# FREIGHT TRAIN SCHEDULING ON A SINGLE LINE CORRIDOR

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**ABSTRACT**

In many countries, rail network consists of single lines with sidings where interactions between trains occur (meet, pass). In this paper, we study two problems: First, scheduling freight trains in a single line corridor while ensuring that the interactions happen safely. Second, allocation of freights to the scheduled freight trains with respect to due and release date of freights, weight of freights and finally, weight capacity of trains. The objective is to minimize the train's travel time, allocating more freights to trains and minimizing the tardiness of freights in their final estination. For this timetabling and allocation problem, we present an integer linear programming formulation. Computational results confirm the validation of our model for freight trains scheduling and allocation of freights to the scheduled freight train.

**Keywords:** *Freight Trains Scheduling, Single Line Corridor, Minimizing Total Tardiness, Minimizing Train's Travel Time.*

## 1. Introduction

Rail transportation is one of the transportation systems in which there are several different problems that can be modeled and solved. Based on the survey of Assad (1980), the issues of rail modeling addressed by the literature can be categorized in the main following categories: Institutional Background, Facilities Location, Yard and Terminal Models, Line Models, Rail Network Model, Blocking and Train Formation, Train Schedules and Timetables and finally, Car and Engine Distribution. One of the most important aforementioned categories which is addressed in this paper, is the train scheduling and timetabling. Some other literature summaries have published by Haghani (1987), Cordeau et al. (1998), Lusby et al. (2011) and Harrod and Gorman (2011). The first work on finding an optimum solution for the train scheduling problem has started by Szpigel (1973). He has represented a linear programming model with a branch and bound method to minimize the sum of travel times and solve the problem. Higgins et al. (1996), considered the train scheduling problem on a single line track. They proposed a multi-objective mathematical programming model in a branch and bound procedure, where the objective is to minimize the deviation from scheduled arrival time and fuel consumption costs. Kuo et al. (2010) discussed that the most common objective functions of freight train scheduling and time tabling might be to minimize deviation from the schedule, operating cost, train delay and average travel time. However, in this paper, we have considered minimizing the total train's travel time as an objective function.

Freight scheduling problems is one of the most difficult among transportation problems (Bożejko et al. 2017). Due to its complexity, Pashchenko et al. (2015), considered freight scheduling problems as three sub problems: freight train scheduling problem, locomotive assignment problem and finally locomotive team assignment problem.

Although there has been a vast research on the passenger train scheduling, only a few researchers focused on the freight train scheduling problem. Godwin et al. (2007) addressed the problem of scheduling freight trains in a passenger rail network. They have showed that freight train scheduling in a passenger rail network is NP- complete and specifically, even for two freight train, the problem is still difficult. They developed a stepwise dispatching heuristic with considering several objectives (i.e., percentage deviation of sum of travel times from lower bound, percentage standard mean tardiness, percentage tardy trains, percentage conditional mean tardiness and percentage maximum tardiness). Cacchiani et al. (2010) presented an integer linear programming formulation to address the same problem with the objective of scheduling and assigning as many new freight trains as possible on railway networks. In their study, they have just considered the constraint of freight trains' capacity. At the 12$^{th}$ workshop on algorithmic approaches for transportation modelling, optimization and systems, Jaumard et al. (2012), presented a research on freight trains scheduling problem. More comprehensive constraints (i.e., travel and dwell time constraints, safety distance constraints, segment conflict constraints and capacity constraints) with the usage of mixed integer programming for addressing the problem is developed. One of the recent study dates back to 2014, when Rahman and Froyland, represented an integer programming formulation for the freight trains scheduling problem in a single line corridor. By considering the safe interactions between trains as constraints, the objective is minimizing the arrival time of the last train at its last station. Finally, Ke et al. (2015) addresses the problem of freight train timetabling on a

single-track railway system to minimize train waiting times. With the advantage of both fixed-block signaling systems and fuzzy logic systems, they develop a new method to improve freight train timetabling performance.
This paper, described two main models, namely: scheduling freight train in a single line corridor to minimize the total train's travel time; and allocating freights to scheduled freight trains in order to maximize the allocation of freights and minimizing the tardiness of freights in their final destination.

## 2. Methodology

In this section, we formally describe the problem and proposed models. First, the developed model for solving the problem of scheduling freight trains on a single line corridor will be described. Second, the extended formulation and model of allocating freights to the scheduled freight trains will be described.

### 2.1 The scheduling problem

In this study, we consider a single line corridor where both passenger and freight trains can traverse. Start and end stations are located at the start and end of this corridor, respectively. As the real world problem, the corridor is divided into segments which are separated by stations. At most one pair of trains can cross and overtake one another at these stations. Two naming assumption is assumed for the sections, i.e., one for heading trains and one for returning trains. Two trains can follow each other with a minimum distance which is dependent to their speed.
In real world transportation problems, freight rail transportation is used for transferring variety of goods. Due to the differences between these goods (i.e., rottenness, value, due date and etc.), their importance will be different. Therefore, in this study, we consider that the trains are different based on their freights and considered different priorities for them. Also, we assume that the rail road is single line corridor in which both passenger and freight trains are in run. On the other hand, same as Rahman and Froyland (2014), we consider some safety and operational constraints which prevent the collision of trains as follows: Proximity conflicts: is considered for two trains which are traversing in the same way and back of each other and collision conflicts: is considered for two trains which are traversing to each other at the same segment.
With all aforementioned constraints, a mathematical model with usage of the following notation is proposed.

*Table 1: Subscripts and Parameters*

| Symbol | Definition |
|---|---|
| $S$ | = {1,2,3}, the set of all sections |
| $T$ | = {1,2,3,4 ,5 }, the set of trains |
| $t$ | = {1,2,3}, the set of heading trains |
| $r$ | = {1,2}, the set of returning trains |
| $pd$ | = {1,2,3}, the set of sections traversing heading trains |
| $pdr$ | = {1,2,3}, the set of sections traversing returning trains |
| $AP$ | Start station |
| $DP$ | End station |
| $\pi_t$ | Priority of heading trains |
| $v_s^t$ | Average speed of train $t$ at segment $pd$ |
| $dw_{pd}^t$ | Dwell time of train $t$ at segment $pd$ |
| $mr_{pd}^t$ | Minimum time for train $t$ to travel segment $pd$ |
| $ST_{pd}^t$ | Safe time of traversing train $t$ at segment $pd$ |
| $Lo_{pd}^t$ | Loading time of train $t$ at segment $pd$ |
| $Ul_{pd}^t$ | Unloading time of train $t$ at segment $pd$ |
| $M$ | Sufficiently large constant |
| $n$ | Number of sections |

*In all train related parameters, symbol $t$ and $pd$ will be replaced with $r$ and $pdr$ for the returning trains.

*Table 2: Decision and binomial Variables*

| **Symbol** | **Definition** |
|---|---|
| $d^{t}_{pd}$ | Departure time of heading train $t$ from segment $pd$ |
| $a^{t}_{pd}$ | Arrival time of heading train $t$ to segment $pd$ |
| $\alpha^{tt'}_{pd}$ | =1 if train $t$ departures segment $pd$ before train $t'$<br>=0 otherwise |
| $\beta^{rr'}_{pdr}$ | =1 if train $r$ departures segment $pdr$ before train $r'$<br>=0 otherwise |
| $\gamma^{tr}_{pd,pdr}$ | =1 if train $t$ departures segment $pd$ before train $r$<br>=0 otherwise |

*In all train related decision variables, symbol $t$ and $pd$ will be replaced with $r$ and $pdr$ for the returning trains.
The above binomial variables are considered to prevent the safety conflicts, i.e., the first two binomial variables for proximity conflicts and the last binomial variable for collision conflicts.
The mathematical model of scheduling freight trains in a single line corridor is shown below:

$$Min \sum_{t \in T} (\pi^{t} (a^{t}_{AP(t)} - d^{t}_{DP(t)})) \quad (1)$$

$S.t.$

$$a^{t}_{pd} - d^{t}_{pd} \geq mr^{t}_{pd} \quad , t \in T; pd \in S \quad (2)$$

$$a^{r}_{pdr} - d^{r}_{pdr} \geq mr^{r}_{pdr} \quad , r \in T; pdr \in S \quad (3)$$

$$d^{t'}_{pd} - a^{t}_{pd-1} \geq Ul^{t}_{pd} + Lo^{t}_{pd} + dw^{t}_{pd} \quad , t \in T, pd \in S \quad (4)$$

$$d^{r'}_{pdr} - a^{r}_{pdr-1} \geq Ul^{r}_{pdr} + Lo^{r}_{pdr} + dw^{r}_{pdr} \quad , r \in T, pdr \in S \quad 5$$

$$d^{t'}_{pd} - d^{t}_{pd} \geq ST^{t}_{pd} - M(1 - \alpha^{tt'}_{pd}) \quad \text{if } \alpha^{tt'}_{pd} = 1 \quad 6$$

$$d^{t}_{pd} - d^{t'}_{pd} \geq ST^{t}_{pd} - M\alpha^{tt'}_{pd} \quad \text{if } \alpha^{tt'}_{pd} = 0 \quad 7$$

$$dr^{r'}_{pdr} - dr^{r}_{pdr} \geq ST^{r}_{pdr} - M(1 - \beta^{rr'}_{pdr}) \quad \text{if } \beta^{rr'}_{pdr} = 1 \quad 8$$

$$d^{r}_{pdr} - d^{r'}_{pdr} \geq ST^{r}_{pdr} - M\beta^{rr'}_{pdr} \quad \text{if } \beta^{rr'}_{pdr} = 0 \quad 9$$

$$\alpha^{tt'}_{pd} + \alpha^{t't}_{pd} \leq 1 \quad \forall t, t' \in T, pd \in S \quad 10$$

$$\beta^{rr'}_{pdr} + \beta^{r'r}_{pdr} \leq 1 \quad \forall r, r' \in T, pdr \in S \quad 11$$

$$\gamma^{tr}_{pd,pdr} + \gamma^{rt}_{pd,pdr} \leq 1 \quad \forall t, r \in T, pd, pdr \in S, pd + pdr = n \quad 12$$

$$ar^{r}_{pdr} \leq d^{t}_{pd} + M(1 - \gamma^{rt}_{pd,pdr}) \quad \forall t, r \in T, pd, pdr \in S, pd + pdr = n \quad 13$$

$$a^{t}_{pd} \leq dr^{r}_{pdr} + M\gamma^{rt}_{pd,pdr} \quad \forall t, r \in T, pd, pdr \in S, pd + pdr = n \quad 14$$

As mentioned before, in this study, the objective function of scheduling freight trains is to minimize the total travel time of freight trains with respect to their priorities, i.e., equation 1.
Trains must take up a specific time to traverse the segments which is defined by the trains speed and the length of the segment (equation 2 & 3 for heading and returning trains, respectively). Equations 4 & 5 represent that trains are allowed to dwell, load and unload at all stations and their stop time cannot be less than the sum of these amount of times.

Due to the proximity conflicts, trains following one another, must traverse with a safe time at all segments, i.e., the departure time of two following trains must be greater than the safety time of the heading train (set of equations 6 and 7 for heading trains and equations 8 and 9 for returning trains). On Gams programming, each of binomial variables are shown two times, in order to represent equations of two following rains, equations 10 (for heading trains), 11 (for returning trains) and 12 (for all trains) are considered.
To ensure safe operation, the model is subject to more constraints. Safety constraints which are shown at equations 13 and 14 illustrates that the departure time difference between two trains traversing the opposite direction, must be greater than the travel time of the train.

2.2 The allocation problem
In this section, we give a mathematical model for the allocation problem. Allocating freights are divided into two main categories. First category considers the ordinary goods which are not strategic. Second category includes the initial and strategic goods. In this study, we assume that first category of goods are too much in amount, therefore, if the second category could not fulfill the trains capacity, the first category will be loaded without solving the allocation problem for them. In contrast, the second category has higher priority to fulfill trains and if they are available for loading the trains, they will be loaded. As a result, in this study, we consider the allocation problem for the second category of goods, i.e., which second category freight allocates to which train. Also, the following constraints are considered:

- Freights weights are different
- Trains are different in weight capacity
- Due to train's weight capacity constraint and departure time, some freights may not be allocated to the trains
- At least 0.6 of trains weight capacity shall be allocated to second category freight. (The whole weight capacity of trains can be allocated to the second category of freights)
- Freights have due dates at their end-stations (Penalty is considered for the tardiness/lateness)
- Freights have release dates at their start-station

In order to present the allocation model, we begin by stating the notation used in our model.

*Table 3: Sets and Parameters of allocation model*

| Symbol | Definition |
|---|---|
| $j$ | $j = \{1,2,3,4,5\}$, set of freights |
| $w_j$ | Priority of freight $j$ |
| $\delta_j$ | Weight of freight $j$ |
| $\lambda_t$ | Weight capacity of train $t$ |
| $wr_j$ | Arrival time of freight $j$ to the end-station |
| $u_j$ | Due date of freight $j$ |
| $tardi_j$ | Tardiness of freight $j$ |
| $O_j$ | Release date of freight $j$ |
| $M$ | Sufficiently large constant |

*Table 4: Decision Variables of allocation model*

| Symbol | Definition |
|---|---|
| $x_{jt}$ | =1 if freight $j$ allocates to train $t$<br>=0 otherwise |

The allocation model is shown below. The first sum of objective function, minimizes the total penalty for tardy freights. In order to allocate freights with higher priority (higher penalty for tardiness) to the scheduled trains, we added the second sum to the objective function. In other word, based on the freights weight and weight capacity of freight train, the second sum maximizes the allocation of higher priority freights to scheduled trains.

$$Min(\sum_j (w_j.tardi_j) - \sum_j \sum_t (w_j.x_{j,t})) \quad 15$$

$S.t.$

$$(0.6\times\lambda_t) - M.y_1 \le \sum_j (\delta_j.x_{j,t}) \le \lambda_t \quad , t\in T; j\in J \quad 16$$

$$tardi_j \ge wr_j - u_j \quad , t\in T; j\in J \quad 17$$

$$a_{AP}^t \le wr_j + M(1-x_{j,t}) \quad , t\in T; j\in J \quad 18$$

$$\sum_t x_{jt} \le 1 \quad , t\in T; j\in J \quad 19$$

$$d_{DP}^t \ge O_j.x_{j,t} + LO_{DP}^t \quad , t\in T; j\in J \quad 20$$

As mentioned above, equation 16 assures that at least 0.6 and at most total weight capacity of trains is allocated to the second category freights. Constraint 17 defines the tardiness of freight $j$ as the difference between arrival time and due date of freight $j$ . If freight $j$ allocates to train $t$ ( $x_{j,t}=1$), arrival time of freight $j$ equals to the arrival time of freight $j$ to the end-station, otherwise, i.e., ( $x_{j,t}=0$ ), freight $j$ does not allocate to train $t$ (equation 18). Constraint 19 assures that each freight allocates to only one train. For all freight trains, if freight $j$ allocates to train $t$, the departure time of freight train must be greater than release date plus loading time of freight $j$ (constraint 20).

3. Results

This section is organized to describe and illustrates the results of both aforementioned models in two different subsections. In this study, the mathematical model is solved for a single corridor with three segments (4 stations), three heading trains and two returning trains. Gams software has solved the problem in 1 second. However, in order to evaluate the efficiently and time speed of the proposed formulation with Gams software, we added the number of trains in both directions and the results are shown in table 5.

*Table 5: Solving time and efficiency of proposed formulation by Gams software*

| Number of heading trains | Number of returning trains | Solving time of Gams software |
|---|---|---|
| 3 | 2 | 1 second |
| 60 | 20 | 16 minutes |
| 60 | 80 | 16 minutes |
| 120 | 80 | 16 minutes |

Table 5 shows that the proposed model is efficient enough for solving the problem.

In this study, we used generated data, to illustrate the computational results. Trains priority is considered a random constant between 0 and 1. Also, loading, unloading, dwell and safety time of trains are considered as a random constant between 0 and 4 hours. Due dates are considered equal to hour 10 for all trains and the release dates are developed as a random constant between time 0 and 5.

To illustrate the computational results of our proposed model, different figures are presented in this section. The bold green vertical lines at the end of each rectangular, divides the segments. On the other hand, for all trains, stop time is considered as sum of loading, unloading and dwell time, i.e., at some stations, trains may not have loading, unloading or dwell time and the black rectangulars show the stop time of each train at each station. Two types of figures are illustrated in this section; train-time figures and train-location figures. In all figures, heading and returning trains are shown as blue and red color, respectively. In all train-time figures, the vertical axis and horizontal axis are assigned to trains and time, respectively. Also, trains are assigned to vertical axis and locations are assigned to horizontal axis in all train-location figures.

3.1 Computational results of the scheduling problem
This section illustrates the computational results of the scheduling problem.

I. Scheduling Single line with one direction
In order to evaluate the proposed model and its proximity constraints, we defined three different objective functions for one direction of trains, i.e., minimizing the total departure time for all trains from the start-station, minimizing the total arrival times for all trains to the end-station and finally, minimizing total travel time of all trains.
Based on the aforementioned notation, the first objective function for evaluating the proposed model in a single line with one direction is formulated as $Min(\sum_t d_{DP}^t)$ and **Error! Reference source not found.**.a, shows the results.
Train 1 departures from the first station at time 1 and arrives to the end of segment 1, i.e., $pd1$, at time 3. It spends 3 hours (1 hour for each of loading, unloading and dwell times) at station 2 (beginning of segment 2) and finally, its arrival time to the last station is time 13. Since the goal is minimizing the total departure time of all trains, the model reduces the speed of trains two and three at segment one, therefore, the objective will be minimized, i.e., the objective is three and is the least objective value which in best of our knowledge, cannot be achieved with any other scheduling model.
The last but not the least objective function is minimizing the total travel time of trains, i.e., shown in **Error! Reference source not found.**.c. As expected from the objective function, the model minimizes the travel times of trains and the final value is 38.5 which is the least value for the objective. Thus, based on all three evaluation models, the constraints of proximity conflicts give us the best objective value.

II. Scheduling single line corridor (two directions)

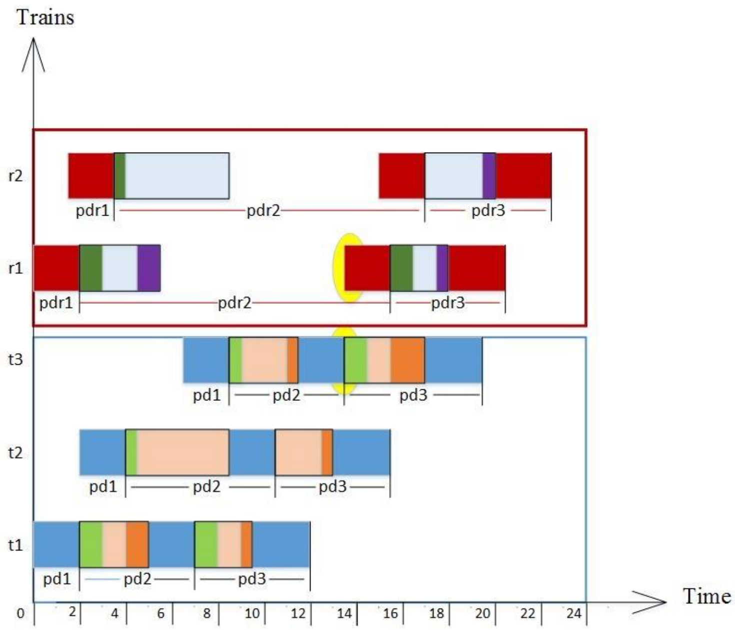


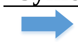

*Figure 1:Gant Chart of scheduling model on a single line corridor (Train-Time)*

Now, we will illustrate the results of the whole scheduling model. In this study, we have considered two names for the segments, one is for heading trains and the other for returning trains. Figure 2, illustrates the names for a single line with three segments. Heading trains traverse segment 1, 2 and 3, respectively, which are shown as $pd1$. Returning trains first traverse segments 3 and then segments 2 and 1, shown as $pdr$ . In other word, segment 1 is the same for both directions of trains, the only difference is between the naming of them for heading and returning trains, i.e., segment 1 is $pd1$ for heading trains and $pdr3$ for returning trains. Computational results for an example of three heading and two returning trains are shown in Figure 2. The above red rectangular shows the returning trains and its under blue rectangular shows the heading trains. Heading trains are traversing segments without any interruption, while the returning trains are interrupted for the safety of heading trains which is shown by the yellow elliptical. This is the consequence of the two following reasons:

1- In the example, the priority of heading trains is considered more than the priority of returning trains and
2- The safety constraints

For further understanding of the model and its result, the train-location graph is followed. Table 7 describes the symbols used in the train-location figure.

*Table 6:Arrows and their definitions used in the train-location figure*

| Symbols | Definition |
|---|---|
|  | Departure of heading trains from a segment |
|  | Arrival of heading trains to a segment |
|  | Departure of returning trains from a segment |
|  | Arrival of returning trains to a segment |

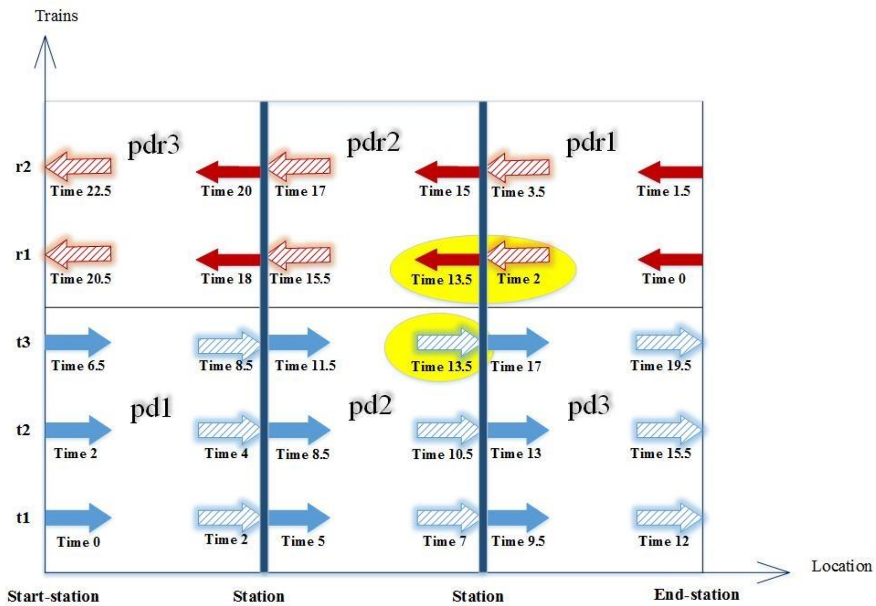



*Figure 2: Train-Loctaion figure ofscheduling freight trains on a single line corridor*

The vertical rectangular shown in Figure 3Figure 2, illustrates the segments, i.e., segment 1 for heading trains, is shown as $pd1$ and this segment for returning trains is $pdr3$. The departure and arrival times of each train to each segment is shown under the arrows. Train $t3$ departures from the segment 2 ( $pd2$ ) at time 11.5 and arrives to the next station at time 13.5. Due to the safety constraints, two facing trains cannot coincidently traverse at the same segment and train $r1$ must stop till train $t3$ traverses the second segment. Therefore, after time 13.5 (when train $t3$ finishes traversing at segment 2, train $r1$ can start its traversing at segment 2). In order to highlight these interruptions, as similar as train-time figures, the interruptions are shown with yellow elliptical.

3.2 computational results of the allocation model

The computational results of the allocation model, have three output variables; allocation of freights to trains ( $x_{j,t}$ ), arrival time of freights to the end-station ( $wr_j$ ) and tardiness of freights at the end-station. An example of five freights is solved with the model and the outputs are shown below (Table 7):

*Table 7: Outputs of the allocation model*

| **Freight** | $j_1$ | $j_2$ | $j_3$ | $j_4$ | $j_5$ |
|---|---|---|---|---|---|
| $wr_j$ | 18.5 | 18.5 | 18.5 | 12 | 23.5 |
| $tardi_j$ | 8.5 | 8.5 | 8.5 | 2 | 13.5 |

*Table 8: Allocation of freights to trains*

| $x_{j,t}$ | $j_1$ | $j_2$ | $j_3$ | $j_4$ | $j_5$ |
|---|---|---|---|---|---|
| $t_1$ | 0 | 0 | 0 | 1 | 0 |
| $t_2$ | 1 | 1 | 1 | 0 | 0 |
| $t_3$ | 0 | 0 | 0 | 0 | 1 |

As mentioned in Table 9, based on the freights weights, weight capacity of trains and the due dates of freights, freights 1, 2 and 3 are allocated to train 2 and freight 4 and 5 are allocated to trains 1 and 3, respectively. As mentioned in the previous section, we liked to allocate each freight to a single train and as the results show in Table 8, this constraint is also satisfied.

## 4. Conclusion

This paper has presented a mathematical model for the scheduling problem of freight trains on a single line corridor and extended the model with the allocation of freight to the scheduled freight trains. The objective function of the scheduling problem is minimizing total travel time of freight trains with considering safety and proximity constraints as well as trains weight constraints. In addition, we solved the model for three heading and two returning trains in a single line corridor with three segments. The allocation model has presented a mathematical model to follow two goals; minimizing the total penalty for tardy freights and maximizing the allocation of higher priority freights to scheduled trains. An example of five freights with different due dates, priorities and weights for allocating to the scheduled freight trains is also represented. Finally, in order to evaluate the models efficiency, the model is solved for 120 heading and 80 returning trains and the Gams software presented the answers in just 16 minutes. It is concluded that the proposed model is able to produce useful results in terms of optimal schedules in a reasonable time. At the end, to settle more constraints, the model and method of this paper can be extended to provide schedules for such problems considering capacity constraints of trains, capacity of freights.

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