EXPONENTIAL STABILIZATION OF UNCERTAIN CHAINED SYSTEMS OF NONHOLONOMIC ROBOTS BASED ON VISUAL FEEDBACK

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ABSTRACT
The exponential stabilization problem for a kind of nonholonomic wheeled mobile robots with uncertain camera parameters is analyzed based on the visual feedback. Firstly, a model of uncertain chained form system of type (2,1) mobile robot is presented by using the state input transformations. Then, two kinds of controllers are designed by using the auxiliary variable method and the input control method. This two kinds of controllers not only overcome the limit of the initial status of the system but also have high unification. Simulation results demonstrate the effectiveness of the proposed strategies.

Keywords: *Nonholonomic mobile robot, Nonholonimic system, Controller design, Stabilization*.

1. INTRODUCTION
The robust stabilization problem of nonholonomic wheeled mobile robots had received a great deal of attention in the past several decades [1,2,3]. In the control of nonholonomic WMR, it is usually assumed that the states are available using sensor measurements. But in practice, there are uncalibrated parameters, mechanical limitations, noise and so on. Many strategies have been investigated to stabilize the uncertain nonholonomic systems [4,5]. Exponential stability is an important performance characteristic for practical application [6,7,8,9]. Robust discontinuous exponential regulation was discussed for the dynamic nonholonomic WMR with parameter uncertainties [5]. By using a smooth time-varying feedback control law, an assistant state variable and a time-varying state transformation based on the concept of minimal dilation degree [6], the asymptotic exponential stability was addressed which could be transformed into linear time-varying control systems for a large class of nonholonomic systems [7]. In order to solve the problems of exponential stabilization, a smooth time-varying feedback control law was proposed by using a new control input [8]. Robust exponential regulations of nonholonomic systems with uncertainties were proposed in [9,10].

Visual feedback is an important approach to improve the control performance of manipulators and mobile robots [11,12,13,14,15,16,17,18,19]. Two possible configurations exist to set up a vision system for visual servoing [11,12,13,14]. One is the camera mounted at the end-effector of the manipulator and robots, called eye-in-hand system [11,12]. The other is the camera fixed at the position near the manipulator [13,14,15,16,17,18,19]. In [19], the feedbacks from uncalibrated and fixed (ceiling-mounted) cameras were used. The adaptive tracking controllers were developed which compensated for the parametric uncertainties in the camera and the mobile robot dynamics. Dynamic feedback robust regulation and dynamic feedback tracking control problems [15] were discussed for nonholonomic type (2,0) mobile robot with visual feedback and uncertain visual parameters. Homography-based visual servoing controllers were discussed in [16] and [17]. Robust stabilization of nonholonomic uncertain chained systems was addressed in [18,19].

In this paper, motivated by the control ideas in [8] and [20], we address the smooth time-varying feedback laws which can exponentially stabilize the nonholonomic uncertain chained model for type (2,1) robot. Two kinds of controllers are designed by using the auxiliary variable method and the input control method.

The rest paper is organized as follows. Section 2 introduces the robot-camera system and the stabilization problem for an uncertain chained model deduced from the robot-camera system with unknown visual parameters. Section 3 addresses two kinds of controllers for the uncertain chained system and give the proofs rigorously. In Section 4, simulation results are provided to illustrate the effectiveness of the proposed control strategies. Finally, the major contributions of the paper are summarized in Section 5.

2. PROBLEM STATEMENT

2.1. System Configuration
In Fig1, a robot-camera system is shown. It is assumed that the camera plane runs parallel to the mobile robot plane, and the camera can capture images throughout the entire robot workspace. There are three coordinate frames,

namely the inertial frame X-Y-Z, the camera frame i-j-k and the image frame i_1-o_1-j_1. Assume that the i-j plane of the camera frame is parallel to the plane of the image coordinate plane. C(c_x, c_y) is the crossing point between the optical axis of the camera and X-Y plane. The coordinate of the original point of the camera frame with respect to the image frame is defined by (O_{c1},O_{c2}). (x,y) is the coordinate of the mass center P for the robot with respective to X-Y plane. β(t) denotes the angle between the orientation of the plane of steering wheel and i_2 axis. θ denotes the angle between the i_2 axis and X axis.

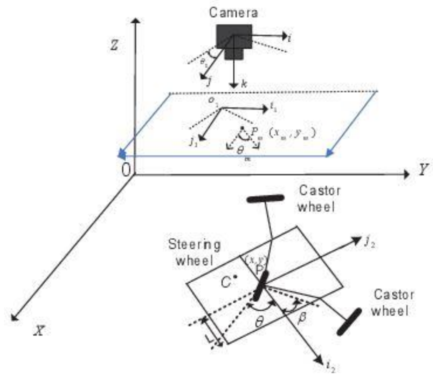

Fig.1 Camera-robot system configuration

Assume that the geometric center point and the mass center point of the robot are identical. In order to avoid slide movement of the wheels on the ground, the nonholonomic constraint is described by

$$\dot{x}\sin(\theta+\beta)+\dot{y}\cos(\theta+\beta)=0 \tag{1}$$

Then the nonholonomic kinematic system of the robot can be given by

$$\begin{cases} \dot{x} &= -v_0\sin(\theta+\beta) \\ \dot{y} &= v_0\cos(\theta+\beta) \\ \dot{\theta} &= v_1 \\ \dot{\beta} &= v_2 \end{cases} \tag{2}$$

where v_0 denotes the velocity of the heading direction of the robot, v_1 denotes the angular velocity of the rotation of the robot and v_2 denotes the angular velocity of the direction of center wheel.

Suppose that (x_m, y_m) is the coordinate of (x, y) relative to the image frame. Pinhole camera model yields

$$\begin{bmatrix} x_m \\ y_m \end{bmatrix} = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} R\left[\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} c_x \\ c_y \end{bmatrix}\right] + \begin{bmatrix} O_{c1} \\ O_{c2} \end{bmatrix} \tag{3}$$

where α_1 and α_2 are positive constants and dependent on the depth information, focal length, scalar factors. In (3)

$$R = \begin{bmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{bmatrix}$$

where θ_0 denotes the angle between j axis and X axis which represents the constant, anticlockwise orientation angle of the camera coordinate system with respective to the task-space coordinate system.

By using (2) and (3), we have

$$\begin{bmatrix} \dot{x}_m \\ \dot{y}_m \\ \dot{\theta} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -v_0\alpha_1 \sin(\theta+\beta-\theta_0) \\ v_0\alpha_2 \cos(\theta+\beta-\theta_0) \\ v_1 \\ v_2 \end{bmatrix} \tag{4}$$

Taking the state and the input transformations below

$$\begin{cases} x_0 &= \theta+\beta \\ x_1 &= x_m \cos(\theta+\beta) + y_m \sin(\theta+\beta) \\ x_2 &= -x_m \sin(\theta+\beta) + y_m \cos(\theta+\beta) \\ x_3 &= \theta-\beta, \end{cases} \tag{5}$$

$$\begin{cases} u_0 &= v_1 + v_2 \\ u_1 &= v_0 - x_1 u_0 \\ u_2 &= v_1 - v_2. \end{cases} \tag{6}$$

We can get the uncertain chained form system as follows

$$\begin{cases} \dot{x}_0 &= u_0 \\ \dot{x}_1 &= x_2 u_0 + (\alpha_{21} \sin(x_0-\theta_0)\cos x_0 + \alpha_2 \sin\theta_0)(x_1 u_0 + u_1) \\ \dot{x}_2 &= -x_1 u_0 + (\alpha_{21} \cos(x_0-\theta_0)\cos x_0 + \alpha_1 \cos\theta_0)(x_1 u_0 + u_1) \\ \dot{x}_3 &= u_2 \end{cases} \tag{7}$$

where x_0, x_1, x_2, x_3 are new state variables. u_0, u_1, u_2 are new state control inputs, and $\alpha_{21} = \alpha_2 - \alpha_1$.

In system (7), three unknown parameters θ_0, α_1 and α_2 exist if the parameters of the camera are not calibrated.

When $\theta_0 \neq 0$, it can turn into $\theta_0 = 0$. Therefore, we just need to discuss $\theta_0 = 0$. The uncertain chained system is as follows

$$\begin{cases} \dot{x}_0 &= u_0 \\ \dot{x}_1 &= x_2 u_0 + \alpha_{21} \sin x_0 \cos x_0 (x_1 u_0 + u_1) \\ \dot{x}_2 &= -x_1 u_0 + (\alpha_{21} \cos x_0 \cos x_0 + \alpha_1)(x_1 u_0 + u_1) \\ \dot{x}_3 &= u_2 \end{cases} \tag{8}$$

3. CONTROLLER DESIGN

In this section, two kinds of controllers are designed for exponential stabilization of the nonholonimic system under the Assumptions and Lemmas presented below.

Assumption 1. θ_0 is known. α_1, α_2 are unknown. there exist four positive known constraints $\underline{\alpha}_1, \bar{\alpha}_1, \underline{\alpha}_2, \bar{\alpha}_2$ such that $0 < \underline{\alpha}_2 \le \alpha_2 \le \bar{\alpha}_2 < \underline{\alpha}_1 \le \alpha_1 \le \bar{\alpha}_1$.

Lemma 1. If $\overline{A} \in R^{3\times3}$, the characteristic polynomial of matrix is denoted as follows:

$$\left|\lambda I - \overline{A}\right| = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3$$

$\overline{A}$ is a Hurwitz matrix only if satisfies the following conditions:

$$a.\ \ a_1 > 0, a_2 > 0, a_3 > 0$$

$$b.\ \ \Delta_3 = a_1a_2 - a_3 > 0$$

Lemma 2. Consider a time-varying linear system defined by

$$\dot{x} = (\overline{A} + \overline{B}(t))x$$

If $\overline{A} \in R^{3\times3}$ is a Hurwitz matrix and for every element in $\overline{B}(t)$ which satisfies the following conditions:

$$b_{ij}(t) \to 0(i, j = 1, ..., n, t \to \infty)$$

Then, the system is exponentially stable.

3.1. Input control method

For system (8), we can obtain $\dot{x}_0 = -\mu x_0 + p(t)$ by taking control input

$$u_0 = -\mu x_0 + p(t)$$

where $p(t) = p_0(0)e^{-\lambda_0 t}(p_0 \neq 0)$ and $\mu > \lambda_0 > 0$ .Then, the solution is

$$x_0(t) = \frac{p_0}{\mu - \lambda_0}e^{-\lambda_0 t} + \tilde{c}e^{-\mu t}(\tilde{c} \in R^1, t \geq 0) \tag{9}$$

Denote $k_0 = \mu - \lambda_0, y_0 = \frac{x_0}{p}, \frac{1}{k_0} = c$, we have

$$y_0 \to \frac{1}{k_0}, u_0 = p(1 - \mu y_0) \to 0, \frac{u_0}{p} \to \frac{-\lambda_0}{k_0} = c_1, t \to \infty \tag{10}$$

where c, c_1 are constants.

For system (8), taking state-scaling transformation and the linear control inputs as follows

$$y_1 = \frac{x_1}{p}, y_2 = x_2, y_3 = x_3, u_0 = -\mu x_0 + p(t), u_1 = k_2\lambda_0 y_2 + k_3\lambda_0 y_3, u_2 = k_1\lambda_0 y_1 \tag{11}$$

The derivative of $y_i(i = 1, 2, 3)$ can be deduced as follows

$$\begin{cases} \dot{y}_1 & = \lambda_0 y_1 + (c_1 + \alpha_{21} k_2 \lambda_0 c) y_2 + \alpha_{21} k_3 \lambda_0 c y_3 + \alpha_{21} \dfrac{\sin 2x_0}{2} u_0 y_1 \\ & + (\dfrac{u_0}{p} - c_1) y_2 + \alpha_{21} k_2 \lambda_0 (\dfrac{\sin 2x_0}{2p} - c) y_2 + \alpha_{21} k_3 \lambda_0 (\dfrac{\sin 2x_0}{2p} - c) y_3 \\ \dot{y}_2 & = \alpha_2 k_2 \lambda_0 y_2 + \alpha_2 k_3 \lambda_0 y_3 - p u_0 y_1 + (\alpha_{21} \cos x_0 \cos x_0 + \alpha_1) p u_0 y_1 \\ & - \alpha_{21} \sin^2 x_0 k_2 \lambda_0 y_2 - \alpha_{21} \sin^2 x_0 k_3 \lambda_0 y_3 \\ \dot{y}_3 & = k_1 \lambda_0 y_1 \end{cases} \tag{12}$$

Hence, system (12) can be written in matrix as

$$\dot{Y} = [A + B(t)] Y \tag{13}$$

where $Y = [y_1, y_2, y_3]^T$ and

$$A = \begin{bmatrix} \lambda_0 & c_1 + \alpha_{21} k_2 \lambda_0 c & \alpha_{21} k_3 \lambda_0 c \\ 0 & \alpha_2 k_2 \lambda_0 & \alpha_2 k_3 \lambda_0 \\ k_1 \lambda_0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$\begin{cases} b_{11} & = & \alpha_{21} \dfrac{\sin 2x_0}{2} u_0 \\ b_{12} & = & \dfrac{u_0}{p} - c_1 + \alpha_{21} k_2 \lambda_0 (\dfrac{\sin 2x_0}{2p} - c) \\ b_{13} & = & \alpha_{21} k_3 \lambda_0 (\dfrac{\sin 2x_0}{2p} - c) \\ b_{21} & = & -p u_0 + (\alpha_{21} \cos x_0 \cos x_0 + \alpha_1) p u_0 \\ b_{22} & = & -\alpha_{21} \sin^2 x_0 k_2 \lambda_0 \\ b_{23} & = & -\alpha_{21} \sin^2 x_0 k_3 \lambda_0 \\ b_{31} & = & 0 \\ b_{32} & = & 0 \\ b_{33} & = & 0 \end{cases}$$

In order to make the system (8) stabilize exponentially, theorem 1 below is introduced.

Theorem 1. Under Assumptions 1 and (11), for system (8), choosing k_1, k_2 and k_3 such that

$$k_2 = n_1, k_1 k_3 = n_2, n_1 < \min(\frac{-1}{\underline{\alpha}_2}, \frac{\alpha_1}{\underline{\alpha}_2(\underline{\alpha}_2 - \overline{\alpha}_1)}), n_2 > \max(\frac{\overline{\alpha}_2 n_1}{(\overline{\alpha}_2 - \underline{\alpha}_1)c}, \frac{-\overline{\alpha}_2 n_1 (\overline{\alpha}_2 n_1 + 1)}{c(\overline{\alpha}_1 - (\overline{\alpha}_2 - \underline{\alpha}_1)\underline{\alpha}_2 n_1)}) \tag{14}$$

System (8) can be stabilized exponentially.

Proof. For system (12), the characteristic polynomial of matrix A is

$$|\lambda I - A| = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3$$

$$\begin{aligned} &a_1 = -\lambda_0(\alpha_2 k_2 + 1), a_2 = \lambda_0^2(\alpha_2 k_2 - \alpha_{21}k_1k_3c), a_3 = \lambda_0^3\alpha_2 k_1 k_3 c \\ &\Delta_3 = a_1a_2 - a_3 = -\lambda_0^3(\alpha_2k_2(\alpha_2k_2+1) + k_1k_3c(\alpha_1 - \alpha_{21}\alpha_2k_2)) \end{aligned} \tag{15}$$

Considering (14) and Lemma 1, we obtain

$$\begin{aligned} a_1 &= -\lambda_0(\alpha_2 k_2 + 1) > -\lambda_0(\alpha_2 \frac{-1}{\underline{\alpha}_2} + 1) > 0 \\ a_2 &= \lambda_0^2(\alpha_2k_2 - \alpha_{21}k_1k_3c) > \lambda_0^2(\alpha_2k_2 - \alpha_{21}c\frac{\bar{\alpha}_2 n_1}{(\bar{\alpha}_2 - \underline{\alpha}_1)c}) = \lambda_0^2 n_1(\alpha_2 - \alpha_{21}\frac{\bar{\alpha}_2}{(\bar{\alpha}_2 - \underline{\alpha}_1)}) > 0 \\ a_3 &= \lambda_0^3\alpha_2k_1k_3c > \lambda_0^3\alpha_2\frac{\bar{\alpha}_2 n_1}{(\bar{\alpha}_2 - \underline{\alpha}_1)} > 0 \\ \Delta_3 &= a_1a_2 - a_3 = -\lambda_0^3(\alpha_2k_2(\alpha_2k_2+1) + k_1k_3c(\alpha_1 - \alpha_{21}\alpha_2k_2)) \\ &> -\lambda_0^3(\alpha_2k_2(\alpha_2k_2+1) + \frac{-\bar{\alpha}_2n_1(\bar{\alpha}_2n_1+1)}{c(\bar{\alpha}_1 - (\bar{\alpha}_2 - \underline{\alpha}_1)\underline{\alpha}_2n_1)}c(\alpha_1 - \alpha_{21}\alpha_2k_2)) \\ &> -\lambda_0^3 n_1(\alpha_2(\alpha_2k_2+1) - \bar{\alpha}_2(\bar{\alpha}_2n_1+1)) > 0 \end{aligned} \tag{16}$$

According to Lemma 1, matrix A is Hurwitz. For system(8), $x_0(t) \to 0$ exponentially as $t \to \infty$. Considering $|\sin x_0| \le |x_0|$, $\cos x_0$ and α_1, α_2, c_1 and c are all bounded, then every element $b_{ij} \in B(t)$ $(i, j = 1,2,3,4)$ converge to zero exponentially as t goes to infinity. x_0, y_1, y_2, y_3, y_4 converge to zero exponentially as t goes to infinity by Lemma 2. (11) can be used to deduce that x_1, x_2, x_3, x_4 converge to zero exponentially as t goes to infinity. Therefore, system (8) can be stabilized exponentially by using (11) and Lemmas above.

3.2. Auxiliary variable method

For system (8), taking auxiliary variable $x_a(t)$ as follows

$$\begin{cases} \dot{x}_a = x_0 \\ \dot{x}_0 = u_0 \end{cases} \tag{17}$$

Taking control input $u_0 = -k_a x_a - k_0 x_0 (k_a > 0, k_0 > 0)$. If k_a, k_0 satisfy $k_0^2 - 4k_a > 0$, the system (17) has two negative characteristic root $-\omega, -\varpi$. Supposing that $-\omega > -\varpi$, we have

$$\begin{cases} x_a = \iota_a e^{-\omega t} + \iota_0 e^{-\varpi t} \\ x_0 = -\omega\iota_a e^{-\omega t} - \varpi\iota_0 e^{-\varpi t} \end{cases}$$

where ι_a, ι_0 are constants, and

$$\iota_a = \frac{-\varpi x_a(0) - x_0(0)}{\omega - \varpi} = \frac{-\varpi x_a(0)}{\omega - \varpi}, \iota_0 = \frac{-\varpi x_a(0) + x_0(0)}{\omega - \varpi} = \frac{\omega x_a(0)}{\omega - \varpi} \tag{18}$$

Defined $z = e^{-\varpi t}$ we can obtain

$$u_0 = zf = z[f_0 + f_1].$$

where $f_0 = (k_0\bar{\omega} - k_a)\iota_0$, $f_1 = (k_0\omega - k_a)\iota_a e^{-(\omega-\bar{\omega})t}$.

According to system (17), we can find

$$\begin{aligned} &k_a = \omega\bar{\omega}, k_0 = \omega + \bar{\omega}, k_0^2 - 4k_a = (\omega - \bar{\omega})^2 > 0 \\ &f_0 = \bar{\omega}^2\iota_0 = -\omega h,\ f_1 = \omega^2\iota_a e^{-(\omega-\bar{\omega})t} \to 0, \frac{\sin x_0 \cos x_0}{z} \to h, t \to \infty \end{aligned} \tag{19}$$

where $h = -\bar{\omega}\iota_0$ and choose $x_a(0) < 0$.

For system (8), taking state-scaling transformation and the linear control inputs as follows

$$y_1 = \frac{x_1}{z}, ty_2 = x_2, pty_3 = x_3, u_0 = -k_a x_a - k_0 x_0, u_1 = k_2\bar{\omega}y_2 + k_3\bar{\omega}y_3, u_2 = k_1\bar{\omega}y_1. \tag{20}$$

The derivative of $y_i (i = 1, 2, 3)$ can be deduced as follows

$$\begin{cases} \dot{y}_1 & = \bar{\omega}y_1 + (-\bar{\omega}h + \alpha_{21}k_2\bar{\omega}h)y_2 + \alpha_{21}k_3\bar{\omega}hy_3 + \alpha_{21}\dfrac{\sin 2x_0}{2}u_0 y_1 \\ & \quad + f_1 y_2 + \alpha_{21}k_2\bar{\omega}(\dfrac{\sin 2x_0}{2z} - h)y_2 + \alpha_{21}k_3\lambda_0(\dfrac{\sin 2x_0}{2z} - h)y_3 \\ \dot{y}_2 & = \alpha_2 k_2\bar{\omega}y_2 + \alpha_2 k_3\bar{\omega}y_3 - zu_0 y_1 + (\alpha_{21}\cos x_0 \cos x_0 + \alpha_1)zu_0 y_1 \\ & \quad -\alpha_{21}\sin^2 x_0 k_2\bar{\omega}y_2 - \alpha_{21}\sin^2 x_0 k_3\bar{\omega}y_3 \\ \dot{y}_3 & = k_1\bar{\omega}y_1, \end{cases} \tag{21}$$

Hence, system (21) can be written in matrix as

$$\dot{Y} = [A_0 + B_0(t)]Y \tag{22}$$

where $Y = [y_1, y_2, y_3]^T$ and

$$A_0 = \begin{bmatrix} \bar{\omega} & -\bar{\omega}h + \alpha_{21}k_2\bar{\omega}h & \alpha_{21}k_3\bar{\omega}h \\ 0 & \alpha_2 k_2\bar{\omega} & \alpha_2 k_3\bar{\omega} \\ k_1\bar{\omega} & 0 & 0 \end{bmatrix}, B_0 = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$\begin{cases} b_{11} &= \alpha_{21}\dfrac{\sin 2x_0}{2}u_0 \\ b_{12} &= f_1+\alpha_{21}k_2\bar{\omega}(\dfrac{\sin 2x_0}{2z}-h) \\ b_{13} &= \alpha_{21}k_3\lambda_0(\dfrac{\sin 2x_0}{2z}-h) \\ b_{21} &= -zu_0+(\alpha_{21}\cos x_0\cos x_0+\alpha_1)zu_0 \\ b_{22} &= -\alpha_{21}\sin^2 x_0k_2\bar{\omega} \\ b_{23} &= -\alpha_{21}\sin^2 x_0k_3\bar{\omega} \\ b_{31} &= 0 \\ b_{32} &= 0 \\ b_{33} &= 0. \end{cases}$$

In order to make system (8) stabilize exponentially, theorem 2 below is introduced.

Theorem 2. Under Assumptions 1 and (11), for system(8)choose k_1, k_2 and k_3 such that

$$\begin{aligned} &k_2=n_1, k_1k_3=n_2, n_1<\min(\frac{-1}{\underline{\alpha}_2},\frac{\underline{\alpha}_1}{\underline{\alpha}_2(\underline{\alpha}_2-\bar{\alpha}_1)}) \\ &n_2>\max(\frac{\bar{\alpha}_2n_1}{(\bar{\alpha}_2-\underline{\alpha}_1)h},\frac{-\bar{\alpha}_2n_1(\bar{\alpha}_2n_1+1)}{h(\bar{\alpha}_1-(\bar{\alpha}_2-\underline{\alpha}_1)\underline{\alpha}_2n_1)}) \end{aligned} \tag{23}$$

System (8) can be stabilized exponentially.

Proof. For system (22), the characteristic polynomial of matrix A is

$$|\lambda I-A|=\lambda^3+a_1\lambda^2+a_2\lambda+a_3$$

$$\begin{aligned} &a_1=-\bar{\omega}(\alpha_2k_2+1), a_2=\bar{\omega}^2(\alpha_2k_2-\alpha_{21}k_1k_3h), a_3=\bar{\omega}^3\alpha_2k_1k_3h \\ &\Delta_3=a_1a_2-a_3=-\bar{\omega}^3(\alpha_2k_2(\alpha_2k_2+1)+k_1k_3h(\alpha_1-\alpha_{21}\alpha_2k_2)) \end{aligned} \tag{24}$$

Considering (24) and Lemma 1, we obtain

$$\begin{aligned} a_1&=-\bar{\omega}(\alpha_2k_2+1)>-\bar{\omega}(\alpha_2\frac{-1}{\underline{\alpha}_2}+1)>0 \\ a_2&=\bar{\omega}^2(\alpha_2k_2-\alpha_{21}k_1k_3h)>\bar{\omega}^2(\alpha_2k_2-\alpha_{21}h\frac{\bar{\alpha}_2n_1}{(\bar{\alpha}_2-\underline{\alpha}_1)h})=\bar{\omega}^2n_1(\alpha_2-\alpha_{21}\frac{\bar{\alpha}_2}{(\bar{\alpha}_2-\underline{\alpha}_1)})>0 \\ a_3&=\bar{\omega}^3\alpha_2k_1k_3h>\bar{\omega}^3\alpha_2\frac{\bar{\alpha}_2n_1}{(\bar{\alpha}_2-\underline{\alpha}_1)}>0 \\ \Delta_3&=a_1a_2-a_3=-\bar{\omega}^3(\alpha_2k_2(\alpha_2k_2+1)+k_1k_3h(\alpha_1-\alpha_{21}\alpha_2k_2)) \\ &>-\bar{\omega}^3(\alpha_2k_2(\alpha_2k_2+1)+\frac{-\bar{\alpha}_2n_1(\bar{\alpha}_2n_1+1)}{h(\bar{\alpha}_1-(\bar{\alpha}_2-\underline{\alpha}_1)\underline{\alpha}_2n_1)}h(\alpha_1-\alpha_{21}\alpha_2k_2)) \\ &>-\bar{\omega}^3n_1(\alpha_2(\alpha_2k_2+1)-\bar{\alpha}_2(\bar{\alpha}_2n_1+1))>0 \end{aligned} \tag{25}$$

According to Lemma 1, matrix A is Hurwitz. For system(8), $x_0(t) \to 0$ exponentially as $t \to \infty$. Considering $|\sin x_0| \le |x_0|$, $\cos x_0$ and α_1, α_2, h are all bounded, then every element $b_{ij} \in B(t)$ $(i, j = 1,2,3,4)$ converge to zero exponentially as t goes toinfinity. x_0, y_1, y_2, y_3, y_4 converge to zero exponentially as t goes to infinity by Lemma 2. (20) can be used to deduce that x_1, x_2, x_3, x_4 converge to zero exponentially as t goes to infinity. Therefore, system (8) can be stabilized exponentially.

4. SIMULATION

In this section, under Assumptions 1 and 2, simulations are conducted for the uncertain chained system (8).

Method 1. For the robot-camera system (4), take state transformation (5) and choose state-scaling transformation and the linear control inputs (11) with parameters $\lambda_0 = 1$, $\mu = 2$ and $\underline{\tilde{\alpha}}_1 = 2.25$, $\alpha_1 = 2.3$, $\bar{\bar{\alpha}}_1 = 2.5$, $\underline{\tilde{\alpha}}_2 = 0.9$ and $\alpha_2 = 1, \bar{\bar{\alpha}}_2 = 1.12, (c_x, c_y) = (2,1), (o_{c1}, o_{c2}) = (-0.2, 0.2)$ We have $[k_0, k_1, k_2, k_3] = [1, -300, 150, -10]$ .Choose the initialstate as $[p_0(0), x_0(0), x_1(0), x_2(0), x_3(0)] = [-1, 0, -1.8, 2.8, -1.6]$ .under Assumptions 1 and 2. The moving paths of (x_m, y_m) in the camera space and (x, y) in the robot space are plotted in Fig.2. The trajectories of inputs $u_i (i = 0,1,2)$ are plotted in Fig.3 respectively. All the trajectories of states $x_i (i = 0,1,2,3)$ are plotted in Fig.4.

Method 2. For the robot-camera system (4), take state transformation (5) and choose state-scaling transformation and the linear control inputs (20) with parameters $\lambda_0 = 1$, $\mu = 2$ and $\underline{\tilde{\alpha}}_1 = 2.25$, $\alpha_1 = 2.3$, $\bar{\bar{\alpha}}_1 = 2.5$, $\underline{\tilde{\alpha}}_2 = 0.9$ and $\alpha_2 = 1.18, \bar{\bar{\alpha}}_2 = 1, (c_x, c_y) = (2,1), (o_{c1}, o_{c2}) = (-0.2, 0.2)$ We have $[k_1, k_2, k_3] = [-300, 125, -10]$ .Choose the initialstate as $[z_0(0), x_0(0), x_1(0), x_2(0), x_3(0)] = [-1, 0, -1.8, 2.8, -1.6]$. under Assumptions 1 and 2. The moving paths of (x_m, y_m) in the camera space and (x, y) in the robot space are plotted in Fig.5. The trajectories of inputs $u_i (i = 0,1,2)$ are plotted in Fig.6 respectively. All the trajectories of states $x_i (i = 0,1,2,3)$ are plotted in Fig.7.

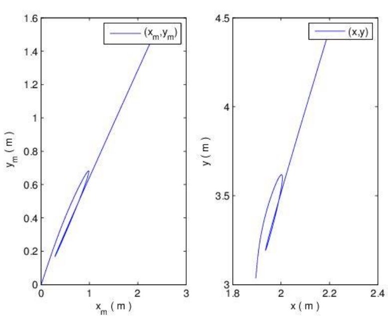

Fig.2 The trajectories of mobile robot

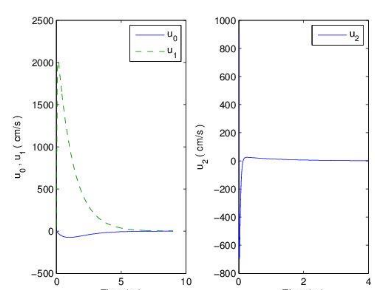

Fig.3 The trajectories of inputs respect to time

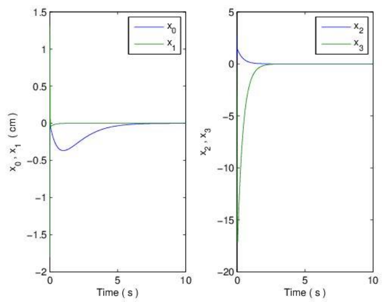

Fig. 4 The trajectories of states respect to time

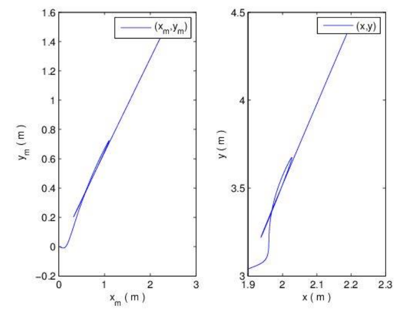

Fig.5 The trajectories of mobile robot

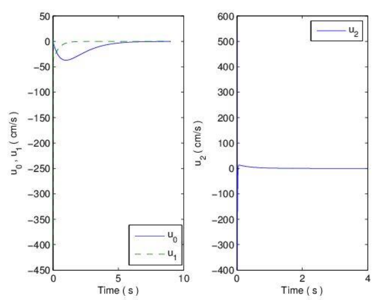

Fig.6. The trajectories of inputs respect to time

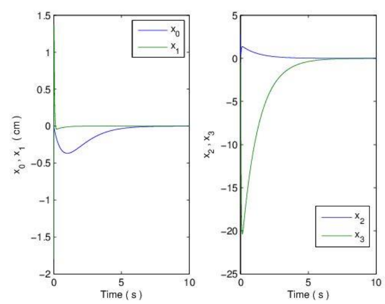

Fig.7 The trajectories of states respect to time

5. CONCLUSION

Based on the visual servoing feedback and state-input transformation, we present a uncertain chained model. Two kinds of smooth time-varying feedback controllers are discussed for the uncertain chained form of type (2,1) mobile robot with uncertainties by using the auxiliary variable method and the control input method. The controllers are proposed which can stabilize the new uncertain chained system exponentially. Simulation results demonstrate the effectiveness of themethod proposed in this paper.

6. ACRNOWLEDGEMENT

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7. REFERENCES

[1] G.Ca mpion, G. Bastin, B. D'Andréa-Novel. Structural properties and classification of kinematic and dynamic models of wheeled mobile robots[J]. IEEE Transactions on Robotics and Automation, 1996, **12**(1): 47-62.

[2] D.G. Zhang, S.F. Zhou. Dynamic analysis of flexible-link and flexible-joint robots[J]. Applied Mathematics and Mechanics, 2006, **27**(5): 695-704.

[3] I.Kolmanovsky, N.H. McClamroch. Developments in nonholonomic control problems[J]. IEEE Control System Magazine, 1995, **15**(6): 20-36.

[4] C.L. Wang, Y.C. Mei, Z.Y. Liang, Q.W. Jia. Dynamic feedback tracking control of non-holonomic mobile robots with unknown camera parameters[J]. Transactions of the Institute of Measurement, 2010, **32**(2): 155-169.

[5] B.L. Ma, S.K. Tso. Robust discontinuous exponential regulation of dynamic nonholonomic wheeled mobile robots with parameter uncertainties[J]. International Journal of Robust Nonlinear Control, 2008, **18**(9): 960-974.

[6] Y.P. Tian, S.H. Li. Exponential stabilization of nonholonomic dynamic systems by smooth timevarying control[J]. Control Theory & Applications, 2001, 1**8**(3): 317-322.

[7] Y.P. Tian, S.H. Li. Smooth time-varying exponential stabilization of nonholonomic Systems in the Vector Power Form[J]. Automatica, 2002, **38**(7): 1139-1146.

[8] B.L. Ma, W. Huo. Smooth time-varying exponential stabilization of nonholonomic chained systems[J]. Acta Automatic Sinica, 2003, **29**(2): 301-305.

[9] Z.P. Jiang. Robust exponential regulation of nonholonomic systems with uncertainties[J]. Automatica, 2000, **36**(2): 189-209.

[10] Z.R. Xi, G. Feng, Z.P. Jiang, D.Z. Cheng. Output feedback exponential stabilization of uncertain chained systems[J]. Journal of Franklin Institute, 2007, **344**(1): 36-57.

[11] J.A. Piepmeier, G.V. McMurray, H. Lipkin. Uncalibrated dynamic visual servoing[J]. IEEE Transactions on Robotics and Automation, 2004, **20**(1): 143-147.

[12] H.S. Wang, Y.H. Liu, D.X. Zhou, Adaptive visual servoing using point and line features with an uncalibrated eye-in-hand camera[J]. IEEE Transactions on Robotics, 2008, **24**(4): 843-857.

[13] W.E. Dixon, D.M. Dawson, E. Zergeroglu. Adaptive tracking control of a wheeled mobile robot via an uncalibrated camera system[J]. IEEE Transactions on Systems Man & Cybernetics Part B Cybernetics A Publication of the IEEE Systems Man & Cybernetics Society, 2001, **31**(3): 341-352.

[14] Y.H. Liu, H.S. Wang, C.Y. Wang, K.K.Lam. Uncalibrated visual servoing of robots using a depth independent interaction matrix[J]. IEEE Transactions on Robotics, 2006, **22**(4): 804-817.

[15] C.L. Wang, Z.Y. Liang, Q.W. Jia. Dynamic feedback robust stabilization of nonholonomic mobile robots based on visual servoing[J]. Control Theory Apply, 2000, **8**(2): 139-144.

[16] Y.C. Fang, W.E. Dixon, D.M. Dawson, P. Chawda. Homography-based visual servo regulation of mobile robots[J]. IEEE Transactions on Systems Man & Cybernetics Part B Cybernetics A Publication of the IEEE Systems Man & Cybernetics Society, 2005, **35**(5): 1041-1050.

[17] G. Hu, W. MacKunis, N. Gans, W.E. Dixon, J. Chen, A. Behal, D. Dawson, Homography-based visual servo control with imperfect camera calibration[J]. IEEE Transactions on Automatic Control, 2009, **54**(6): 1318-1324.

[18] Z.Y. Liang, C.L. Wang. Robust exponential stabilization of uncertain nonholonomic chained systems based on visual feedback[J]. Journal of Systems Science & Complexity, 2012, **25**(3): 441-450.

[19] Z.Y. Liang, C.L. Wang. Robust stabilization of nonholonomic chained form systems with uncertainties[J]. Acta Automatica Sinica, 2011, **37**(2): 129-142.

[20] M.C. Laiou, A. Astolfi. Quasi-smooth control of chained systems[C]. American Control Conference, 1999, **6**: 3940-3944.