IMAGE DENOISING METHODS: LITERATURE REVIEW

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ABSTRACT

Image denoising is a fundamental and important task in image processing and computer vision fields. There are many methods are proposed to reconstruct clean images from noisy versions. These methods differ in both methodology and performance. On one hand, denoising methods can be classified into local and non-local methods. On the other hand, they can be marked as spatial and frequency domain methods. Sparse coding and low-rank are two popular techniques for denoising recently. This paper summarizes existing techniques and provides several promising directions for further studying in the future.

Keywords: *Image denoising, Filtering, Wavelet, Non-local self-similarity, Sparse representation, Low rank.*

1. INTRODUCTION

A picture paints a thousand words. Images are vital and convenient for communication. As the rapid development of imaging technique and mobile devices, numerous images are produced, translated and edited every day. However, as the variety of imaging devices, the quality of images is unstable. For example, cameras on mobile usually produce images with noise, blurry or other spots inevitably. What's more, some information may be damaged during the transition process between different devices. Thus, to remove noise and recover meaningful information from noisy images is a vital problem nowadays.

In fact, image denoising is a classic problem and has been studied for a long time. However, it still remains as a challenging and open task. The main reason is that, in mathematical view, the nature of image denoising is an inverse problem and the solution to it is not unique. Therefore, additional assumptions must be made to get a practical solution. As the ideal assumption is difficult to find for all images, many researches keep working on it and various techniques are developed to promote denoising performance. With the efforts made by researchers in recent years, promising and interesting techniques appear in image denoising field. In this paper, we give the formulation of image denoising problem, summarize the denoising techniques till now, and propose some possible study directions in future.

2. IMAGE DENOISING PROBLEM

Mathematically, image denoising is a typical inverse problem and can be formulated as $y = x + n$. In this formula, y represents the observed noisy image, x represents the unknown clean image, and n is the additional noise. In most cases, additional zero-mean Gaussian white noise is adopted for modeling and evaluating different denoising techniques, i.e., $n \sim N(0, \sigma^2)$.

Now, we can describe image denoising task in a concise way. Given a noisy image y, we'd like to recover the clean image x with noise being assumed as $n \sim N(0, \sigma^2)$, the reconstructed image is denoted as $\hat{x}$.

As mentioned in Section 1, we can't get unique solution from the noisy image model, $y = x + n$, so different assumptions on x are made and induce various denoising methods. Although previous methods differ a lot from various aspects, they explicitly or implicitly adopt the same scheme -averaging. If we classify them considering the spatial scope of pixels involving in averaging operation when denoising a pixel, they can be classified roughly into two groups — local ones and non-local ones.

Quantitatively, PSNR (Peak Signal to Noise Rate) and SSIM (Structure Similarity Index Measurement) are two representative measurements to evaluate the performance of a denoising method [1]. While quantitative measurements can’t reflect the visual quality perfectly, visual quality comparisons on a set of images are necessary. Besides noise removal effect, edges and textures preservation is vital for evaluating a denoising method.

3. LOCAL DENOISING METHODS

Literally, local methods denoise a pixel using its local neighborhood pixels. Traditional methods include spatial domain and frequency domain methods. Recently, sparse representation based and low-rank based methods are popular and the effectiveness of them has been demonstrated.

3.1. Spatial Domain Methods

In early days, denoising is performed in spatial domain, including classic filters, PDE based methods, total variation, neighborhood filters, and so on. These methods are fast, but tend to blur edges as they only use local neighbors to remove noise.

3.1.1. Local filters

Gaussian filter is a classic isotropic filter. It estimates $\hat{x}$ as $\hat{x} = G_h * y$**,** where G_h is a pre-determined $N \times N$ template. The weights in averaging are fixed, and ignore the effect of different local structures. So edges are blurred heavily for removing noise. Different from Gaussian filter, Wiener filter is a data-driven filter [2]. It gives an Image $\hat{x}$ satisfying that the MSE (Mean Squared Error) between the estimated image and original image is minimized, i.e.**,** $\hat{x} = \min_x MSE = \min_x E\{(\hat{x} - x))^2\}$**.** As Wiener filter is data-driven, it is adaptive. For unstable signals, Wiener filter is generalized as Kalman filter [3].

3.1.2. PDE based methods

Perona and Malik proposed anisotropic diffusion Model , $\partial x \,/\, \partial t \, = \mathrm{div}(g(|\nabla_x \cdot \nabla_x\,|),$ where $g(s) = 1/(1 + \lambda s^2)$ [4]. As can be seen, it adjusts diffusion speed in a pixel adaptively according to the magnitude of gradient. In flat zones, diffusion is taken as isotropic, while in edges or textures zones, diffusion is done mainly along tangent direction. Thus noise in flat zones can be removed fully, while edges and textures can be preserved. Curvature motion model another commonly used denoising method, which can be formulated as $\partial x \,/\, \partial t \, = F(k)|\nabla x|$, where $F(k)$ is a function of principle curvature k of image and generally $F(k) = k^{1/3}$. The main advantage of curvature motion is that, for contrast invariant images, it won't lose edge information [5].

3.1.3. Total variation

Total variation is proposed by Rudin and Osher from the view of functional and attributes denoising into a universal optimization framework with regularization [6]. The formulation is $\hat{x} = \min_x ||y - x||_2^2 + \lambda\, |\nabla x|$, where λ is a regularization parameter. The fidelity term is $||y - x||_2^2$, and the regularization term is total variation constraint $TV(x) = |\nabla x|$. It provides an important framework for image denoising, but it is too strict for natural images and tends to give over- smoothed results. Many researchers then propose modified versions of total variation [7] .

3.1.4. Neighborhood filters

In neighborhood filters, the neighborhood of a pixel here is indeed totally different from previous filters. It is the grayscale similarity other than spatial proximity that used to define a neighbor pixel. In the extreme case, all pixels in an image will take part in denoising, and it changes to a nonlocal method. Yaroslavsky neighborhood filters consider grayscale similar pixels from neighborhoods within a certain spatial scope [8] . Tomasi et al. proposed bilateral filter [9]. It considers color similarity and estimates a pixel $\hat{x}(i)$ as the weighted average of its $N * N$ neighborhoods. In neighborhood filters, dissimilar pixels weight less than similar pixels in averaging operation, so they can preserve more details than local filters.

3.2. Transform Domain Methods

Transform domain methods employ the following observation: performances of image/signal information and noise are different in transform domain.

3.2.1. Spatial-frequency filtering

Generally speaking, signals are steady, while noises are unstable. Thus, after being transformed by low-pass filters, such as Fourier transform, image information mainly spreads in low frequency domain, while noise spreads in high frequency domain. Then by selecting certain transform domain features and transforming them back to image domain, we can remove noise. But it is often ignored that edges and textures also spread in high frequency domain, so details are often lost and artifacts are introduced in denoised images [10].

3.2.2. Wavelet domain methods

Wavelet transform is a powerful tool in image processing, and a series of algorithms are proposed for image denoising based on it. These methods can be classified roughly into four classes. First, methods extending Wiener filter into wavelet domain are proposed [11]–[13]. Second, similar to that in section 3.2.1, a given noisy image is transformed by wavelet transform, and coefficients under a threshold are considered as useful, while coefficients above the threshold are regarded as noisy. Hard-thresholding and soft-thresholding are two schemes for deal with these coefficients. Some methods calculate an adaptive threshold other than adopt an empirically pre-chosen threshold, including SURE-based, Bayes-based, cross validation-based methods and so on [14]–[16]. Third, wavelet coefficients are assumed to fit a certain distribution model, including deterministic and statistical models. Forth, non-orthogonal wavelet transforms are employed for denosing.

3.2.3. Data adaptive transform

ICA (Independent Component Analysis) and PCA (Principle Component Analysis) are adopted as transform tools on given noisy images [17]–[19]. They are data adaptive, and the assumptions of difference of signal and noise still hold.

3.3. Sparse Representation

Sparse representation can be traced to compressive sensing and is popular recently [20], [21]. As analyzed above, image denoising can be formulated into an optimization framework, $\hat{x} = \min_x ||y - x||_2^2 + \lambda R(x)$, where $R(x)$a regularization term and λ is is a regularization parameter. Sparse representation assumes that a patch x_i can be sparsely represented by a linear combination of atoms in a redundant dictionary D, i.e., $x_i = Da_i$, where a_i is the sparse codes. Elad et al. proposed K—SVD [22] based on the sparse representation prior, $\hat{x} = \min_{ai} \|y_i - Da_i\|_2^2 + \lambda \, ||a_i||_0$. After $\widehat{\alpha}_i$ is estimated, the patch is calculated as $\widehat{x}_i = D\alpha_i$ and the whole image is estimated as the average of overlapping patches. The dictionary can be chosen as a pre-determined transform, such as DCT and wavelet transform, or be trained from a natural image dataset or the noisy image itself. The sparse representation based method outperforms previous local methods significantly as it doesn't force any smooth assumption on resulted image. But the edges are still blurred. Improved K-SVD is also proposed for preserving details.

In most of the cases, dictionary and sparse codes are updated iteratively. The original model is NP-hard. To solve it efficiently, the commonly used dictionary updating algorithms are K-SVD and MOD [23]. For solving sparse codes, greedy algorithm and l_1 norm convex relaxation are two effective methods. Greedy algorithms include MP [24], OMP [25] and Thresholding method [26] , and l_1 problem can be solved by BP [27] and IRLS [28].

3.4. Low rank

Low-rank is another popular technique in image processing fields [29]. Different from sparse representation, low-rank based model can be formulated as $\min_{A,E} \mathrm{rank}(A) + \lambda ||E||_0 \mathrm{s.t}, D = A + E.$ It indicates a matrix D which can be decomposed as a low-rank matrix and a sparse matrix. It can be applied successfully mainly benefited from RPCA (Robust principle component analysis) [30] .

4. NON-LOCAL DENOISING METHODS

In Section 3, we reviewed the local denoising methods. In this section, we will discuss a transformative technique which changes the denoising field totally — non-local scheme.

4.1. Non-Local Means

Non-local means is proposed by Buades et al., and it is the first method exploiting non-local self-similarity provided by the redundancy information in natural images to promote denoising performance [31] . The idea of non-local means is simple but effective. It estimates a pixel in $\hat{x}$ as the similarity weighted average of all pixels or pixels in a limited search window. Different from neighborhood filters, the similarity of two pixels is not computed only by the grayscale of the two single pixels, but is measured by Gaussian distance of two image patches centering at the two pixels respectively. It is robust to noise and can make full use of information provided by the given image to get accurate result to a large extent. The visual quality of denoised results by non-local means is much more pleasant than traditional local methods. As summarized in section 4.2, inspired by non-local means, many promising denoising methods combining non-local scheme with both traditional local methods and recently popular methods are proposed and achieve state-of-the-art performances.

4.2. Non-Local Scheme Induced Methods

BM3D (Block Matching 3-D) is a two-stage non-locally collaborative filter in transform domain [32]. In each stage, firstly, it collects similar patches by block matching for each patch in the image and forms 3-D groups. Secondly, each group is transformed by wavelet or so into transform domain. Thirdly, hard-thresholding or Wiener filter is taken on the coefficients. Finally, new coefficients are transformed back into image region, all recovered patches are aggregated and the whole image is estimated. BM3D-SAPCA is an improved version of BM3D [33]. They perform quite well especially for small noise levels. However, for large noise levels, the visual quality of the recovered images is imperfect. Artifacts are introduced especially in flat areas.

LSSC (Learned Simultaneous Sparse Coding) combines nonlocal self-similarity with sparse coding into one unified framework [34]. A grouped-Sparsity regularizer is enforced on the codes of the group of similar patches. Based on the idea, the formulation is $\min_{Ai}||A_i||_{p,q}, \text{s.t.} \Sigma_{J\in S_i}||y_i - D_{\alpha_{ij}}||_2^2 \leq \varepsilon_i$, where A_i is a sparse code matrix of all patches in group S_i. Thus codes of all similar patches are forced to own non-zero codes in the same positions and achieve a grouped simultaneous sparsity. NCSR (Non-locally Centralized Sparse Representation) is another successful method exploiting nonlocal scheme and sparse representation [35]. Different from LSSC, NCSR doesn't force the same positions of non-zero codes in a group. It encourages each patch to centralize on the weighted average of codes of its similar patches. It can be modeled as $\alpha_i = \text{argmin}_{\alpha_i}\{||\, y_j - D_{\alpha_{ij}}||_2^2 + \Sigma_j\, \lambda_{ij} \cdot |\alpha_i(j) - \beta_i(j)|\}$, where β_i is the weighted center of similar patches. It performs well even for large noise levels, but it will lose too many details for texture-redundant images. Texture preserving and edge restoration are main topics in developing new denoising methods.

5. CONCLUSION

In this paper, we listed some representative algorithms of two classes of image denoising methods (local and non-local ones). As analyzed above, local methods are fast, but tend to bring blurring especially for edges and textures. Non-local methods are slow, but can give visually pleasant results. Sparse representation and low-rank are two powerful and promising techniques recently.

As the complexity and higher requirement of image denoising, researches on it are still in high demand. We propose several study directions in performance improvement and speed acceleration respects. Most sparsity based models can remove noise to a large extent, but they can't preserve edges well. On the one hand, to preserve edges and textures well, the properties of them in noisy images should be studied further. For example, models of natural image edges can be constructed and guide the denoising procedure. On the other hand, to make non-local methods more practical, the complexity of non-local similarity measurement should be reduced.

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