CHANCE CONSTRAINED PROGRAMMING (CCP) WITH GENERALIZED EXPONENTIAL (GE) DISTRIBUTED RANDOM PARAMETERS

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ABSTRACT

The three-parameter generalized exponential distribution $GE(\lambda,\mu,\alpha)$ is considered more flexible and applicable than three-parameter Gamma and Weibull distributions, where their Shape Parameter (α) may be not Integer.

The purpose of this paper is to transform CCP linear model to an equivalent deterministic linear programming model also, in turn can be solved by simplex method. Two cases are considered:

(i) When some right hand side parameter ($\tilde{b}_i$) have $GE(\lambda_i,\mu_i,\alpha_i)$ or

(ii) When one coefficient parameter ($\tilde{a}_{ij}$) have $GE(\lambda_{ij},\mu_{ij},\alpha_{ij})$.

Finally a numerical example is presented to illustrate the transformation and solution.

Keywords: *CCP, Cumulative dist. Function,* $GE(\lambda,\mu,\alpha)$ *distribution, Inverse function, Log-convex function, Probabilistic programming, Simplex method.*

1. INTRODUCTION

For the first time, Charnes and Cooper 1995, presented the CCP technique to transform the probabilistic models to an equivalent deterministic models by using the inverse cumulative distribution functions of the random parameters. Then, they and other applied the technique for solving many different problems [5,7].

Sengupta and Gruver 1969 introduced CCP with the random parameters are distributed normal or exponential, and they derived nonlinear approximate deterministic models when the coefficient parameters are random [12]. Then, they used it to study the reliability analysis of systems.

Sengupta 1972 derived a deterministic approximate nonlinear programming model when the coefficient parameters are distributed chi-square that by using non-central chi-square distribution [8].

El-Dash 1984 transformed CCP linear and linear goal models to deterministic exact nonlinear programming models when the coefficient parameters are distributed chi-square or exponential with 2 parameters ($Exp.(\lambda,\mu)$) by using Box's theorem [7].

Biswal and Biswal 1998 transformed CCP linear model with the coefficient parameters are distributed exponential with single parameter $Exp.(\lambda)$. Nada 2018 extended Biswal & Biswal's approach when coefficient parameters are distributed exponential with 2 parameters ($Exp.(\lambda,\mu)$). She introduced deterministic exact non-linear programming model also [4].

Usually, when the coefficient parameters are distributed exponential with 2 parameters or gamma or chi-square or non-central chi-square distribution, the transforming CCP model to an equivalent deterministic model, there are two problems: (1) the degree of freedom must be integer, (2) the degree of freedom are function in unknown decision's variables (X).

Gupta and Kundu (1999) introduced generalized exponential distribution with 3 parameters $GE(\lambda,\mu,\alpha)$ where λ,μ,α are scales, location, and shape parameters respectively, they proved that $GE(\lambda,\mu,\alpha)$ is more flexible and applicable than three-parameters gamma and weibull distributions where their shape parameters (α) may be not integer [9].

From above, it is important to fit $GE(\lambda,\mu,\alpha)$ distributions for nonnegative random parameters.

Consider the following CCP linear model:

$$\text{Max.}Z = \sum_{j=1}^{n} c_j x_j \tag{1.1}$$

$$\text{S.t.} \quad \sum_{j=1}^{n} a_{ij} x_j \le b_i \quad , \quad i = 1,2,\ldots.,m' \tag{1.2}$$

$$P_r\Big(\sum_{j=1}^{n} a_{ij} x_j \le \tilde{b}_i\Big) \ge \gamma_i \quad , \quad i = m'+1, m'+2,\ldots.,m \tag{1.3}$$

$$P_r\Big(\tilde{a}_{i1} x_1 + \sum_{j=2}^{n} a_{ij} x_j \le b_i\Big) \ge \gamma_i \quad , \quad i = m+1 \tag{1.4}$$

$$x_j \ge 0 \quad , \quad j = 1,2,\ldots.,n \tag{1.5}$$

Where $x_j \ge 0$, $j = 1,2,\ldots.,n$ are decision variables, c_j, a_{ij}, b_i , $i = 1,2,\ldots.,m+1$ are constants, $\tilde{b}_i$ and $\tilde{a}_{i1}$ are random parameters with $GE(\lambda_i, \mu_i, \alpha_i)$ and $GE(\lambda_{ij}, \mu_{ij}, \alpha_{ij})$ respectively and γ_i, $i = m'+1, m'+2,\ldots.,m$ are the tolerance measures.

The main objective of this paper is to transform CCP linear model in (1.1)-(1.5) to equivalent deterministic linear model, which can be solved by simplex method [8].

2. THE EQUIVALENT DETERMINISTIC CONSTRAINTS

Lemma (2.1): Suppose the chance constraints in (1.3) and $\tilde{b}_i \sim GE(\lambda_i, \mu_i, \alpha_i)$, $i = m'+1, m'+2,\ldots.,m$ then the equivalent deterministic linear constraints as following:

$$\sum_{j=1}^{n} a_{ij} x_j \le \mu_i - \lambda_i \ln[1-(1-\gamma_i)^{1/\alpha_i}] \quad , \quad i = m'+1, m'+2,\ldots.,m \tag{2.1}$$

Where $\ln[1-(1-\gamma_i)^{1/\alpha_i}]$ is constant.

Proof: let F_i and F_i^{-1} are the cumulative distribution function of $\tilde{b}_i$ and its inverse function respectively [8], then

$$P_r\Big(\sum_{j=1}^{n} a_{ij} x_j \le \tilde{b}_i\Big) \ge \gamma_i \longrightarrow 1 - F_i\Big(\sum_{j=1}^{n} a_{ij} x_j\Big) \ge \gamma_i$$

$$F_i\Big(\sum_{j=1}^{n} a_{ij} x_j\Big) \ge 1-\gamma_i \longrightarrow \sum_{j=1}^{n} a_{ij} x_j \le F_i^{-1}(1-\gamma_i)$$

$$\text{Where } F_i(b_i) = \big[1 - e^{-(b_i-\mu_i)/\lambda_i}\big]^{\alpha_i} \tag{2.2}$$

and F_i be log convex function, in turn

$$F_i^{-1}(1-\gamma_i) = \{\, \mu_i - \lambda_i \ln[1-(1-\gamma_i)^{1/\alpha_i}]\} \tag{2.3}$$

Lemma (2.2): Suppose the chance-constraint (1.4) and $\tilde{a}_{i1} \sim GE(\lambda_{ij}, \mu_{ij}, \alpha_{ij})$, then its equivalent deterministic constraint as following:

$$\sum_{j=2}^{n} a_{ij} x_j + x_1\{\, \mu_{i1} - \lambda_{i1} \ln[1-(1-\gamma_i)^{1/\alpha_{i1}}]\} \le b_i \quad , \quad i = m+1 \tag{2.4}$$

Proof: $P_r(\tilde{a}_{i1}x_1 + \sum_{j=2}^{n} a_{ij}x_j \le b_i) \ge \gamma_i \quad , \quad i = m+1 \quad \longrightarrow$

$$P_r(\tilde{a}_{i1} \le \frac{b_i - \sum_{j=2}^{n} a_{ij}x_j}{x_1}) \ge \gamma_i \quad \longrightarrow$$

$$F_i\left(\frac{b_i - \sum_{j=2}^{n} a_{ij}x_j}{x_1}\right) \ge \gamma_i \quad \longrightarrow \quad \frac{b_i - \sum_{j=2}^{n} a_{ij}x_j}{x_1} \le F_i^{-1}(\gamma_i)$$

$$b_i - \sum_{j=2}^{n} a_{ij}x_j \le x_1\, F_i^{-1}(\gamma_i) \quad \longrightarrow \quad \sum_{j=2}^{n} a_{ij}x_j + x_1\, F_i^{-1}(\gamma_i) \le b_i \quad \longrightarrow$$

$$\sum_{j=2}^{n} a_{ij}x_j + x_1\{\mu_{i1} - \lambda_{i1} \ln[1-(1-\gamma_i)^{1/\alpha_{i1}}]\} \le b_i \quad , \quad i = m+1$$

3. NUMERICAL EXAMPLE

Consider the following CCP linear model and

$$\tilde{b}_1 \sim GE(\lambda_1 = 2, \mu_1 = 5, \alpha_1 = 1.5), \ \gamma_1 = 0.7$$

$$\tilde{a}_{21} \sim GE(\lambda_{21} = 1, \mu_{21} = 10, \alpha_{21} = 2), \ \gamma_2 = 0.6, \ b_2 = 15$$

$$Max.Z = 5\,x_1 + 2\,x_2 - x_3$$

$$P_r(x_1 + 2\,x_2 + x_3 \le \tilde{b}_1) \ge 0.7 \qquad (1)$$

$$P_r(\tilde{a}_{21}\,x_1 + 3\,x_2 - x_3 \le 15) \ge 0.6 \qquad (2)$$

$$-x_1 + x_2 + 2\,x_3 \le 1 \qquad (3)$$

$$x_1, x_2, x_3 \ge 0$$

The following deterministic linear constraints (4) and (5) are equivalent to chance constraints (1) and (2) respectively:

$$x_1 + 2\,x_2 + x_3 \le 6.190 \qquad (4)$$

$$11.490\,x_1 + 3\,x_2 - x_3 \le 15 \qquad (5)$$

By simplex method, the optimal solution is:

$$Z^* = 7.81 \ , \quad x_1^* = 0.83 \ , \quad x_2^* = 1.83 \ , \quad x_3^* = 0$$

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