{P}_R(s) = \frac{\lambda_R}{A_1(s)}.\frac{1}{s + \mu_R}$$ …(56)

$$\overline{P}_F(s)=\frac{\lambda_F}{A_1(s)}.\frac{1}{s+\mu_F} \quad \ldots(57)$$

$$\overline{P}_T(s)=\frac{\lambda_T}{A_1(s)}.\frac{1}{s+\mu_T} \quad \ldots(58)$$

$$\overline{P}_{C_1}(s)=\frac{1}{A_1(s)}.B_1(s) \quad \ldots(59)$$

$$\overline{P}_E(s)=\frac{1}{A_1(s)}.\frac{[1+B_1(s)]\lambda_E}{s+\mu_E} \quad \ldots(60)$$

$$\overline{P}_{C_2}(s)=\frac{B_1(s)}{A_1(s)}.\frac{\lambda_{C_2}}{s+\mu_{C_2}} \quad \ldots(61)$$

$$\overline{P}_{AC_1}(s)=\frac{B_1(s)}{A_1(s)}.\frac{\lambda_A}{s+\mu_A} \quad \ldots(62)$$

$$\overline{P}_{RC_1}(s)=\frac{B_1(s)}{A_1(s)}.\frac{\lambda_R}{s+\mu_R} \quad \ldots(63)$$

$$\overline{P}_{FC_1}(s)=\frac{B_1(s)}{A_1(s)}.\frac{\lambda_F}{s+\mu_F} \quad \ldots(64)$$

$$\overline{P}_{TC_1}(s)=\frac{B_1(s)}{A_1(s)}.\frac{\lambda_T}{s+\mu_T}\text{, where} \quad \ldots(65)$$

$$A_1(s)=s+\lambda_A+\lambda_R+\lambda_F+\lambda_E+\lambda_{C_1}-\lambda_A\frac{\mu_A}{s+\mu_A}-\lambda_R\frac{\mu_R}{s+\mu_R}-\lambda_F\frac{\mu_F}{s+\mu_F}-\lambda_T\frac{\mu_T}{s+\mu_T}$$
$$-\lambda_E B_1(s)\frac{\mu_E}{s+\mu_E}-\lambda_{C_2}B_1(s)\frac{\mu_{C_2}}{s+\mu_{C_2}} \quad \ldots(66)$$

and

$$B_1(s)=\frac{\lambda_{C_1}}{s+\lambda_A+\lambda_R+\lambda_F+\lambda_T+\lambda_E+\lambda_{C_2}-\lambda_A\frac{\mu_A}{s+\mu_A}-\lambda_R\frac{\mu_R}{s+\mu_R}-\lambda_F\frac{\mu_F}{s+\mu_F}-\lambda_T\frac{\mu_T}{s+\mu_T}}$$
$$-\lambda_E\frac{\mu_E}{s+\mu_E}-\lambda_E B_1(s).\frac{\mu_E}{s+\mu_E}-\lambda_{C_2}B_1(s)\frac{\mu_{C_2}}{s+\mu_{C_2}} \quad \ldots(67)$$

(b) Availability and profit function for the system:

We have, $\bar{P}_{up}(s) = \dfrac{1}{s+\lambda_A+\lambda_R+\lambda_F+\lambda_T+\lambda_E+\lambda_{C_1}}\left[1+\dfrac{\lambda_{C_1}}{s+\lambda_A+\lambda_R+\lambda_F+\lambda_T+\lambda_{C_2}+\lambda_E}\right]$

or, $\bar{P}_{up}(s) = \dfrac{1}{s+a}\left[1+\dfrac{\lambda_{C_1}}{s+b}\right]$,

where $a = \lambda_A+\lambda_R+\lambda_F+\lambda_T+\lambda_E+\lambda_{C_1}$

$b = \lambda_A+\lambda_R+\lambda_F+\lambda_T+\lambda_{C_2}+\lambda_E$

On taking inverse Laplace transform, we obtain

$$P_{up}(t) = e^{-at} - \frac{\lambda_{C_1}}{a-b}e^{-at} + \frac{\lambda_{C_1}}{a-b}e^{-bt} \qquad \ldots(68)$$

It is interesting to note that

$$P_{up}(0) = 1$$

Also, $P_{down}(t) = 1 - P_{up}(t)$ …(69)

Again, the profit function for the system is given by

$$G(t) = C_1\int_0^t P_{up}(t)dt - C_2t - C_3,$$

Where C_1 revenue cost per unit time is, C_2 is repair cost per unit time and C_3 is system establishment cost for one period, so here,

$$G(t) = C_1\left\{\frac{1-e^{-at}}{a}\left(1-\frac{\lambda_{C_1}}{a-b}\right)+\frac{1-e^{-bt}}{b}\frac{\lambda_{C_1}}{a-b}\right\} - C_2t - C_3 \qquad \ldots(70)$$

where *a* and *b* have mentioned earlier.

(c) Numerical Computation

For a numerical computation, let us consider the data $\lambda_A = .01$, $\lambda_R = .02$, $\lambda_F = .03$, $\lambda_T = .04$, $\lambda_E = .05$, $\lambda_{C_1} = .06$, $\lambda_{C_2} = .07$, $C_1 = Rs50/\,unit\ time$, $C_2 = Rs20/\,unit\ time$, $C_3 = Rs50/$ setup and t=0,1,2,--------. By putting these values in equations (68), (69) and (70), we plot various graphs shown in figures (3), (4) and (5), to observe the changes in values of availability and profit function. Tables 1,2, and 3 show the value of $P_{up}(t), P_{down}(t)$ and G(t), respectively.

7. RESULTS AND DISCUSSION

All the transition state probabilities, in case repairs follow exponential time distribution, have been computed to improve practical utility of the model. Steady-state behaviour of the system has also been obtained. One numerical computation with its graphical illustration has mentioned in the end to highlight important results of study.
An examination of *table -1* and *fig-3* reveals that the availability of the system decreases catastrophically in the beginning but after t =4, it decreases in a constant manner. Also, *fig-4* is the graph 'Profit function vs t' and the corresponding values are given in the *table -2*. A critical examination of *fig-4* yields that profit function is negative in starting because, initially we invest money to establish the new system but G(t) becomes positive and its value starts increasing up to t=6 and thereafter it again decreases. Maximum value of G (t) is 44.5488 and it is for t=6.

t	$\mathbf{P_{up}(t)}$
0	1
1	.858976934
2	.735109212
3	.6270346
4	.533276193
5	.452337741
6	.382766373
7	.323191787
8	.272348649
9	.229087238
10	.192376047
11	.161249056

Table-1

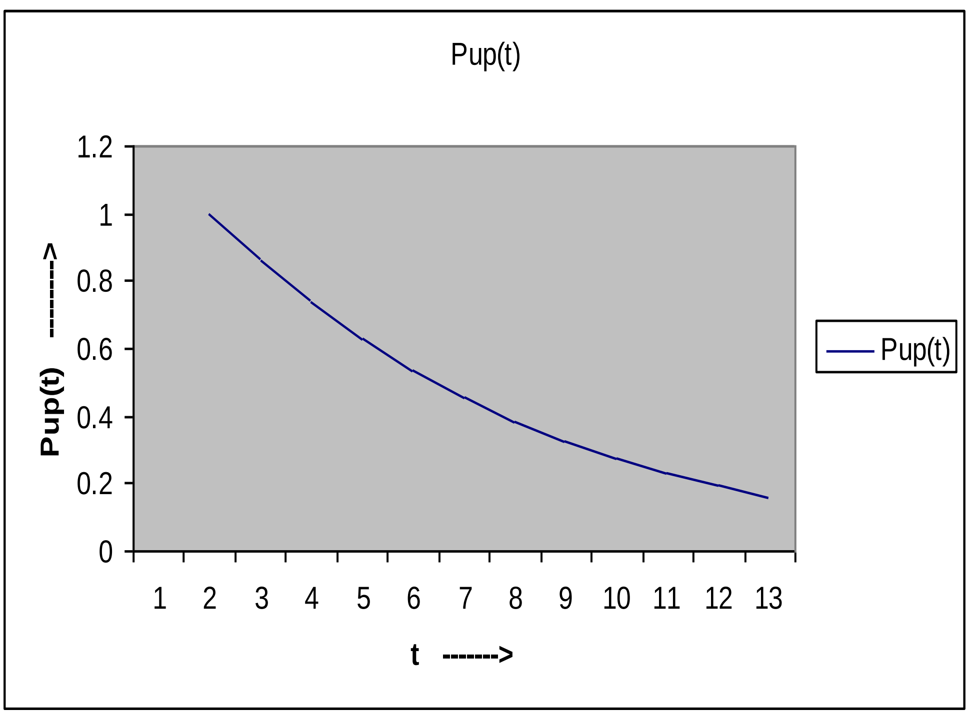

Fig- 3

t	G(t)
0	-30
1	-3.59965
2	16.1837
3	30.17445
4	39.12565
5	43.7157
6	44.5488
7	42.1588
8	32.3679
9	29.51995
10	20.03125

Table-2

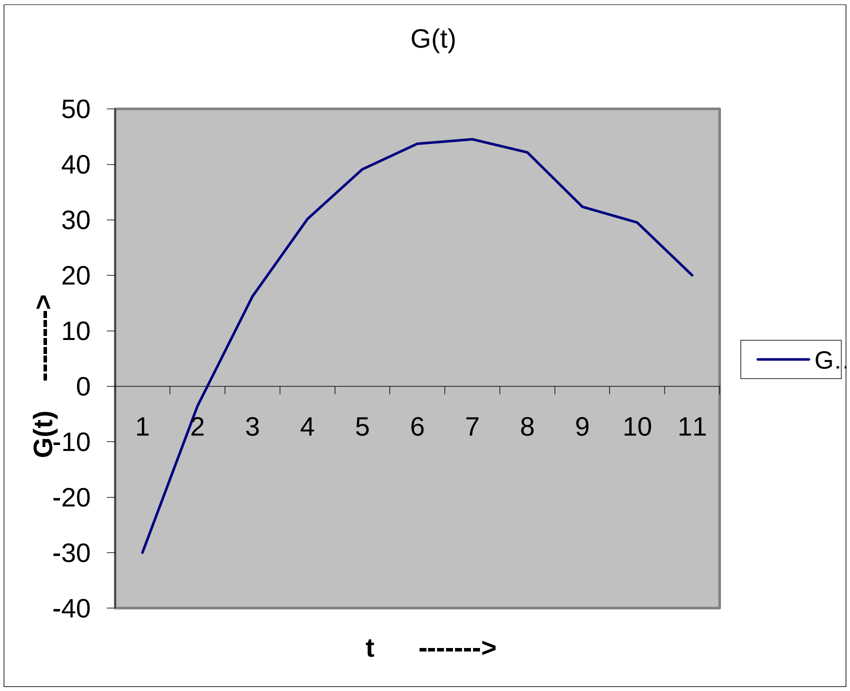

Fig- 4

8. REFERENCES

[1]. **Satellite Communications Systems Engineering**, Prentice Hall, Inc., New Jersey, 1993. G.D. Gordon and W.L. Morgan:

[2]. **Principles of Communication Satellites**, John Wiley & Sons, Inc., N.Y., 1993, 533 pp.

[3]. **Hyderi, P. D.; Joorel, J. P. S.: "**Stochastic Behaviors of a Two Unit Cold Standby Redundant System Subject to Random Failure", Microelectronic Reliab. , Vol. 36(2), pp 243 – 246, 1996.

[4]. **Satellite Communications Systems,** John Willey and Sons, Fourth Edition, 2003, 757pp. Satellite and satellite subsystems reliability: Statistical data analysis and modeling

[5]. **Capes', V.; Bakstein**, D.: "An Introduction to Continuous-time Stochastic Processes: Theory, Models and Applications", Birkhauser Publication, 2004

[6]. **Sharma, Deepankar, Sharma, Jyoti;** "Estimation of reliability parameters for telecommunication system, Journal of combinatorics, information and system sciences," Vol.29, Nos.1-4, pp151-160(2005)

[7]. **Sharma, Deepankar, Geol, C.K.; Sharma, Vinit:** "Reliability and MTTF evaluation of telecommunication system, Bulletin of pure and applied Sciences," Vol.24 (E) No.2, pp349-354(2005).