GENERALIZED LIU-TYPE ESTIMATOR FOR LINEAR REGRESSION

Rasha A. Farghali

Department of Mathematics, Insurance and Applied Statistics, Faculty of Commerce and Business Administration, Helwan University, Cairo, Egypt.

ABSTRACT

Multicollinearity in linear regression affects negatively on the variance of the ordinary least squares estimators. That leads to inflated confidence intervals and theoretically important variables become insignificant in testing hypotheses. In this article, the generalized Liu-type estimator is proposed, that has smaller mean squared error than the ordinary least squares estimators. The proposed estimator is a general biased estimator which includes other generalized biased estimators such as generalized Liu estimators and generalized ridge regression estimators as special cases of the proposed estimator. A Monte Carlo Simulation study is given to evaluate the performance of this estimator. Portland cement data is used to illustrate the results. Simulation results showed that generalized Liu-type estimators with different shrinkage parameters are robust to correlation between the independent variables. Therefore, the OLS should not be used in the presence of severe multicollinearity. Also, application results showed that the suggested estimator outperforms OLS estimates in terms of smaller mean squared errors.

Keywords and phrases: *Generalized Liu estimator, Liu estimator, Liu-type estimator, Multicollinearity, Ridge regression estimator, Generalized ridge regression*.

1. INTRODUCTION

Multicollinearity means that there is a near dependency between the independent variables $x_1, x_2, \ldots, x_p$. In linear regression models, ordinary least squares (OLS) method is used to estimate the regression parameters, $\hat{\beta}_{OLS}$ are unbiased estimators with minimum variances but their performance becomes poor when multicollinearity exists. Thus, $\hat{\beta}_{OLS}$ still unbiased but with large variances, inflated confidence intervals and theoretically important variables become insignificant in testing hypotheses.

Applying biased estimators is a remedy to the negative results of multicollinearity problem. Many shrinkage estimators are introduced to correct the multicollinearity problem in linear regression, logistic regression and multinomial logistic regression see [1,4,5,6 and 7]. Such as ridge regression and generalized ridge regression estimators see [16,17,18,19 and 20], Liu estimator and Liu-Type estimator see [8, 21, 24,25 and 26].

Motivated by [16,17, 18] and [2, 8,12,24,25, and 26]. This paper aims to introduce the generalized Liu-type estimator to deal with multicollinearity in linear regression model. A Monte Carlo simulation is conducted to study the properties of the proposed estimator.

The rest of the paper is organized as follows. In section 2, the new shrinkage estimator in linear regression model is proposed and some methods for determining the shrinkage parameters are explained. In section 3, the performance of the suggested estimators is evaluated using a Monte Carlo simulation study. A real data has been applied in section 4. Finally, a brief summary and conclusions are provided in section 5.

2. GENERALIZED LIU-TYPE ESTIMATOR

2.1. Methodology

Consider the real relation:

$$Y^* = X^*\beta + \varepsilon \qquad (1)$$

Where X^* is a $n \times p$ matrix of the standardized independent variables, which is predetermined, Y^* is a $n \times 1$ vector of the standardized dependent variable, ε is a $n \times 1$ vector of errors with $E(\varepsilon) = 0$ and $V(\varepsilon) = \sigma^2 I$. The ordinary least squares estimators are obtained through the following model:

$$Min \quad s = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} \left(y_i{}^* - \sum_{j=1}^{p} \beta_j x^*{}_{ij}\right)^2$$

s.t.

$$\frac{\partial s}{\partial \beta_j} = 0 \qquad j = 1,2, \ldots, p \qquad (2)$$

Thus, $\hat{\beta}_{OLS} = (X^{*\prime}X^*)^{-1}(X^{*\prime}Y^*)$, and $Var(\hat{\beta}_{OLS}) = \hat{\sigma}^2 \, (X^{*\prime}X^*)^{-1}$, and $\hat{\sigma}^2$ is the residual mean square and it is defined as $\hat{\sigma}^2 = \frac{(Y^*-X^*\hat{\beta}_{OLS})\prime(Y^*-X^*\hat{\beta}_{OLS})}{n-p}$. $\hat{\beta}_{OLS}$ are the best estimators to estimate β, but their performance becomes poor when multicollinearity exists. When the information matrix $(X^{*\prime}X^*)$ is ill-conditioned, the values of $(X^{*\prime}X^*)^{-1}$ become large. Thus, $\hat{\beta}_{OLS}$ still unbiased but with large variances, large mean squared errors $MSE(\beta)$, inflated confidence intervals and theoretically important variables have insignificant variables in testing hypotheses.

Regularization methods using penalization are based on penalized least squares (PLS), this method helps to deal with the issue of multicollinearity, by putting constraints on the values of the estimated parameters, thus, the entries of the variance-covariance matrix metric is also reduced. In penalized least squares a penalty function $Pen(\beta)$ is added to model (2) as follows:

$$Min \quad LS = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} \left(y^*{}_i - \sum_{j=1}^{p} \beta_j x^*{}_{ij}\right)^2$$

s.t.

$$\frac{\partial s}{\partial \beta_j} = 0 \qquad j = 1,2, \ldots, p$$

$$Pen(\beta) \quad \leq \quad t \qquad (3)$$

Where, (β) , is a specific penalty function of the estimates $\hat{\beta}_j$, $j = 1,2, \ldots, p$ that penalizes the size of the parameter, t , is a tuning parameter. This restricted optimization is solved by Lagrange method. Penalized estimators were used for compacting multicollinearity in linear and nonlinear regression models. One of the oldest penalization methods for linear models was introduced in [16, 17] which is ridge penalty, known as L_2 penalized regression, on the L_2 norm of the regression parameters. Thus, the ridge regression estimates $\hat{\beta}_{RR}$ are obtained by minimizing the ridge regression criterion see [18]. (RR) dealt with this problem by adding a single shrinkage parameter k to the main diagonal of the matrix $(X^{*\prime}X^*)$, diagonal so that,

$$\|\hat{\beta}_{RR}\| < \|\hat{\beta}_{OLS}\| \qquad (4)$$

this led to biased estimates with smaller values and smaller standard errors than those of $\hat{\beta}_{OLS}$. Ridge regression estimator is defined as:

$$\beta_{RR} = (X^{*\prime}X^* + k\, I)^{-1}(X^{*\prime}X^*)\, \hat{\beta}_{OLS} \quad , \quad k \geq 0 \quad (5)$$

But, in (GRR) a shrinkage parameter k_j , $j = 1,2, \ldots, p$ is specified for each independent variable $x^*{}_j$, $j = 1,2, \ldots, p$. It is defined as:

$$\beta_{GRR} = (X^{*\prime}X^* + K)^{-1}(X^{*\prime}X^*)\, \hat{\beta}_{OLS} \quad (6)$$

Where: $K = diag(k_1, k_2, \ldots, k_p), k_j \geq 0$, $j = 1,2, \ldots, p$.

If $k_1 = k_2 = \cdots = k_p = k$, $k > 0$, then $\beta_{GRR} = \beta_{RR}$, and if $k = 0$, then $\beta_{RR} = \beta_{OLS}$.

Following [16,17] a new biased estimator [23] was introduced, it is defined as:

$$\hat{\beta}_{LE} = (X^{*\prime}X^* + I)^{-1}(X^{*\prime}X^* + d\, I)\hat{\beta}_{OLS} \qquad 0 < d < 1 \quad (7)$$

He also introduced a its generalized form by replacing the single shrinkage parameter d by multiple shrinkage parameters, d_j, $j = 1,2, \ldots, p$, the generalized Liu estimator is defined as:

$$\hat{\beta}_{GLE} = (X^{*\prime}X^* + I)^{-1}(X^{*\prime}X^* + D)\hat{\beta}_{OLS} \qquad (8)$$

Where: $D = diag(d_1, d_2, \dots, d_p), 0 < d_j < 1 \ , j = 1,2, \dots, p$.

If $d_1 = d_2 = \cdots = d_p = d$, $0 < d < 1$, then $\beta_{GLE} = \beta_{LE}$ and if $d = 1$, then $\beta_{LE} = \beta_{OLS}$.

It has been noticed that the existence of multicollinearity is accompanied by high R^2 and most of the independent variables are insignificant, so that, methods for diagnosing multicollinearity should be used. Condition index $(C.I.)$ is commonly used to diagnose multicollinearity, it is defined as:

$$C.I. = \sqrt{\frac{\lambda_{max}}{\lambda_{min}}} \qquad (9)$$

Where, λ_{max} and λ_{min} are the largest and the smallest eigenvalues of $(X^{*\prime}X^{*})$, in [10] it was suggested that: if $C.I. \leq 10$, then there is no multicollinearity among $x_1, x_2, \dots, x_p$, if $10 < C.I. < 30$, then the multicollinearity is moderate and may be corrected, but if $C.I. \geq 30$, then it means that there is a severe multicollinearity and corrective actions must be taken. In practical, the shrinkage parameter in ridge regression k, is usually small close to zero and $C.I.$ of the matrix $(X^{*\prime}X^{*} + k\,I)$ is relatively large, this means that the conditioning problem still exist . This problem was discussed in [25,26], he ensured that if we want to control the condition index to a small level, k should be large, else the obtained ridge regression estimators will be unstable. To overcome this problem, he suggested another parameter d adjust the ridge regression estimator (4) to make the equation still give good fit.

He proposed a two-parameter estimator known as (Liu-type).The two parameters (k, d) have different functions; $k, k > 0$, aims to reduce the condition index of the matrix $(X^{*\prime}X^{*} + k\,I)$, $k \geq 0$ to a desired level, and $d, -\infty < d < \infty$, aims to reduce $MSE(\hat{\beta}_{LTE})$, see [13] . The two parameter estimator is a generalization of Liu estimator, introduced by [24], it is defined as:

$$\hat{\beta}_{LT} = (X^{*\prime}X^{*} + k\,I)^{-1}(X^{*\prime}X^{*} - d\,I)\hat{\beta}_{OLS} \quad k \geq 0 , -\infty < d < \infty \quad (10)$$

The generalized Liu estimator and introduced the feasible generalized Liu estimator and almost unbiased feasible generalized Liu estimator was extended by using the balanced loss function as a basis of evaluating the performance of these estimators [3].

Following [16, 17] and [24, 25 and 26], this study aims to introduce the generalized Liu-type estimator $(GLTE)$ which is a generalization for Liu-type estimator, it is defined as:

$$\hat{\beta}_{GLTE} = (X^{*\prime}X^{*} + K)^{-1}\,(X^{*\prime}X^{*} - D)\hat{\beta}_{OLS} \qquad (11)$$

Where:

$K = diag(k_1, k_2, \dots, k_p)$, $k_j \geq 0$, $D = diag(d_1, d_2, \dots, d_p)$, $-\infty < k_j < \infty$, $j = 1,2, \dots, p$. With a variance-covariance matrix is defined as follows:

$$Var(\hat{\beta}_{GLTE}) = \hat{\sigma}^2 (X^{*\prime}X^{*} + K)^{-1}\,(X^{*\prime}X^{*} - D)(X^{*\prime}X^{*})^{-1}(X^{*\prime}X^{*} - D)(X^{*\prime}X^{*} + K)^{-1} \qquad (12)$$

Lemma 1.

The proposed biased estimator represents a general case, as it is easy to see that

$$\lim_{K \to I} \hat{\beta}_{GLTE} = (X^{*\prime}X^{*} + I)^{-1}\,(X^{*\prime}X^{*} - D)\hat{\beta}_{OLS} = \hat{\beta}_{GLE} \,,$$

$$\lim_{D \to O} \hat{\beta}_{GLTE} = (X^{*\prime}X^{*} + K)^{-1}\,(X^{*\prime}X^{*})\hat{\beta}_{OLS} = \hat{\beta}_{GRR},$$

$$\lim_{K \to O, D \to O} \hat{\beta}_{GLTE} = (X^{*\prime}X^{*})^{-1}\,(X^{*\prime}X^{*})\hat{\beta}_{OLS} = \hat{\beta}_{OLS},$$

2.2. Obtaining the shrinkage parameters of the proposed estimator:

It was proved that $\hat{\beta}_{LT}$ has two advantages over ridge regression, first, it has less MSE, second, it allows larger k and thus the ill-conditioning problem is fully addressed see [25 ,26]. In order to provide the explicit form of the function $MSE(\hat{\beta}_{LT})$, we use the following transformations, Suppose that there exists a matrix T such that:

$$T'(X'X)T = \Lambda = diag(\lambda_1, \lambda_2, \dots, \lambda_p) \qquad (13)$$

Where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p$, are the ordered eigenvalues of $(X^{*\prime}X^*)$ and T is a $(p \times p)$ orthogonal matrix whose columns are the corresponding eigenvectors of $\lambda_1, \lambda_2, \dots, \lambda_p$. Rewrite model (1) in canonical form:

$$Y^* = Z\alpha + \epsilon \qquad (14)$$

Where $Z = X^*T$, $\alpha = T'\beta$, for model (14), the ordinary least squares estimator is:

$$\hat{\alpha}_{OLS} = \Lambda^{-1}Z'Y^* \qquad (15)$$

The corresponding estimates $\hat{\beta}$ can be obtained as:

$$\hat{\beta}_{OLS} = T\,\hat{\alpha}_{OLS},\ \hat{\beta}_{GRR} = T\,\hat{\alpha}_{GRR}\ \text{, and } \hat{\beta}_{GLE} = T\,\hat{\alpha}_{GLE}$$

The suggested estimator is defined as:

$$\hat{\alpha}_{GLTE} = (\Lambda + K)^{-1}\,(\Lambda - D)\hat{\alpha}_{OLS} \qquad (16)$$

Where:

$K = diag(k_1, k_2, \dots, k_p)$, $k_j \geq 0$, $D = diag(d_1, d_2, \dots, d_p)$, $-\infty < k_j < \infty$, $j = 1,2, \dots, p$ and its variance-covariance matrix is defined as:

$$Var(\hat{\alpha}_{GLTE}) = \hat{\sigma}^2(\Lambda + K)^{-1}\,(\Lambda - D)\Lambda^{-1}(\Lambda - D)(\Lambda + K)^{-1} \qquad (17)$$

$$Var(\hat{\alpha}_{GLTE}) = \hat{\sigma}^2 \sum_{j=1}^{p} \frac{(\lambda_j - d_j)^2}{\lambda_j(\lambda_j + k_j)^2} \qquad (18)$$

Since that, (GLTE) is a biased estimator, so the bias will be:

$$E(\hat{\alpha}_{GLTE}) = (\Lambda + K)^{-1}\,(\Lambda - D)\alpha$$

$$bias^2 = (E(\hat{\alpha}_{GLTE}) - \alpha)^2$$

$$= \left(\sum_{j=1}^{p} \frac{(\lambda_j - d_j)}{(\lambda_j + k_j)}\,\alpha_j - \alpha_j \right)^2$$

$$= \left(\sum_{j=1}^{p} \frac{(\lambda_j - d_j)\,\alpha_j - (\lambda_j + k_j)\,\alpha_j}{(\lambda_j + k_j)} \right)^2$$

$$\therefore \quad bias^2 = \sum_{j=1}^{p} \frac{(d_j + k_j)^2\alpha_j^2}{(\lambda_j + k_j)^2} \qquad (19)$$

As in [24, 25 and 26] the shrinkage parameters d_j are chosen to minimize $MSE(\hat{\alpha}_{GLTE})$:

$$MSE(\hat{\alpha}_{GLTE}) = E[(\hat{\alpha}_{GLTE} - \alpha)'(\hat{\alpha}_{GLTE} - \alpha)] = Var(\hat{\alpha}_{GLTE}) + bias^2 \qquad (20)$$

Substituting equations (17) and (18), in equation (19), it can be written be as:

$$MSE(\hat{\alpha}_{GLTE}) = \hat{\sigma}^2 \sum_{j=1}^{p} \frac{(\lambda_j - d_j)^2}{\lambda_j(\lambda_j + k_j)^2} + \sum_{j=1}^{p} \frac{\alpha_j^2(d_j + k_j)^2}{(\lambda_j + k_j)^2} \qquad (21)$$

$$MSE(\hat{\alpha}_{GLTE}) = \gamma_1(k_j, d_j) + \gamma_2(k_j, d_j) \quad , \quad j = 1,2, \dots, p \qquad (22)$$

The first term $\gamma_1(k_j, d_j)$ is the variance and the second term $\gamma_2(k_j, d_j)$ is the squared bias. The objective of the biased estimator is to choose appropriate values of k_j and d_j such that the reduction in the variance term is greater than the increase of the squared bias. Thus, $MSE(\hat{\alpha}_{GLTE})$ will be less than $MSE(\hat{\beta}_{OLS})$. It is obvious that $\gamma_1(k_j, d_j)$ and

$\gamma_2(k_j, d_j)$ are two continuous functions of k_j *and* d_j so we can obtain the optimal d_j that minimizes $MSE(\hat{\alpha}_{GLTE})$, by differentiating equation (21) with respect to d_j as follows:

$$\frac{\partial\, MSE(\hat{\alpha}_{GLTE})}{\partial d_j} = -2\,\hat{\sigma}^2 \sum_{j=1}^{p} \frac{(\lambda_j - d_j)}{\lambda_j(\lambda_j + k_j)^2} + 2\sum_{j=1}^{p} \frac{\alpha_j^2(d_j + k_j)}{(\lambda_j + k_j)^2}$$

$$= -2\,\hat{\sigma}^2 \sum_{j=1}^{p} \frac{\lambda_j}{\lambda_j(\lambda_j + k_j)^2} - \frac{d_j}{\lambda_j(\lambda_j + k_j)^2} + 2\sum_{j=1}^{p} \frac{\alpha_j^2 d_j}{(\lambda_j + k_j)^2} + \frac{\alpha_j^2 k_j}{(\lambda_j + k_j)^2}$$

$$= -2\sum_{j=1}^{p} \frac{\hat{\sigma}^2}{(\lambda_j + k_j)^2} - \frac{d_j\hat{\sigma}^2}{\lambda_j(\lambda_j + k_j)^2} + 2\sum_{j=1}^{p} \frac{\alpha_j^2 d_j}{(\lambda_j + k_j)^2} + \frac{\alpha_j^2 k_j}{(\lambda_j + k_j)^2}$$

$$\frac{\partial\, MSE(\hat{\alpha}_{GLTE})}{\partial d_j} = -2\sum_{j=1}^{p} \frac{(\hat{\sigma}^2 - \alpha_j^2 k_j)}{(\lambda_j + k_j)^2} + 2\sum_{j=1}^{p} \frac{d_j\,(\hat{\sigma}^2 + \alpha_j^2\lambda_j)}{\lambda_j(\lambda_j + k_j)^2} \qquad (23)$$

Then equating the derivative (23) to zero, and solving the equation :

$$-\sum_{j=1}^{p} \frac{(\hat{\sigma}^2 - \alpha_j^2 k_j)}{(\lambda_j + k_j)^2} + \sum_{j=1}^{p} \frac{d_j(\hat{\sigma}^2 + \alpha_j^2\lambda_j)}{\lambda_j(\lambda_j + k_j)^2} = 0$$

$$\sum_{j=1}^{p} \frac{(\hat{\sigma}^2 - \alpha_j^2 k_j)}{(\lambda_j + k_j)^2} = \sum_{j=1}^{p} \frac{d_j(\hat{\sigma}^2 + \alpha_j^2\lambda_j)}{\lambda_j(\lambda_j + k_j)^2}$$

$$\frac{(\hat{\sigma}^2 - \alpha_j^2 k_j)}{(\lambda_j + k_j)^2} = \frac{d_j(\hat{\sigma}^2 + \alpha_j^2\lambda_j)}{\lambda_j(\lambda_j + k_j)^2}\,, \qquad j = 1,2,\dots,p$$

$$\therefore \qquad d_j = \frac{\hat{\sigma}^2 - \alpha_j^2 k_j}{\alpha_j^2 + \dfrac{\hat{\sigma}^2}{\lambda_j}}\,, \qquad j = 1,2,\dots,p \qquad (24)$$

It is obvious that, $d_j = f(k_j)$, $j = 1,2,\dots,p$, so that firstly we select k_j , then optimal value of d_j can be chosen. There are many methods to find the shrinkage parameters k_j , $j = 1,2,\dots,p$ of generalized ridge regression estimator see [8, 10, 11, 14, 16, 17 and 23, 29], some of these methods are used in determining Liu-type estimators, see [27]. Now we will review some of these methods in the literature that are used in the simulation and the application to find the shrinkage parameters k_j , $j = 1,2,\dots,p$ of (GRR):

(1) $k_j(HK) = \frac{\hat{\sigma}^2}{\hat{\alpha}_j^2}$, $j = 1,2,\dots,p$ (25)

(2) $k_j(TC) = \frac{\lambda_j\,\hat{\sigma}^2}{\lambda_j\,\hat{\alpha}_j^2 + \hat{\sigma}^2}$, $j = 1,2,\dots,p$ (26)

(3) $k_j(Fir) = \frac{\lambda_j\,\hat{\sigma}^2}{\lambda_j\,\hat{\alpha}_j^2 + (n-p)\hat{\sigma}^2}$, $j = 1,2,\dots,p$ (27)

(4) $k_j(Dor) = \frac{2\,\hat{\sigma}^2}{\lambda_{Max}\,\hat{\alpha}_j^2}$, $j = 1,2,\dots,p$ (28)

(5) $k_j(Nom) = \frac{\hat{\sigma}^2}{\hat{\alpha}_j^2}\left\{1 + \left[1 + \lambda_j\sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_j^2}}\right]\right\}$, $j = 1,2,\dots,p$ (29)

(6) $k_j(SB) = \frac{\lambda_j\,\hat{\sigma}^2}{\lambda_j\,\hat{\alpha}_j^2 + \hat{\sigma}^2} + \frac{1}{\lambda_{Max}}$, $j = 1,2,\dots,p$ (30)

The performance of the suggested estimator with these different methods for calculating k_j and d_j , $j = 1,2,\dots,p$ is considered via a simulation study and an application.

3. MONTE CARLO SIMULATION STUDY

This study aims to evaluate the performances of the suggested generalized Liu-type estimators, it has been conducted using R x64 3.4.4.

3.1. Design of the simulation

The design of this simulation study is based on defining the factors that are expected to affect proposed estimators properties and choosing a criteria to evaluate the results. Following [18] the independent variables are generated using the following relation:

$$x_{ij} = \sqrt{(1-\rho^2)}\, z_{ij} + \rho\, z_{ij} \qquad i = 1,2,\dots,n\,,\ j = 1,2,\dots,p \qquad (31)$$

Where z_{ij} are independent standard normal pseudo random numbers, ρ is specified such that ρ^2 represents the correlation between any two independent variables and p is the number of independent variables. These independent variables are standardized so that, the information matrix $X'X$ is the correlation matrix. The values of Y are determined by the real relation:

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip} + \varepsilon_i \qquad i = 1,2,\dots,n\,,\ j = 1,2,\dots,p \qquad (32)$$

Where ε_i, $i = 1,2,\dots,n$ are i.i.d and follows $N(0\,,\sigma^2)$ with true parameters $\beta = (1,1,1)'$ at $p = 3$ and $\beta = (1,1,1,1,1,1,1)'$ at $p = 7$.

The simulation was carried out for samples of size (n) = 25, 50, 100, 200 and the variance of the disturbance term (σ^2) = 1, 5, 15, 25 with varying degrees of correlation as ρ = 0.75, 0.85, 0.95, 0.99. Generalized Liu-type estimators are computed by considering different estimators for calculating k_j and d_j , $j = 1,2,\dots,p$ as defined in equations (24) – (30).

The experiments are repeated 10000 times each and the average mean squared error (AMSE) was computed as follows:

$$AMSE = \frac{1}{10000}\sum_{i=1}^{10000}\sum_{j=1}^{p}(\hat{\beta}_{ij} - \beta_j)^2 \qquad (33)$$

Where, $\hat{\beta}_{ij}$ is the estimated j^{th} parameter in the i^{th}replication and β_j is the j^{th} true parameter value.

3.2. Results and Discussion

Results of the Monte Carlo simulation are displayed in this subsection. In tables (1-8) the estimated AMSE values of OLS, and the suggested generalized Liu-type estimator with different methods for calculating shrinkage parameters k_j , $j = 1,2,\dots,p$ are presented.

Table (1): the estimated average mean squared errors (AMSE) when: p =3, $\sigma^2 = 1$

	$\rho = 0.75$				$\rho = 0.85$			
	Sample size (n)				Sample size (n)			
Estimators	20	50	100	200	20	50	100	200
MSE.OLS	16.25530	16.44388	16.39037	16.073499	16.76029	16.39874	16.23317	16.24038
MSE.Fir	15.91348	16.32544	16.33429	16.051320	16.28989	16.24835	16.15731	16.21503
MSE.HK	15.88310	16.31951	16.33556	16.054354	16.17748	16.20825	16.14438	16.21465
MSE.TC	15.87711	16.31396	16.33230	16.051791	16.19141	16.20928	16.14542	16.21243
MSR.SB	15.86347	16.31194	16.33182	16.051677	16.17801	16.20767	16.14506	16.21235
MSE.Nom	10.30196	10.41095	10.40907	10.324226	10.33975	10.36082	10.35241	10.36607
MSE.Dor	10.20058	10.11623	10.06227	9.973081	10.23008	10.08135	10.02246	10.00858
Estimators	$\rho = 0.95$				$\rho = 0.99$			
MSE.OLS	17.54864	16.66262	16.22323	15.998488	22.43261	18.45009	16.83282	16.34689
MSE.Fir	16.56724	16.33902	16.09060	15.939212	18.36168	17.09780	16.20889	16.06945
MSE.HK	16.23892	16.19915	16.04210	15.912564	16.81786	16.41691	15.89580	15.91168
MSE.TC	16.29374	16.21760	16.04684	15.913407	17.08710	16.51392	15.94189	15.92885
MSR.SB	16.26989	16.21574	16.04653	15.913334	16.79666	16.49765	15.94035	15.92860
MSE.Nom	10.33190	10.34257	10.32488	10.272094	10.44718	10.37381	10.27143	10.27150
MSE.Dor	10.49831	10.15701	10.06610	9.951418	12.43565	10.90505	10.43423	10.12296

Table (2): the estimated average mean squared errors (AMSE) when: p =3, $\sigma^2 = 5$

	$\rho = 0.75$				$\rho = 0.85$			
	Sample size (n)				Sample size (n)			
Estimators	20	50	100	200	20	50	100	200
MSE.OLS	22.43261	19.059647	17.526341	16.674247	28.384870	20.477586	17.977898	17.134645
MSE.Fir	18.36168	18.093614	17.087108	16.483218	24.125189	19.155084	17.378046	16.874434
MSE.HK	16.81786	17.482184	16.771820	16.338216	22.128273	18.271359	16.907961	16.655110
MSE.TC	17.08710	17.537317	16.797706	16.345261	22.419785	18.358314	16.953344	16.671072
MSR.SB	16.79666	17.532375	16.796942	16.345110	22.248193	18.350256	16.952357	16.670907
MSE.Nom	10.44718	8.732302	8.672898	8.672224	9.434657	8.773401	8.676956	8.678849
MSE.Dor	12.43565	9.062345	8.427115	8.129350	12.563663	10.155835	9.006151	8.349583
Estimators	$\rho = 0.95$				$\rho = 0.99$			
MSE.OLS	48.69194	27.855148	21.153535	18.536836	171.85842	72.72777	41.88761	28.676855
MSE.Fir	38.14415	24.618451	19.801621	17.916917	123.26831	57.98385	35.49155	25.789011
MSE.HK	33.08777	22.265386	18.634626	17.319561	99.52041	46.89434	29.68225	22.835074
MSE.TC	33.83706	22.522879	18.745831	17.368871	103.06465	48.13673	30.28169	23.110937
MSR.SB	32.75436	22.475657	18.740864	17.368235	81.92967	47.09821	30.17192	23.097914
MSE.Nom	12.66541	9.304472	8.751336	8.722279	44.70284	17.54384	11.19758	9.426623
MSE.Dor	22.10023	14.727986	11.580561	9.730681	79.36921	43.34534	26.67619	17.439170

Table (3): the estimated average mean squared errors (AMSE) when:p =3, $\sigma^2 = 15$

	$\rho = 0.75$				$\rho = 0.85$			
	Sample size (n)				Sample size (n)			
Estimators	20	50	100	200	20	50	100	200
MSE.OLS	87.10102	41.27120	27.67117	21.777239	124.07323	54.40415	33.377264	24.851512
MSE.Fir	75.06488	37.63223	26.09252	21.069941	106.16953	49.03567	31.078452	23.838387
MSE.HK	70.32737	35.20746	24.69816	20.328636	99.16021	45.42243	29.013739	22.749219
MSE.TC	70.94540	35.42181	24.79657	20.367161	100.05057	45.74485	29.167271	22.811436
MSR.SB	70.01737	35.38204	24.79218	20.366579	98.08006	45.66235	29.158544	22.810350
MSE.Nom	18.55598	9.80088	8.62596	8.498443	28.88581	11.41646	8.853052	8.438365
MSE.Dor	23.04277	17.90892	14.37068	11.660272	33.35703	25.15358	18.732427	14.097011
Estimators	$\rho = 0.95$				$\rho = 0.99$			
MSE.OLS	307.3413	120.72978	63.69953	39.890492	1416.0208	522.6965	251.82302	131.20745
MSE.Fir	259.4059	106.41085	57.90097	37.298288	1188.4131	455.0993	223.81590	118.93971
MSE.HK	240.6093	96.57382	52.64178	34.443305	1099.5672	408.8519	198.18148	105.23721
MSE.TC	243.0211	97.47187	53.02778	34.619796	1110.7456	413.0230	200.13317	106.14961
MSR.SB	229.7807	96.90364	52.96806	34.612653	849.3345	400.2791	198.76583	105.98977
MSE.Nom	98.5483	27.24371	12.98788	9.376448	669.6438	194.8908	74.38875	30.50047
MSE.Dor	85.9485	59.83457	39.89136	26.226115	401.3190	269.1756	167.49115	97.90716

Table (4): the estimated average mean squared errors (AMSE) when:p =3, $\sigma^2 = 25$

Estimators	$\rho = 0.75$				$\rho = 0.85$			
	Sample size (n)				Sample size (n)			
	20	50	100	200	20	50	100	200
MSE.OLS	213.31943	85.87150	48.251557	32.02469	314.94290	122.22993	64.54454	40.398671
MSE.Fir	190.16282	79.00126	45.368205	30.73675	280.16524	111.92306	60.25060	38.519078
MSE.HK	182.42976	74.88827	42.967731	29.39592	268.63208	105.73254	56.63639	36.527824
MSE.TC	183.36104	75.22814	43.122989	29.46479	270.00910	106.23542	56.87623	36.628752
MSR.SB	180.59109	75.11184	43.110568	29.463259	264.10720	105.98962	56.85039	36.625634
MSE.Nom	49.87294	14.96416	9.796425	8.958005	89.37574	22.60604	11.47476	9.043901
MSE.Dor	40.12394	29.41354	23.703708	18.13563	59.70592	44.79605	34.18441	24.576137
Estimators	$\rho = 0.95$				$\rho = 0.99$			
MSE.OLS	824.2636	306.30912	149.41729	82.68384	3903.8942	1421.6289	672.1848	336.17473
MSE.Fir	730.1512	278.16994	138.18769	77.72974	3454.8524	1287.8420	617.3574	312.54014
MSE.HK	698.9427	260.97341	128.73601	72.42511	3307.1283	1206.7289	571.0028	287.11822
MSE.TC	702.6996	262.41887	129.34586	72.70887	3324.4615	1213.3787	574.0992	288.55904
MSR.SB	662.7909	260.70790	129.16568	72.68744	2536.2350	1174.9843	569.9709	288.07717
MSE.Nom	337.9777	85.32873	30.59068	14.13722	2251.5527	671.9918	257.5680	98.48811
MSE.Dor	162.2448	120.95920	85.40847	56.25920	777.7179	574.8086	395.6365	244.20703

Table (5): the estimated average mean squared errors (AMSE) when: p =7, $\sigma^2 = 1$

Estimators	$\rho = 0.75$				$\rho = 0.85$			
	Sample size (n)				Sample size (n)			
	20	50	100	200	20	50	100	200
MSE.OLS	89.53444	88.54495	87.67995	87.35094	89.96543	88.55886	87.81853	87.63712
MSE.Fir	88.46897	88.26118	87.56996	87.31346	88.53982	88.24992	87.65800	87.60727
MSE.HK	88.13692	88.18579	87.53178	87.30857	88.06766	88.16683	87.58098	87.61884
MSE.TC	88.20110	88.19291	87.53334	87.30587	88.16170	88.16970	87.59360	87.60950
MSR.SB	88.18159	88.19073	87.53283	87.30575	88.14170	88.16818	87.59324	87.60942
MSE.Nom	64.63148	64.93369	64.66641	64.58212	64.68856	65.00744	64.69775	64.75542
MSE.Dor	69.28661	69.23281	68.75129	68.60670	69.52635	69.42756	68.82255	68.83395
Estimators	$\rho = 0.95$				$\rho = 0.99$			
MSE.OLS	92.30127	87.85921	87.22528	86.14227	111.03422	92.06770	89.11801	86.42680
MSE.Fir	88.72551	87.12255	86.81007	86.03092	94.65498	88.07803	87.07782	85.82250
MSE.HK	87.44834	86.84888	86.59157	86.00403	88.84085	86.11858	85.96045	85.57827
MSE.TC	87.71392	86.88030	86.62343	85.99604	89.96151	86.40955	86.13188	85.56865
MSR.SB	87.65445	86.87888	86.62310	85.99598	89.02049	86.38947	86.12934	85.56844
MSE.Nom	64.40037	64.48285	64.21623	64.04265	64.50613	63.82385	63.78082	63.89794
MSE.Dor	70.61659	69.35377	68.38056	68.04031	79.01542	70.96166	68.90219	68.47070

Table (6): the estimated average mean squared errors (AMSE) when: p =7, $\sigma^2 = 5$

	$\rho = 0.75$				$\rho = 0.85$			
	Sample size (n)				Sample size (n)			
Estimators	20	50	100	200	20	50	100	200
MSE.OLS	117.57156	96.37774	91.15695	88.87059	134.21438	100.49002	93.55329	89.89633
MSE.Fir	106.93504	93.82301	90.12399	88.43092	118.62566	96.96810	91.98244	89.29222
MSE.HK	102.75524	92.15602	89.34703	88.09777	112.32234	94.56711	90.67429	88.77912
MSE.TC	103.44772	92.34327	89.42115	88.11879	113.35603	94.80317	90.80457	88.81591
MSR.SB	103.17158	92.33575	89.42011	88.11861	112.80526	94.79077	90.80313	88.81572
MSE.Nom	53.36036	52.04052	52.16249	52.10628	55.74325	52.23408	52.10051	52.27230
MSE.Dor	60.55517	49.42738	46.72201	45.26285	70.87308	53.35776	48.29708	46.16265
Estimators	$\rho = 0.95$				$\rho = 0.99$			
MSE.OLS	217.9632	120.54394	103.50363	93.07562	719.2411	259.39399	170.04472	121.20647
MSE.Fir	175.5795	111.13750	99.34197	91.37325	513.5438	213.22220	149.55965	112.92770
MSE.HK	157.9319	104.32015	95.63697	89.72297	427.8690	179.04738	131.05988	104.61538
MSE.TC	160.8611	105.08184	96.01625	89.86670	442.2742	182.83789	132.99677	105.36167
MSR.SB	157.2770	105.00911	96.00873	89.86592	373.7940	181.17126	132.83137	105.34505
MSE.Nom	72.352	53.49720	52.26039	52.14466	215.1186	79.87486	60.63445	54.00530
MSE.Dor	122.7658	71.19723	56.30711	50.50587	433.8956	171.19410	105.23117	75.17469

Table (7): the estimated average mean squared errors (AMSE) when:p =7, $\sigma^2 = 15$

	$\rho = 0.75$				$\rho = 0.85$			
	Sample size (n)				Sample size (n)			
Estimators	20	50	100	200	20	50	100	200
MSE.OLS	353.89541	162.12763	120.99573	102.61258	502.7912	202.70375	141.36497	111.13652
MSE.Fir	306.77949	151.27219	116.56301	100.76000	431.8763	186.68661	134.66620	108.42772
MSE.HK	291.55485	144.41810	112.91876	98.94358	409.3707	176.64939	128.91948	105.62053
MSE.TC	293.74190	144.97471	113.19152	99.05059	412.4687	177.43843	129.34128	105.79067
MSR.SB	290.52096	144.90407	113.18447	99.04978	405.8946	177.29623	129.32743	105.78920
MSE.Nom	97.62266	48.99063	45.75318	45.23610	148.6136	54.78963	46.42708	45.32538
MSE.Dor	139.40001	79.12120	58.58317	46.17835	213.8671	110.60831	74.35267	54.27583
Estimators	$\rho = 0.95$				$\rho = 0.99$			
MSE.OLS	1262.1298	403.8931	238.11188	154.66813	5789.177	1663.1463	836.4769	423.5849
MSE.Fir	1064.4640	360.1057	219.91721	147.09817	4812.400	1448.3000	746.3750	387.2046
MSE.HK	1000.8088	331.5732	203.58708	138.85727	4490.021	1307.6020	665.1617	346.8123
MSE.TC	1009.9645	334.0261	204.81070	139.40626	4538.884	1319.8863	671.5091	349.4864
MSR.SB	966.1726	333.0948	204.72044	139.39678	3694.765	1298.9784	669.4736	349.2741
MSE.Nom	477.0952	106.3078	60.56336	47.86370	2926.981	645.6995	260.1722	110.7498
MSE.Dor	582.7897	260.2576	151.69877	94.65351	2836.049	1147.4150	622.7115	331.8856

Table (8): the estimated average mean squared errors (AMSE) when: p =7, $\sigma^2 = 25$

Estimators	$\rho = 0.75$				$\rho = 0.85$			
	Sample size (n)				Sample size (n)			
	20	50	100	200	20	50	100	200
MSE.OLS	827.6922	293.83170	181.04350	130.55865	1239.8742	408.32585	236.99779	154.65126
MSE.Fir	737.2533	272.99964	172.52737	127.03693	1102.4009	376.88179	224.02176	149.41532
MSE.HK	712.5951	261.17744	165.74423	123.50411	1065.9350	359.30744	213.55465	144.02752
MSE.TC	715.9570	262.04906	166.21095	123.69135	1070.6978	360.59126	214.24004	144.30949
MSR.SB	706.2877	261.83960	166.19074	123.68915	1050.9631	360.16323	214.19899	144.30517
MSE.Nom	252.0561	64.77279	47.07382	43.68082	438.0915	91.59958	52.69568	44.31349
MSE.Dor	242.6387	138.71337	95.63667	66.22688	401.9364	215.86534	138.29390	88.42019
Estimators	$\rho = 0.95$				$\rho = 0.99$			
MSE.OLS	3349.170	975.2957	506.8806	279.48796	15929.712	4474.665	2165.931	1033.7193
MSE.Fir	2961.793	888.5509	471.3484	264.78927	14003.452	4047.868	1989.139	963.2474
MSE.HK	2856.627	838.8018	441.7477	249.22682	13460.352	3802.762	1841.875	887.6193
MSE.TC	2871.098	842.6315	443.6735	250.11983	13540.007	3822.294	1852.088	891.8756
MSR.SB	2739.374	839.8044	443.4017	250.09108	10995.252	3759.094	1845.951	891.2315
MSE.Nom	1575.323	296.1655	115.6228	59.41235	9685.063	2173.371	845.896	312.4483
MSE.Dor	1189.394	585.1659	344.3685	198.13876	6065.095	2796.649	1599.138	849.5371

The effective factors are chosen to be the number of independent variables p, the variance of the disturbance term (σ^2) , the sample size n and the correlation among the explanatory variables ρ .

It is noted that, in the case of high multicollinearity, the suggested estimator using equations (24) and (29) showed its best performance by means of the reduction of the AMSE values and it is not affected by multicollinearity as much as other estimators.

For all of the cases, all of the proposed estimators have better performance than OLS such that the suggested estimator with all methods has less value of AMSE than OLS has. Moreover, all of the estimators have monotonic behaviors according to the AMSE, namely, when the sample size increases, the estimated AMSE values decrease. It is obvious from tables that, increasing the sample size affect positively on the performance of all estimators (including OLS).

Also, it can be noted that, when p and ρ are fixed, increasing the variance of the disturbance term (σ^2) causes an increase in the AMSE values of all estimators without exception. This increase is much larger in OLS than other estimators.

Furthermore, when (σ^2) and p are fixed, the increase in the correlation ρ affect negatively on the AMSE values of all estimators, especially on OLS. In other words, this increase is much larger in OLS than other estimators.

Also, increasing (σ^2) , ρ and p with small sample size n, inflates the AMSE values of all estimators. That is, the AMSE has its largest value when $n = 200, p = 7, (\sigma^2) = 25\ , and\ \rho = 0.99$, with and it has its smallest value when $n = 20, p = 3, (\sigma^2) = 1, and\ \rho = 0.75$ with k_{Dor} . According to the tables, there is some difference between the performances of the suggested estimators according to the shrinkage parameter that is used. From the tables, it may be concluded that, k_{Nom} is the best shrinkage parameter among others in most cases.

4. REAL DATA APPLICATION

To evaluate the performance of the suggested estimator using different methods for calculating the ridge parameters, we consider Portland cement data that was widely used as [23 ,26] among others. The experiment aimed to investigate the heat evolved during the setting and hardening of Portland cement. There were four independent variables:

X_1 : amount of tricalcium aluminate, X_2 : amount of ticalcium silicate, X_3 : amount of tetracalcium alumino ferrite, and X_4 : amount of dicalcium silicate. The dependent variable is: Y: heat evolved in calories per gram of cement.

Consider the following linear relation:

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_1 x_{i2} + \beta_1 x_{i3} + \beta_1 x_{i4} + \varepsilon \qquad i = 1,2, \dots ,13$$

The variables were standardized, then the parameters were estimated using the OLS method and the fitted model is:

$$\hat{y}_i = 0.607\ x_{i1}^* + 0.528\ x_{i2}^* + 0.043\ x_{i3}^* - 0.160\ x_{i4}^* \qquad i = 1,2,\dots,13$$

The sample correlation matrix of the independent variables was:

$$R = \begin{bmatrix} 1 & 0.229 & -0.824 & -0.246 \\ 0.229 & 1 & -0.139 & -0.973 \\ -0.824 & -0.139 & 1 & 0.030 \\ -0.246 & -0.973 & 0.030 & 1 \end{bmatrix}$$

Although the coefficient of determination $R^2 = 0.982$, yet all the independent variables were insignificant at ($\alpha = 5\%$), that indicated multicollinearity. Diagnostic measures of multicollinearity as: variance inflation factor (VIF) and condition index (CI) assured that the model suffered from severe multicollinearity as shown in table (9), since that, $VIF_j > 1\ \ ,\ j = 1,2,3,4$ and $CI > 30$.

Table (9): multicollinearity diagnostics of Portland cement data

Multicollinearity diagnostics	VIF_1	VIF_2	VIF_3	VIF_4	CI
	38.496	254.423	46.868	282.513	37.57

The suggested estimator was applied with different $k_j\ and\ d_j\ \ ,\ j = 1,2,3,4$ for correcting multicollinearty in linear regression. The parameter estimates and its standard errors are shown in table (10) as follows:

Table (10): estimated regression parameters using generalized Liu-type, its standard errors and theoretical MSE

Estimators	OLS	HK	TC	Fir	Dor	Nom	SB
$\hat{\beta}_1$	0.607	0.189	0.048	-0.219	1.322	**0.188**	0.057
$\hat{\beta}_2$	0.528	-0.674	-0.937	-1.647	-4.709	**-0.451**	-0.912
$\hat{\beta}_3$	0.043	-0.441	-0.576	-0.858	-2.123	**-0.447**	-0.557
$\hat{\beta}_4$	-0.160	-1.367	-1.708	-2.447	-5.648	**-1.131**	-1.667
$SE(\hat{\beta}_1)$	0.275	0.352	0.467	0.800	2.346	**0.172**	0.452
$SE(\hat{\beta}_2)$	0.706	1.004	1.247	2.179	6.418	**0.593**	1.224
$SE(\hat{\beta}_3)$	0.303	0.402	0.521	0.894	2.623	**0.262**	0.507
$SE(\hat{\beta}_4)$	0.744	1.007	1.310	2.291	6.674	**0.614**	1.280
$MSE(\hat{\beta})$	1.249	0.0627	0.0582	0.0845	0.0579	**0.0560**	0.0616

It is clear from table (10) that all shows that generalized Liu-type estimators outperform the OLS estimator because they have smaller $MSE(\hat{\beta})$, but the estimates using k_{Nom} have the best performance among others since that it have the smallest standard errors and the smallest $MSE(\hat{\beta})$, which agree with the simulation results.

5. CONCLUSIONS

In this article, a new biased estimator, named, ($GLTE$) which provides an alternative method for dealing with multicollinearity in linear regression, is introduced. It is a generalization of Liu-type estimator in linear regression. Generalized biased estimators such as generalized ridge regression and generalized Liu estimator are special cases of the suggested estimator.

A Monte Carlo experiment is designed by generating pseudo random numbers for the independent variables and the dependent variable. Several sample sizes, degrees of correlation and different numbers of the independent variables are considered. A comparison was made between generalized Liu-type estimators that were computed using different methods for calculating shrinkage parameters $k_j\ , j = 1,2,\dots,p$. $AMSE$ is used as performance criterion. The results showed that generalized Liu-type estimators with different shrinkage parameters are robust to correlation between the independent variables. Therefore, the OLS should not be used in the presence of severe multicollinearity, as it becomes instable with large variances and it has larger $AMSE$. It is noted that, in the case of high multicollinearity, the suggested generalized Liu-type estimator using equations (24) and (29) showed its best performance by means of the reduction of the AMSE values and it is not affected by multicollinearity as much as other estimators.

According to the tables, there is some difference between the performances of the suggested estimators according to the shrinkage parameter that is used and it may be concluded that, k_{Nom} is the best shrinkage parameter among others in most cases.

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