NONPARAMETRIC ESTIMATION FOR ABSOLUTE POVERTY LINE WITH APPLICATION TO EGYPT

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ABSTRACT

The absolute poverty line is the income or the expenditure necessary to meet basic human needs (food, clothing, housing) Ravallion [27]. Two approaches have been used to estimate the absolute poverty line; the cost of basic needs and the food energy intake. According to the cost of basic needs approach; the absolute poverty line is estimated using a multiple linear regression model. The Traditional Least Square method (parametric technique) is often used to estimate the absolute poverty line. In this paper we will suggest the rank-based estimation as a robust nonparametric technique to estimate the absolute poverty line. The parametric and robust nonparametric techniques will be used to estimate the absolute poverty line for Egypt using the Egypt Household income, spending and consumption survey (HIECS). The real data set has been collected by CAPMAS from 2010/2011 to 2014/2015. This data will be used to comparing the different techniques that are used to estimate the absolute poverty line.

Keywords: *Absolute Poverty Line, The Cost of Basic Needs, Rank-Based Estimation, Weighted Wilcoxon Estimation.*

1. INTRODUCTION

Recently, poverty analysis has become one of the most important issues on the international scene where it associated with weak human resources, poor economies, poor social services and weak policies addressing the challenges to human, social and economic development. Among the multiple poverty definitions, Adam Smith defined poverty as "the inability to purchase necessities required by nature or custom" Smith [30]. Rowntree [29] defined poverty as "earnings insufficient to obtain the minimum necessaries for the maintenance of merely physical efficiency". The World Bank [33] introduced the definition of poverty where "poverty is pronounced deprivation in wellbeing". The framework to measure poverty is summarize in 4 steps as the following: defining an indicator of the standard of living (income or expenditure), determine the living unit used to measure poverty (individual or household) , estimate the minimum level of income or expenditure to meet the basic needs or estimate the poverty line to separate the poor from the non-poor and create an index through which the poverty level can be measured statistically, Ravallion [28].

In this paper we focus on the poverty line because it is the most important step in measuring poverty. There are three types of poverty lines: Absolute poverty line that is defined by Ravallion [28] as "the minimum spending or consumption (or income) required meeting the basic needs (food, clothing, health and housing ...)". The relative poverty line that is defined by Townsend [32] as "the level of income or expenditure that allows to obtain the economic and social resources necessary to engage in the types of behavior expected of members of a particular community (e.g. education, social or recreational activities; ensuring adequate housing,…)". The subjective poverty line that is based on asking people what is the minimum income or spending required covering their basic needs. The absolute poverty line is the poverty line that we will estimate for Egypt in this paper.

The two approaches that used to estimate the absolute poverty line are the cost of basic needs (CBN) and the food energy intake (FEI). CBN approach is often used to estimate the absolute poverty line; it estimates the cost of getting enough food per person and then adds the cost of other necessities such as clothing and housing. Therefore in this approach the absolute poverty line estimates as the sum of two poverty line; food poverty line and non-food poverty line. The food poverty line is the cost of obtaining the food requirements for daily activity by knowing the amount of money needed to purchase the basket of food commodities that provides the energy between 2,000 and 2,200 calories. Non-food poverty line is the cost of obtaining basic non-food needs such as clothing, housing, learning… etc. The difficulty in estimating the spending on basic non-food needs due to the lack of accurate data on non-food goods and services to differ from one place to another and from one person to another. Therefore to estimate the spending on non-food basic needs we estimate first the relative spending on food for the percentage of the poorest household using multiple linear regression model and subtracted the spending on food from the total spending of these households to get the spending on non-food that is correspond non-food poverty line. When the information of prices are unavailable,

the FEI approach can be used, this approach determine the level of income and expenditure that allows the individual or household to get enough food to meet their energy needs sufficient to engage in daily activity.
The multiple linear regression models that used to estimate the relative spending on food to estimate the non-food poverty line and then estimate the absolute poverty line will be estimated using parametric and robust nonparametric techniques.
As a Parametric Technique, Traditional Least Squares (LS) or L_2 method is the most widely for statistical analysis of linear regression models and its theoretical properties were widely studied and fully understood, as example it's BLUE (Best Liner Unbiased Estimation) under regularity all conditions or assumptions (normality, no outlier …). Despite its great advantages, the LS is not reliable in case of violation in assumptions, where it can be sensitive to outliers (non-robust) and Its performance may be compromised in terms of accuracy and statistical inferences when errors are large and heterogeneous.
For Robust Nonparametric Techniques, there is no strict parametric assumptions are made on the underlying probability model that generated the data (normality) and can reveal structure in data that might be missed by parametric methods. Unlike LS, Robust Nonparametric Technique (rank-based estimation) is not sensitive to outliers therefore it's robust, obtain robust estimates and more efficient in case of non-normality.
Robust is a statistical term defined as insensitive against violation of a specified assumption, there are robust methods have been developed since 1960 to be less sensitive to outliers Huber and Ronchetti (2009). Rank-based estimators were developed as a robust nonparametric alternative to traditional LS or likelihood estimators. Rank-based regression was first introduced by Jurečková [21] and Jaeckel [20]. Hettmansperger and Mckean [14] presented a Newton step algorithm of these rank-based estimates, a rank-based inference for linear models has been developed that is based on rank-based estimation similar to the way that traditional analysis is based on LS estimation Hollander and Wolfe [17] and Hettmansperger and Mckean [15]. This rank-based analysis generalizes Wilcoxon procedures for simple location models and, further, it inherits the same high efficiency that these simple nonparametric procedures possess. In addition, weighted versions can have high breakdown (up to 50%) in factor space Chang et al. [3]. Also, the Weighted Wilcoxon (WW) as a rank-based estimator to estimate linear regression model, Mckean [24] will be introduced in this paper.
In this paper, in section ii, the traditional techniques for setting absolute poverty line using LS method will be presented. In Section iii, we will show the robust nonparametric technique for setting absolute poverty line using rank-based estimation that used WW estimator. In section iv, the absolute poverty line will be estimated using the proposed techniques to the real survey data on household expenditures collected in Egypt by CAPMAS from year 2010/2011 to year 2014/2015 [6,7,8]and we will compare between these techniques.

2. TRADITIONAL TECHNIQUES FOR SETTING ABSOLUTE POVERTY LINE

For setting the absolute poverty line, we will use the CBN approach, where the absolute poverty line (PL) consists of two parts: food poverty line (PL_F) and non-food poverty line (PL_N). There are many studies estimated the absolute poverty line using CBN approach and traditional LS method in the literature such as; Bidani and Ravallion [2] as the following:

Food poverty line (PL_F)

The food poverty line has estimated as the following:

1. Arrange sample households according to consumer spending. Then determinant a specific percentage of the sample (15%), which includes households with the least consumption spending to be used to build the food poverty line.

2. Distribute of various food commodities on a certain number of categories. Obtain the X_q vector, which represents the average of the actual quantities consumed per household per year for the different categories of food commodities for the families defined by q, where q represents percentage of the sample (15%).

3. Determine the total number of calories TC_q as follows:

$$TC_q = X_q * C \tag{1}$$

Where C is the vector of the number of calories per kilogram of different food commodities.

4. Obtain X^* as follows:

$$X^* = \frac{C^*}{TC_q} * X_q \tag{2}$$

Where X^* represents a vector whose components are the average relative quantities of goods to be consumed by poor households, and C^* represents the number of calories required by the Food and Agricultural Organization of the United Nations, it's 2,100 Calories per person per day.

Then the food poverty line PL_F estimated as:

$$PL_F = X^* * P \tag{3}$$

Where, P Price vector for various food commodities.

Non-food poverty line

The annual spending on food is estimated using the following linear regression model:

$$FRS_i = \beta_\circ + \beta_1 \ln(\frac{TS_i}{PL_F}) + \beta_2[\ln(\frac{TS_i}{PL_F})]^2 + L_i + \varepsilon_i \quad , i = 1, \dots . n \tag{4}$$

Where n is the number of households, FRS_i is annual relative spending on food for household number i, TS_i is the total spending of the household number i, L_i is Place of residence for household number i, ε_i is random error term. The parameters are estimated using the LS method that minimizes the Euclidean norm as the following:

$$\widehat{FRS} = \hat{\beta}_\circ + \hat{\beta}_1 \ln(\frac{\overline{TS}}{PL_F}) + \hat{\beta}_2[\ln(\frac{\overline{TS}}{PL_F})]^2 \tag{5}$$

Where $\overline{TS}$ is the annual average of total spending of the household, $\widehat{FRS}$ is estimation of relative spending on food. The estimation of annual spending on food for household $\widehat{FS}$ on poverty line will be:

$$\widehat{FS} = \widehat{FRS} * \overline{TS} \tag{6}$$

Then the annual spending on non-food for household $\widehat{NS}$ which is equivalent to a non-food poverty line will be:

$$\widehat{NS} = \overline{TS} - \widehat{FS} = \overline{TS} * \left(1 - \widehat{FRS}\right) \tag{7}$$

And the non-food poverty line PL_N will be estimated as follows:

$$PL_N = \overline{TS} * \left(1 - \widehat{FRS}\right) = \overline{TS} * \left(1 - (\hat{\beta}_\circ + \hat{\beta}_1 \log(\frac{\overline{TS}}{PL_F}) + \hat{\beta}_2\, S)\right) \tag{8}$$

The absolute poverty line PL according to CBN approach will be:

$$PL = PL_N + PL_F \tag{9}$$

Datt and Gunewardena [4] estimate the absolute poverty line for Sri Lanka at (1985-1990) where the food poverty line has been estimated for five specific areas for more details see; Datt and Gunewardena [4]. Non-food poverty line was estimated in the same way as Bidani and Ravallion [2] using the following linear regression model to estimate the annual spending on food:

$$FRS_{ia} = \beta_{\circ a} + \beta_{1a}S_{ia} + \beta_{2a}C_{ia} + \beta_{3a} \ln\frac{\overline{TS}_{ia}}{PL_{Fa}} + \varepsilon_{ia} \quad , i = 1, \dots . n \tag{10}$$

Where a is the number of the area, S_{ia} is the household size for household number i and area number a, C_{ia} is number of children under 10 years for household number i and area number a.

Assumed,

$$\overline{TS}_a = PL_{Fa} \tag{11}$$

The linear regression model is estimated using the LS method as:

$$\widehat{FRS}_a = \hat{\beta}_{\circ a} + \hat{\beta}_{1a}\,\bar{S}_a + \hat{\beta}_{2a}\,\bar{C}_a \tag{12}$$

Where $\bar{S}_a$: the average household size in area number a, $\bar{C}_a$: average number of children under 10 years in area number a.

El-Laithy and Osman [9] estimate absolute poverty line for Egypt at (1995/1996) using the same methodology that Bidani and Ravallion [2] presented, where the food poverty line is estimated using equation (3) and non-food poverty line is estimated using the following linear regression model:

$$FRS_i = \beta_\circ + \beta_1 \log\left[\frac{\overline{TS}_i}{PL_F}\right] + \beta_2 h_i + \varepsilon_i \quad , i = 1, \dots . t \tag{13}$$

Where, t is a number of annual spending categories of the individual, FRS_i is annual relative spending on food in category i, $\overline{TS}_i$ is annual average of total spending of the individual in category i, h_i is household size in category i, ε_i is random error term.

The estimation of this linear regression model using traditional LS method that minimizes the square of error will be:

$$\widehat{FRS} = \hat{\beta}_\circ + \hat{\beta}_1 \log\left(\frac{\overline{TS}}{PL_F}\right) + \hat{\beta}_2\,S \tag{14}$$

Where S that is the average household size and then the non-food poverty line will be:

$$PL_N = \overline{TS} * \left(1 - \widehat{FRS}\right) = \overline{TS} * \left(1 - \left(\hat{\beta}_\circ + \hat{\beta}_1 \log\left(\frac{\overline{TS}}{PL_F}\right) + \hat{\beta}_2\,S\right)\right) \tag{15}$$

3. ROBUST NONPARAMETRIC TECHNIQUE FOR SETTING ABSOLUTE POVERTY LINE

The robust nonparametric technique to estimate the absolute poverty line in Egypt will be introduced. The robust analysis has presented by Mckean [24] that offers a methodology for multiple linear regression models similar to Least Square and it is not sensitive to outliers in the response space and highly efficient compared to the LS analysis in non-normality. LS estimators based on minimize the Euclidean norm, while rank-based estimators based on minimize pseudo-norm that similar to Euclidean norm that is illustrated as following:

Let $X_{1,\dots,}X_p$, p be the explanatory variables, and Y be a response variable, a set of data and consider the regression of Y on X at the n points, let Y be the $n \times 1$ vector of responses and let X be the $n \times p$ matrix. The linear regression model is:

$$Y = 1\alpha + X_c\beta + \epsilon \tag{16}$$

Where β is a $p \times 1$ vector of regression coefficients, α is an intercept, X_c is the centered design matrix with $x_{cij} = x_{ij} - \bar{x}_j$, $\bar{x}_j = \sum_{i=1}^{n} x_{ij}/n$ and ϵ is a $n \times 1$ vector of random independent errors. The least square estimator for the regression coefficients will be:

$$\hat{\beta}_{Ls} = \text{Arg min}_\beta \parallel Y - X_c\beta \parallel^2 \tag{17}$$

$$\hat{\alpha}_{Ls} = \bar{Y} \tag{18}$$

Where $\parallel Y - X_c\beta \parallel^2$ is the squared Euclidean norm. Under regularity conditions, the large sample distribution of the least square estimator follows the normal distribution as the following:

$$\begin{pmatrix} \hat{\alpha}_{Ls} \\ \hat{\beta}_{Ls} \end{pmatrix} \approx N\left(\begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \begin{bmatrix} \sigma^2/n & 0' \\ 0 & \sigma^2(\grave{X}_c X_c)^{-1} \end{bmatrix}\right)$$

Where, σ^2 is the variance of errors for least square method.

But the rank-based estimator for the regression coefficients will be as the following:

$$\hat{\beta}_W = \text{Arg}\min_{\beta\in\mathbb{R}^n} \parallel Y - X_c\beta \parallel_w \quad (19)$$

Where $\parallel Y - X_c\beta \parallel_w$ is pseudo-norm the norm of rank-based estimator Jaeckel [20]. Assume that

$$U = Y - X_c\beta \quad , U \in \mathbb{R}^n \quad (20)$$

The pseudo-norm $\parallel U \parallel_w$ will be

$$\parallel U \parallel_w = \sum_{i=1}^{n} a(R(u_i))\, u_i = \sum_{i=1}^{n} a(i)\, u_{(i)} \quad (21)$$

Where $R(u_i)$ is the rank of u_i, $a(i)$ is a function of φ as the following:

$$a(i) = \varphi(i / (n + 1)) \quad (22)$$

Where φ(g) is the score function and it is non-decreasing function on (0, 1). There are many score functions, but in this study Mckean [24] was used the linear Wilcoxon score function. So the rank-based estimator becomes Weighted Wilcoxon estimator (WW). The linear Wilcoxon score function is

$$\varphi(g) = \sqrt{12}(g - 0.5) \quad (23)$$

Then,

$$\varphi\left(\frac{i}{(n + 1)}\right) = \sqrt{12}\left(\frac{i}{(n + 1)} - 0.5\right)$$

And using the median of the residuals, the estimate of α will be

$$\hat{\alpha}_W = \text{med}_{1\le i\le n}\{Y_i - x'_{ci}\,\hat{\beta}_W\} \quad (24)$$

Under regularity conditions, the large sample distribution of the Wilcoxon estimator follows the normal distribution as the following:

$$\begin{pmatrix} \hat{\alpha}_W \\ \hat{\beta}_W \end{pmatrix} \approx N\left(\begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \begin{bmatrix} \frac{\tau_s^2}{n} & 0' \\ 0 & \tau^2(\grave{X}_c X_c)^{-1} \end{bmatrix}\right)$$

Where τ^2 and τ_s^2 are the scale parameters, τ^2 is the variance of error for Wilcoxon estimation, τ^2 and τ_s^2 are estimated by Hettmansperger and Mckean [14] where:

$$\tau = (\sqrt{12}\int f^2(t)dt)^{-1} \quad (25)$$

$$\tau_s = (2f(0))^{-1} \quad (26)$$

$f(t)$ is the probability density function of the errors.

Here lies the appeal of nonparametric techniques that have the power to reveal the structure of factual data that may be ignored by traditional parametric techniques. In the traditional parametric techniques, the estimate of the probability density function expressed by the observed data is based on the assumption that the model is a parameter, but if this assumption is not true, all the results and conclusions of this analysis lead to misleading interpretations of the observed data. However, in the nonparametric techniques, the density function is estimated according to the nature of the data and what it encodes. In this case, the density function is more flexible and the data can express itself. This is a powerful tool for analyzing real-world data.

The asymptotic relative efficiencies (ARE) has been used by Mckean [24] to compare between the β's using their variances, ARE between $\hat{\beta}_W$ and $\hat{\beta}_{LS}$ as the following:

$$\text{ARE}\left(\hat{\beta}_w, \hat{\beta}_{LS}\right) = \frac{\text{var}\left(\hat{\beta}_{LS}\right)}{\text{var}\left(\hat{\beta}_w\right)} = \frac{\sigma^2}{\tau^2} \quad (27)$$

In case of the error distribution is normal with variance σ^2

$$\tau^2 = \frac{\sigma^2}{\left(\frac{3}{\pi}\right)} \ , \ \text{ARE} = \frac{3}{\pi} = 0.955$$

Under the assumption of normality, the Weighted Wilcoxon estimator is 95% as efficient as LS estimator. And therefore the efficient of the Wilcoxon estimation comparing with another estimation method depend on the true distribution for the real data. On the other hand, if the true distribution is not normal (has tails heavier than the normal), then ARE is usually larger than 1.

The influence function (IF) has been competed for LS estimator and WW estimator by Mckean [24]. The influence function (IF) was introduced by Hampel [11, 12] for estimators, where IF measures the effect of an additional observation in any y or x (y or x may be an outlier). The IF for LS estimator and WW estimator will be presented as follows:

Assume that

$$n^{-1}\grave{X}_c X_c \rightarrow \Sigma$$

Then

$$\text{IF}\left(\hat{\beta}_{LS}\right) = \Sigma^{-1}\epsilon\, x \quad (28)$$

$$\text{IF}\left(\hat{\beta}_W\right) = \tau\, \Sigma^{-1}\varphi(F(\epsilon))\, x \quad (29)$$

It is shown that the change in the LS estimator is unbounded in y and x spaces, then the LS estimator is not robust to outliers in y and x. on the other hand, the change in the WW estimator is bounded in y space, then the WW estimator is robust to outliers in y.

4. REAL DATA APPLICATION

The data are collected from the Household income, expenditure and consumption survey (HIECS) in Egypt for three rounds 2010/2011, 2012/2013 and 2014/2015, this surveys were presented by "Central Agency for Public Mobilization and Statistics (CAPMAS)" that is the largest government agency to collect data in Egypt. As we mentioned, we will use the household spending (or expenditure) as indicator of welfare to estimate the absolute poverty line. For estimate the absolute poverty line we use CBN approach and the mechanism that presented by El-Laithy and Osman [9] will be used to estimate the food and non-food poverty line for rural and urban Egypt and the household allowance instead of the individual as an indicator of welfare will be used.

The food poverty line: $PL_F = X^* * P$ using the same basket of food commodities that presented by El-Laithy and Osman [9] with adjusted prices consistent with the survey years using the consumer price index (CPI) for this year is estimated. In table 1, the estimation of the food poverty line for rural and urban households in years 2010/2011, 2012/2013 and 2014/2015 is presented.

Table 1: the food poverty line for rural and urban households for Egypt in 2010/2011, 2012/2013 and 2014/2015

Years of surveys	2010/2011	2012/2013	2014/2015
Rural	8965	11507	14167
Urban	11597	14600	18367

In table 1, we note that the food poverty line for urban and rural households is increasing over time due to inflation in the prices of food and drink. And the food poverty line for the urban household is greater than the food poverty line for the rural household, although the average household size in the rural area is higher than the average size of the household in urban areas due to the difference in the prices of food commodities.

As we mentioned, the difficulty of obtaining the estimate of spending on basic non-food needs to estimate the non-poverty line, we will estimate the spending on food needs and subtract it from the total spending to obtain the spending on basic non-food needs. The linear regression model in equation 13 will be used using parametric (LS technique) and robust nonparametric (weighted Wilcoxon technique) techniques. In table 2, we present the results of using the traditional least square method that is used to estimate the regression coefficients, relative spending on food, non-food poverty line and absolute poverty line for rural and urban households in years 2010/2011, 2012/2013 and 2014/2015.

Table 2: the LS estimation for absolute poverty line for rural and urban households for Egypt in years 2010/2011, 2012/2013 and 2014/2015.

Years	2010/2011		2012/2013		2014/2015	
	Rural	urban	Rural	Urban	Rural	Urban
$\hat{\beta}_{\circ}$	0.08171	0.47936	-0.4098	0.27595	0.408482	0.376429
$\hat{\beta}_1$	-1.08183	-0.52297	-1.89	-0.6111	-0.45443	-0.31163
$\hat{\beta}_2$	0.14675	0.01914	0.3107	0.07012	0.033561	0.01571
$\widehat{FRS}$	0.423338	0.3689412	0.4513106	0.3637207	0.3992995	0.3257608
PL_N	10742	16828	12623	19136	19110	28667
PL	19707	28425	24130	33736	33277	47034

As shown in table 2, both the non-food poverty line and the absolute poverty line of the household are increasing in urban and rural areas over time. Also Non-food poverty line and the absolute poverty line for the household per year for rural less than urban due to the different non-food basic needs for both rural and urban and the cost of those needs. We note that, the value of intercept in rural 2012/2013 is negative and this is not logical as that value reflects the relative spending on food, this means there is a problem of this estimation.

In case of violation in normality assumption or existence of outlier values, the least square method (LS) is not reliable. So in Figure 1, 2, 3, 4, 5 and 6 we will check normality and outliers on the data collection for rural and urban in each year of survey using histogram, normal Q-Q plot and box plot for the response variable as the following:

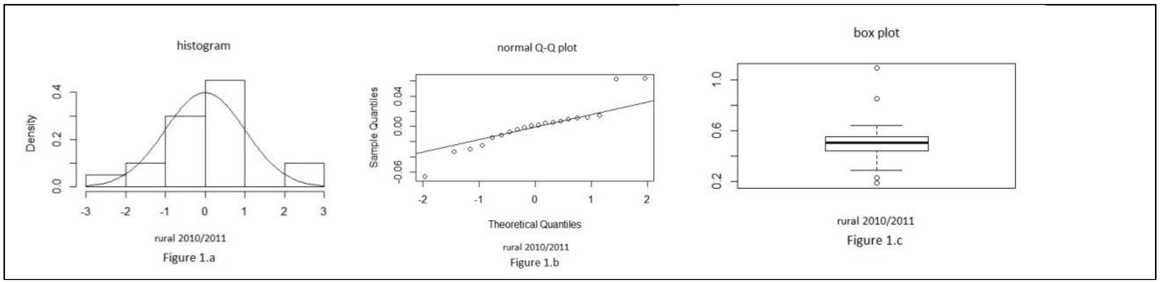

Figure 1: histogram, normal Q-Q plot and box plot for rural Egypt 2010/2011

As shown in Figure1.b, the observations do not follow the normal distribution. Also, in Figure 1.c, there are two outliers in response variable.

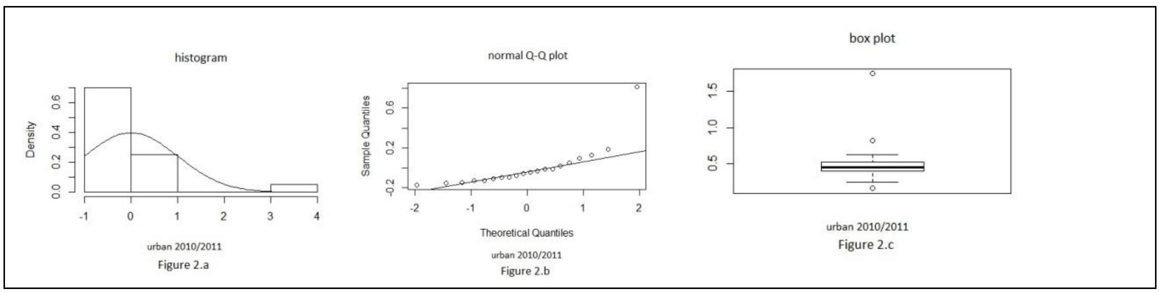

Figure 2: histogram, normal Q-Q plot and box plot for urban Egypt 2010/2011

In Figure 2.b, Errors do not follow normal distribution and there is a very right heavy tail distribution as shown in Figure 2.a. In addition, there are three outliers in response variable in Figure 2.c.

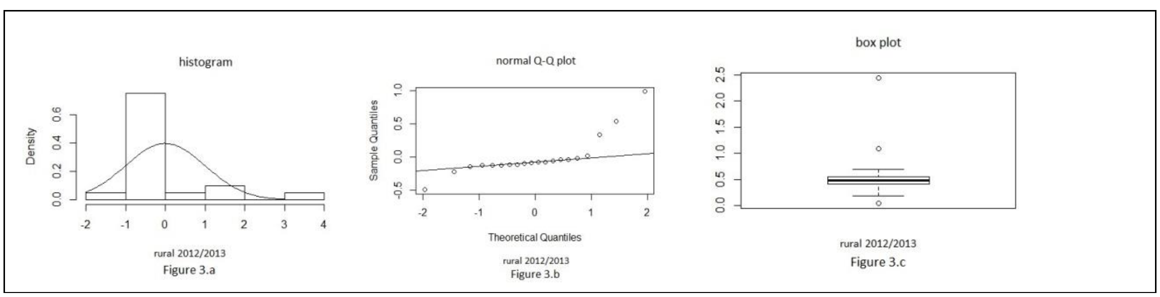

Figure 3: histogram, normal Q-Q plot and box plot for rural Egypt 2012/2013

As shown in Figure 3.a and 3.b, there is a violation in normality where the data follows a distribution has tails heavier than the normal. Also, in Figure 3.c, there are three outliers in response variable.

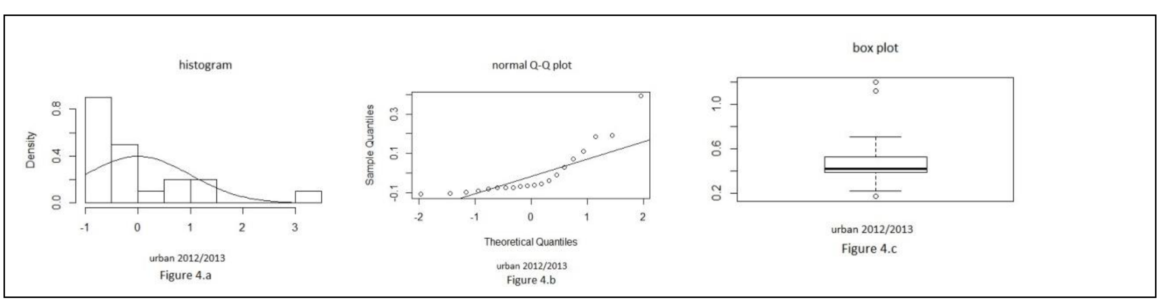

Figure 4: histogram, normal Q-Q plot and box plot for urban Egypt 2012/2013

In Figure 4.b, errors do not follow normal distribution and a right heavy tail distribution shown in Figure 4.a. In addition, there are three outliers in response variable shown in Figure 4.c.

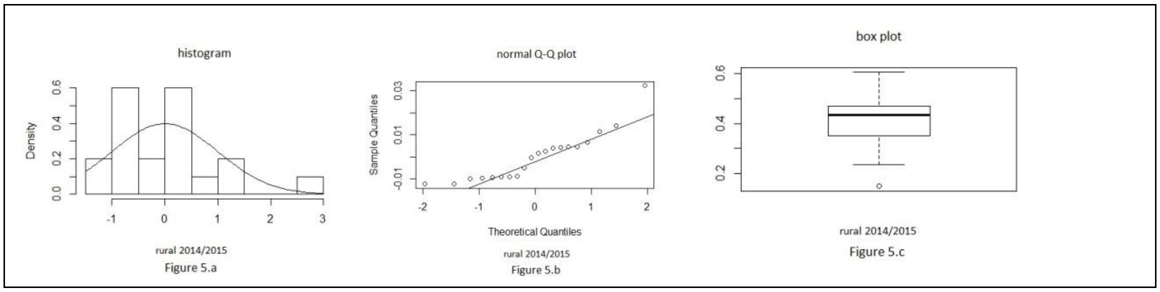

Figure 5: histogram, normal Q-Q plot and box plot for rural Egypt 2014/2015

In Figure 5.b there is a violation in normality and in Figure 5.a the real data distribution shows heavier than the normal distribution. Also, there is one outlier in the response variable in Figure 5.c.

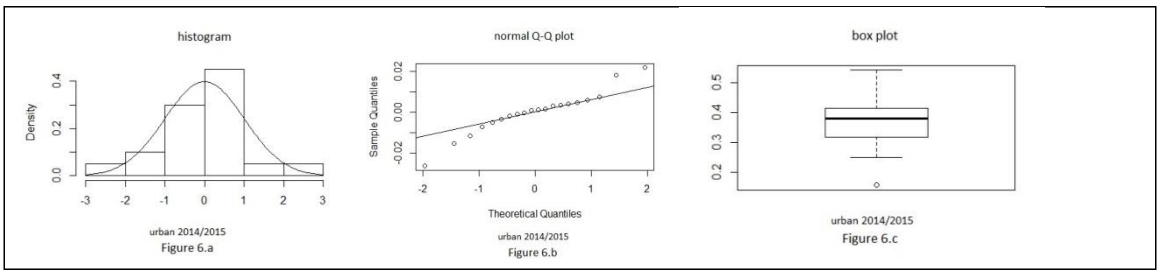

Figure 6: histogram, normal Q-Q plot and box plot for rural Egypt 2014/2015

As shown in Figure 6.b, the observations do not follow the normal distribution, and in Figure 6.c. there is one outlier in response variable.

We can conclude from the previous figures that histograms, normal Q-Q plots and box plots showed that all the data collected from rural or urban in all year have violation in normality that appear in histogram and normal Q-Q plot figures. In addition, the data have some outliers.

In this paper, the rank-based estimator presented by Mckean [24] for linear regression model will be used using the Weighted Wilcoxon (WW) score function discussed in section iii. In table 3, the parameters of linear regression model of relative spending on food are estimated using Weighted Wilcoxon. Also, the non-food poverty line and absolute poverty line for rural and urban for Egypt in years 2010/2011, 2012/2013 and 2014/2015 of survey are estimated.

Table 3: the WW estimation for absolute poverty line for rural and urban households for Egypt in years 2010/2011, 2012/2013 and 2014/2015.

Years	2010/2011		2012/2013		2014/2015	
	Rural	Urban	Rural	Urban	Rural	Urban
$\hat{\beta}_\circ$	0.1323149	0.441302	0.183807	0.223945	0.3885082	0.3673853
$\hat{\beta}_1$	-0.98896	-0.292851	-0.7991	-0.52009	-0.46601	-0.31144
$\hat{\beta}_2$	0.13044	0.009901	0.098529	0.066941	0.038801	0.018375
$\widehat{FRS}$	0.4272685	0.5879109	0.3968525	0.3276072	0.3987456	0.3274594
PL_N	10666	10989	13876	20222	19127	28595
PL	19634	22586	25383	33736	33295	46962

In table 3, the non-food poverty line and the absolute poverty line of the household are increasing in urban and rural areas over time, and non-food poverty line and the absolute poverty line for the household per year for rural less than urban. We note that, WW estimation fixed the LS mistake about the value of intercept in rural 2012/2013.

To compare between LS estimators and WW estimators, the asymptotic relative efficiency (ARE) that in equation (27) will be used. In table 4, σ^2 and τ^2 have been estimated for our data set and ARE is computed in each year as the following:

Table 4: the estimation of σ^2, τ^2 and ARE for rural and urban

Years	2010/2011		2012/2013		2014/2015	
	Rural	Urban	Rural	Urban	Rural	Urban
$\hat{\sigma}$	0.03078	0.2272	0.3248	0.1373	0.01176	0.01118
$\hat{\tau}$	0.019425	0.029254	0.077991	0.062668	0.009172	0.008393
$\hat{\sigma}^2$	0.000947	0.05162	0.105495	0.018851	0.000138	0.000125
$\hat{\tau}^2$	0.000377	0.000856	0.006083	0.003927	8.41E-05	7.04E-05
ARE $(\hat{\beta}_w, \hat{\beta}_{LS})$	2.51	60.32	17.34	4.80	1.64	1.77

In table 4, there are violation in normality and outlier, so the WW estimators are better than the LS estimators. Where ARE is bigger than 1 in all years for rural and urban that means that WW estimators is more efficient than LS estimators in all years. As an example, ARE = 2.510895 in rural at 2010/2011 then the Wilcoxon analysis is estimated to be 2.51 times as efficient as the LS analysis for these data. Interestingly, ARE equal 60.32 (the largest value) in urban 2010/2011 when the distribution very right heavy tail, and there is one of outliers too far from the data, and ARE equal 17.34 in rural 2012/2013 when distribution has tails heavier than the normal, there is one of outliers the farthest value from the data in all years.

5. REFRENCES

[1]. B. Bibani, M. Ravallion. "Poverty in Indonesia 1990". World Bank, Washington, DC (1990).

[2]. B. Bibani, M. Ravallion. "Poverty in Indonesia 1990". World Bank, Washington, DC (1992).

[3]. W.H. Chang, J.W. Mckean, J.D. Naranjo, S.J. Sheather. "High-breakdown rank regression". *Journal of the American Statistical Association*, 94, 205–219 (1999).

[4]. G. Datt, D. Ganewardena. "Some Aspects of Poverty in Srilanka (1985-1990). Poverty and Human Resources Division Policy Research Department". World Bank, Washington, DC (1997).

[5]. G. Datt, D. Jolliffe, M. Sharma. "Poverty Profile in Egypt: 1997". IFPRI Food Security Research Unit of the Agricultural Policy Reform Program in Egypt. IN Collaboration With The Ministry of Agriculture and Land Reclamation and The Ministry of Trade abd Supply USIAD Grant No. 263–9–00–96-0030-00(1998).

[6]. CAPMAS (Egyptian Central Agency for Public Mobilization and Statistics). Income, Consumption and Expenditure Research (2010/2011). Cairo. Egypt (2011).

[7]. CAPMAS (Egyptian Central Agency for Public Mobilization and Statistics). Income, Consumption and Expenditure Research (2012/2013). Cairo. Egypt (2013).

[8]. CAPMAS (Egyptian Central Agency for Public Mobilization and Statistics). Income, Consumption and Expenditure Research (2014/2015). Cairo. Egypt (2015).

[9]. H. EL-Laithy, M. Osman. "Profile and Trend of Poverty and Economic Growth in Egypt". UNDP and Institute of National Planning. Cairo. Egypt (1997).

[10]. H. EL-Laithy, O. EL-Khawaga, N. Raid. "Poverty Assessment in Egypt (1991-1996)". Research Papers Series Economic Department, Faculty of Economics and Political Science. Cairo University (1999)

[11]. F. R. Hampel. "Contributions to the theory of robust estimation". Unpublished dissertation, University of California, Berkeley (1968).

[12]. F. R. Hampel. "The influence curve and its role in robust estimation". *Journal of the American Statistical Association,* 69, 383-393 (1974).

[13]. J. Haughton, S. R. khandker. "Handbook on Poverty and Inequality". The International Bank for Reconstruction and Development. World Bank, Weshington, DC (2009).

[14]. T. P. Hettmansperger, J.W. Mckean. Robust Nonparametric Statistical Methods. Arnold, London (1998).

[15]. T. P. Hettmansperger, J. W. Mckean. Robust Nonparametric Statistical Methods, Chapman Hall, New York (2011).

[16]. T. P. Hettmansperger, J. W. Mckean, S. J. Sheather. "Robust nonparametric methods". *Journal of the American Statistical Association*, 95, 1308-1312 (2000).

[17]. M. Hollander, D. A. Nonparametric Statistical Methods, Wiley, New York (1999).

[18]. M. Hollander, D. A. Wolfe, E. Chicken, E. Nonparametric Statistical Methods. John Wiley and sons, Inc, Hoboken, New Jersey (2014).

[19]. P. J. Huber, E. M. Ronchetti. Robust Statistic. John Wiley & Sons, Inc, Hoboken, New Jersey (2009).

[20]. A. Jaeckel. "Estimating regression coefficients by minimizing the dispersion of the residuals". *The Annals of Mathematical Statistics*, 43:1449–1458 (1972).

[21]. J. Jurecková. "Nonparametric estimate of regression coefficients". *The Annals of Mathematical Statistics*, 42:1328–1338 (1971).

[22]. K. Korayem. Poverty and Income Distribution in Egypt. The Third World Form, Cairo, Egypt (1994).

[23]. S. J. Malik. "Poverty in Pakistan (1984-1985) to (1987-1988)". Proceedings of A Symposium Organized by the World Bank and The International Food Policy Research Institute. World Bank, Washington, DC (1990).

[24]. J. W. Mckean. "Robust Analysis of Linear Models". *Institute of Mathematical statistics, Statistical Science*,19, 4, 562–570 (2004).

[25]. J. W. Mckean, R. M. Schrader. "The Geometry of Robust Procedures in Linear Models". *Journal of the Royal Statistical Society. Series B*, 42, 366-371 (1980).

[26]. J. W. Mckean, T. P. Hettmansperger. "A robust analysis of the general linear model based on one step R-estimates". Biometrika, 65:571-579 (1978).

[27]. 27. M. Ravallion. "Poverty Comparisons: A Guide to Concepts and Methods". LSMS Working Paper, 88. World Bank, Washington, DC (1992).

[28]. M. Ravallion. "Poverty Lines in Theory and Practice". LSMS Working Paper, 133, World Bank, Washington, DC (1998).

[29]. B. S. Rowntree. Poverty, a study of town life. MacMillan, London (1901).

[30]. A. Smith, A. An enquiry into the nature and causes of the wealth of nations. London: Methuen & Co., Ltd (1776).

[31]. 32. P. Townsend. The international analysis of poverty. Milton Keynes: Harvester Wheatsheaf (1993).

[32]. 33. World Bank. World Development Report 2000/2001: Attacking Poverty. World Bank, Washington, DC (2000).