RELIABILITY CHARACTERISTIC OF COLD-STANDBY REDUNDANT SYSTEM

S.C. Agarwal[1], Mamta Sahani[2] & Shikha Bansal[3]

[1] Reader, Department. of Mathematics, M.M (P.G) College, Modinagar, India
[2] Lecturer,D.J college of Engineering and Technology, Modinagar, India
[3] Lecturer, Krishna Institute of Engineering and Technology, Ghaziabad, India

ABSTRACT

The objective of this paper is to improve the system reliability through append the redundant component. In this paper, two units in cold-standby is considered. Each unit of the system has two modes via; operable and failed. The failures of units are of two types: minor and major. After major failure, cold standby unit replaces the failed unit after a random amount of time. The failure and repair times follow exponential and general time distributions respectively. We transform the basic equations of the proposed model into an integro-differential equation and solve it using Supplementary variables techniques, various reliability parameters have been computed and analyzed by tabular and graphical illustrations.

Keywords: *Steady-state behaviour, Redundancy, Cold-standby*

1. INTRODUCTION

Redundancy plays an important role in enhancing system reliability. The redundancy allocation problem has been analyzed for many different system structure [6 ,8,11]. One of the commonly used form of the redundancy is the standby redundancy. Standby system often finds applications in various industrial and other setups. In a standby redundant system, some additional paths are created for the proper functioning of the system. Standby unit is support to increase the reliability of the system.

In general there are 3-types of standby i.e. cold, hot and warm standby. Cold standby means that the redundant components cannot fail while they are waiting. Earlier many researchers [1, 3, 12] have discussed when an operative unit fails, its repair starts immediately but in practical problem it may not possible always. In practice, system do not always fail with major breakdown. Igichart and Igichart and lemoline; Arti R.(1993); Singh, S.B.(1998) developed various mathematical models consists two types of failure namely major and minor.

A maintenance policy that suits a system presenting two types of failure represented by many earlier researchers [2,10]. Revealed minor failures are removed by a minor repair that brings the system back to the operating condition just previous to failure. A major repair is restore the system as good as new. Keeping this fact in view, the author has considered a complex system consisting of an operative unit and cold-standby unit having two repair facilities. Initially the system starts functioning in a normal efficiency state. When the operative unit fails, the standby units starts functioning. Due to imperfect switching system stops working and goes to failed state P_A then the standby unit starts functioning and goes to the state P_B ,further if there is major failure in A, system goes to the state P_{W_A} .moreover if the unit B breakdown then system goes to failed state P_F and it will repair. Using Supplementary variables technique Laplace transforms of various state probabilities have been evaluated. Numerical examples have also been added to highlight the important results.

2. ASSUMPTIONS

1) Initially system is in operable state.
2) The system consists of two unit one operative and other cold-standby.
3) If both units fail, the whole system fails completely.
4) The repair facility is not always available with the system, but is called upon whenever the operative units fails
5) The system has two repair facilities.
6) Failure and repair time follow exponential and general time distribution respectively.

3. NOTATIONS

λ_A, λ_B = constant failure rate of units A and B

$\mu_A(x), \mu_{A_M}$ =Minor and Major repair rate of unit A

$\mu(x)$ =Repair rate of both units A and B

W_S =Time for inspecting the switch

W_A =weighting time for calling the repairman for unit A

$P_0(t)$ = Probability of the system in operable state

$P_F(t)$ = Probability of the system in failed state

$P_A(t)$ = Probability of the unit A in failed state

$P_B(t)$ = Probability of the unit B In operable state

$P_{W_A}(t)$ = Probability of weighting time

State Transition Diagram

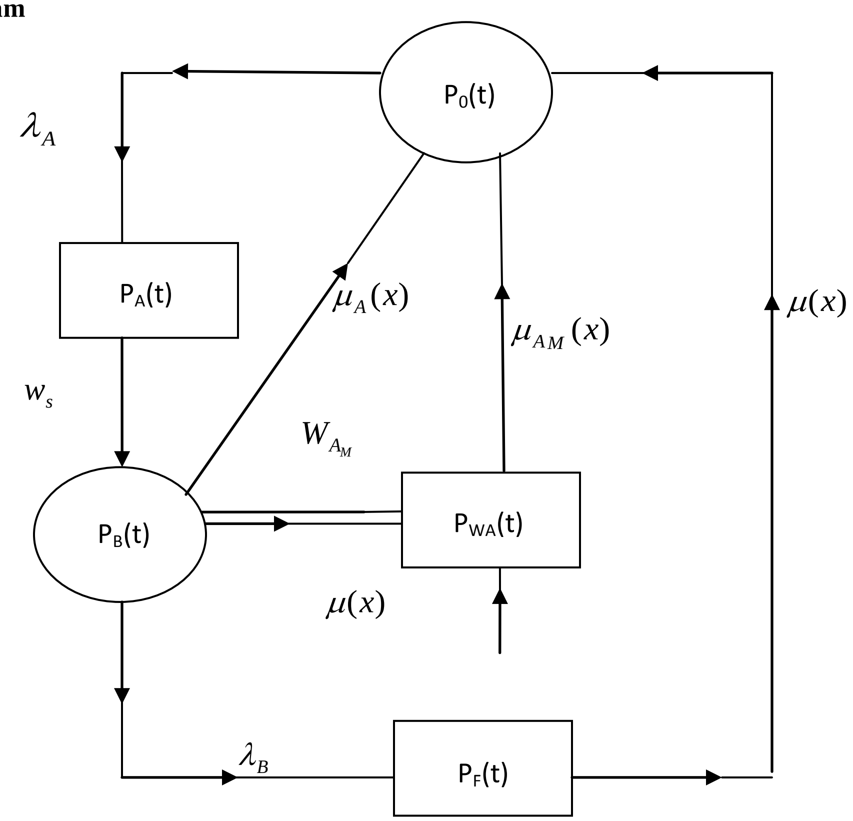

Fig. 1

4. FORMULATION OF THE MATHEMATICAL MODEL

By elementary and continuity arguments, the difference-differential equations governing the stochastic Behaviour of the complex system are:

$$\left[\frac{\partial}{\partial t}+\lambda_A\right]P_0(t)=\int_0^\infty \mu_A(x)P_B(x,t)dx+\int_0^\infty \mu_{A_m}(x)P_{W_A}(x,t)dx+\int_0^\infty \mu_A(x)P_f(x,t)dx \quad \text{....(1)}$$

$$\left[\frac{\partial}{\partial t}+w_s\right]P_A(t)=\lambda_A P_0(t) \quad \text{...(2)}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_A(x)+w_A+\lambda_B\right]P_B(x,t)=0 \quad \text{...(3)}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_{A_m}(x)\right]P_{W_A}(x,t)=0 \quad \text{...(4)}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\mu(x)\right]P_f(x,t)=0 \quad \text{...(5)}$$

BOUNDARY CONDITIONS

$$p_B(0,t)=W_s P_A(t)+\int_0^\infty \mu(x)P_f(x,t)dx \quad \text{...(6)}$$

$$P_{W_A}(0,t)=W_A P_B(t) \quad \text{...(7)}$$

$$P_f(0,t)=\lambda_B P_B(t) \quad \text{...(8)}$$

INITIAL CONDITIONS

$$P_0(t)=1, \text{otherwise zero} \quad \text{...(9)}$$

5. SOLUTION OF THE MODEL

Taking Laplace transform of (1) to (9) and on further simplification, one may obtain

$$\overline{P}_0(s)=\frac{1}{A(s)} \quad \text{...(10)}$$

$$\overline{P}_A(s)=\frac{\lambda_A}{(s+W_s)}\cdot\frac{1}{A(s)} \quad \text{...(11)}$$

$$\overline{P}_B(s)=\frac{W_s\lambda_A}{(s+W_s)}\left\{\frac{1-\overline{S}_A(s+W_A+\lambda_B)}{(s+W_A+\lambda_B)}\right\}\cdot\frac{1}{B(s)}\cdot\frac{1}{A(s)} \quad \text{...(12)}$$

$$\overline{P}_{W_A}(s)=\left[\frac{W_A W_s\lambda_A}{(s+W_s)}\left\{\frac{1-\overline{S}_A(s+W_A+\lambda_B)}{(s+W_A+\lambda_B)}\right\}\left\{\frac{1-\overline{S}_{\mu_{A_m}}(s)}{s}\right\}\frac{1}{B(s)}\right]\frac{1}{A(s)} \quad \text{...(13)}$$

$$\bar{p}_f(s)=\left[\frac{W_s\lambda_A\lambda_B}{(s+W_s)}\left\{\frac{1-\bar{S}_A(s+W_A+\lambda_B)}{(s+W_A+\lambda_B)}\right\}\left\{\frac{1-\bar{S}_{2\mu}(s)}{s}\right\}\frac{1}{B(s)}\right]\frac{1}{A(s)} \qquad ...(14)$$

Where,

$$A(s)=s+\lambda_A-\frac{W_s\lambda_A\bar{S}_A(s+W_A+\lambda_B)}{(s+W_s)}-\left[\begin{matrix}\left\{\frac{W_s\lambda_A}{(s+W_s)}\cdot\frac{1-\bar{S}_A(s+W_A+\lambda_B)}{(s+W_A+\lambda_B)}\frac{1}{B(s)}\right\}\\ \left\{W_A\bar{S}_{\mu_{A_M}}(s)+\frac{\lambda_B}{2}\bar{S}_{2\mu}(s)+\frac{\lambda_B}{2}\bar{S}_{2\mu}(s)\bar{S}_A(s+W_A+\lambda_B)\right\}\end{matrix}\right]$$

… (15)

It is worth nothing that

$$\bar{P}_0(s)+\bar{P}_A(s)+\bar{P}_B(s)+\bar{P}_{W_A}(s)+\bar{P}_f(s)=\frac{1}{s}$$

6. EVALUATION OF L.T. OF UP AND DOWN STATE PROBABILITIES

The L.T.of the probabilities that the system is in operable and down state at time ‘t’ can be are evaluated as follows:

$$\bar{P}_{UP}(s)=\bar{P}_0(s)+\bar{P}_B(s)$$
$$=\left[1+\frac{W_s\lambda_A}{(s+W_s)}\left\{\frac{1-\bar{S}_A(s+W_A+\lambda_B)}{(s+W_A+\lambda_B)}\right\}\frac{1}{B(s)}\right]\frac{1}{A(s)} \qquad ...(16)$$

$$\bar{P}_{down}(s)=\bar{P}_A(s)+\bar{P}_{W_A}(s)+\bar{P}_f(s)$$
$$=\left[\frac{\lambda_A}{(s+W_s)}+\frac{W_s\lambda_A}{(s+W_s)}\left\{\frac{1-\bar{S}_A(s+W_A+\lambda_B)}{(s+W_A+\lambda_B)}\right\}\left\{W_A\left(\frac{1-\bar{S}_{\mu_{A_m}}(s)}{s}\right)+\lambda_B\left(\frac{1-\bar{S}_{2\mu}(s)}{s}\right)\right\}\frac{1}{B(s)}\right]\frac{1}{A(s)} \qquad ...(17)$$

7. ERGODIC BEHAVIOUR

Using Abel’s Lemma

$$\underset{s\to 0}{Lt}\left(s\bar{f}(s)\right)=\underset{t\to\infty}{Lt}f(t)=f(say)$$

Provided that the limit on the right hand side exists, the time independent up and down state probabilities are obtained ass follows:

$$P_{UP} = \left[1 + \frac{\lambda_A}{(W_A + \lambda_B + \mu_A)B(0)}\right]\frac{1}{A'(0)} \quad ...(18)$$

$$P_{DOWN} = \left[\frac{\lambda_A}{W_s} + \frac{\lambda_A}{(W_A + \lambda_B + \mu_A)}\left\{\frac{W_A}{\mu_{A_m}} + \frac{\lambda_B}{2\mu}\right\}\frac{1}{B(0)}\right]\frac{1}{A'(0)} \quad ..,(19)$$

8. RELIABILITY

The reliability of the system is given by

$$R(s) = \frac{1}{A(s)}\left[1 + \frac{w_s \lambda_A}{(s + w_s)(s + \lambda_B + w_A)}\right]$$

Where A(s)=s+ λ_A

Assuming λ_A =0.01, λ_B =0.02, w_s=0.03, w_A=0.04, one may obtain

R (t) =1.3 $e^{-0.01t} + 0.2e^{-0.06t}$ --0.5 e $^{-0.03t}$

Time	R(t)
0	1
1	0.990195
2	0.98076
3	0.971668
4	0.962892
5	0.954408
6	0.946194
7	0.938229
8	0.930494
9	0.92297
10	0.915462
11	0.908493
12	0.901509
13	0.894677
14	0.887984

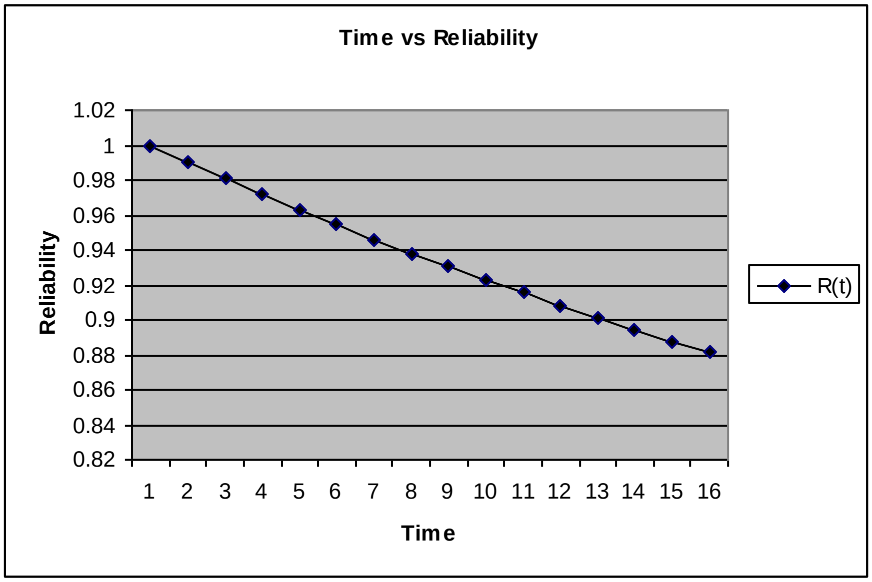

Fig:2

9. MEAN TIME TO FAILURE

The mean time to failure (MTTF) of the system is given by

$$\text{MTTF} = \frac{\lambda_B + w_A + \lambda_A}{(\lambda_B + w_A)\lambda_A}$$

λ_A	**MTTF** $w_A = 0.015$	**MTTF** $w_A = 0.035$	**MTTF** $w_A = 0.055$
0.001	0.001025	0.001017	0.001013
0.002	0.0021	0.002067	0.00205
0.003	0.003225	0.00315	0.003113
0.004	0.0044	0.004267	0.0042
0.005	0.005625	0.005417	0.005313
0.006	0.0069	0.0066	0.00645
0.007	0.008225	0.007817	0.007613
0.008	0.0096	0.009067	0.0088
0.009	0.011025	0.01035	0.010013
0.01	0.0125	0.011667	0.01125

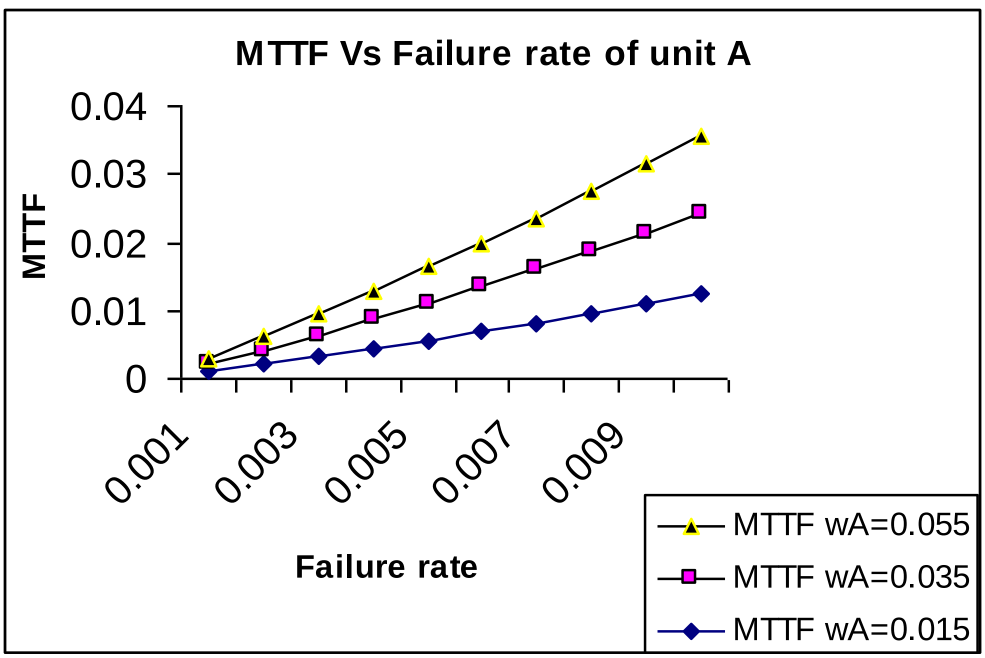

Fig:3

CONCLUSION

From fig: 2 it is clear that the Reliability of the system decreases as the time period increases and fig: 3 display the variation in MTTF as the minor failure of unit A increases. The series of curve represent that MTTF increases as the failure rate increases.

REFERENCES

[1] G.S. Mokaddis, 2M. Salah EL-Sherbeny and 3Y. Mahmoud Ayid, Stochastic Behavior of a Two-Unit Warm Standby System with Two Types of Repairmen and Patience Time, Journal of Mathematics and Statistics 5 (1): 42-46, 2009ISSN 1549-364 © 2009 Science Publications

[2] Bruno Bieth, Liang Hong and Jyotirmoy Sarkar, A standby system with two types of repair persons, Appl. Stochastic Models Bus. Ind. (2009)

[3] Xian-yun Meng, Li Yuan, Rui-ling Yin, The Reliability Analysis of a Two-unit Cold Standby System with FailableSwitch and Maintenance Equipment, 1-4244-0605-6/06/$20.00 C2006 IEEE.

[4] Srinivasan, S.[1]; Subramanian, R. Reliability analysis of a three unit warm standby redundant system with repair Volume 143, Number 1, March 2006 , pp. 227-235(9)

[5] Mettas and W. Zhao, Modeling and analysis of complex repairable systems with general repair , in Proceedings of R A M S, Jan. 2005

[6] Amir Azaron ,Hideki Katagiri ,Kosuke Kato, Masatoshi Sakawa, Reliability Evaluation and Optimization Of Dissimilar-Component Cold-Standby Redundant Systems, Journal of the Operations Research Society of Japan 2005, Vol. 48, No. 1, 71-8

[7] Kuo-Hsiung Wang*, Yu-Ju Lai, and Jyh-Bin Ke, Reliability and Sensitivity Analysis of a System with Warm Standbysand a Repairable Service Station, International Journal of Operations Research Vol. 1, No. 1, 61−70 (2004)

[8] David w. coit, maximization of system reliability with a choice of redundancy strategies,IIE transactions(2003)35,535-543

[9] K.H. Wang, C.C . Kuo , Cost and probabilistic analysis of series systems with mixed standby components , Applied Mathematical Modelling 24 (2000) 957- 967

[10] G.S.,Mokaddis , M.L. Tawfek and S.A.M, Elhssia, Cost Analysis of a two dissimilar – unit cold standby redundant system subject to inspection and two types of repair. Microelectron. Reliab ,1997, 37, 335 -340.

[11] g. s. mokaddis and m. l. tawfek, some characteristics of a two-dissimilar-unit cold standby redundant system with three modes**,** *Microelectron. Reliab.,* Vol. 36, No. 4, pp. 497 503, 1996

[12] Nianfu yang and b. s. dhillon, stochastic analysis of a general standby system with constant human error and arbitrary system repair rates, *Microelectron. Reliab.,* Vol. 35, No. 7, pp. 1037-1045, 1995