EVALUATION OF THE CONTRIBUTORS OF OVERALL EQUIPMENT EFFECTIVENESS USING SHAPLEY VALUE

[1]Muhamad Arifpin Mansor & [2]Ario Ohsato

[1]Faculty of Mechanical Engineering, University Malaysia Pahang, Kuantan, Malaysia.
[2]Graduate School of Engineering, Nagaoka University of Technology, Nagaoka, Japan.

ABSTRACT

Maintenance is an indispensable factor in the current manufacturing industries and Overall Equipment Effectiveness is a key performance indicator that provides the overall performance of a single piece of machinery, or for an entire factory. Three measurable components: Performance Rate (PR), Quality Rate (QR) and Availability(A) are working together to achieve an OEE and their combinations in each period are always different. The best tool to evaluate the contribution of these components or contributors is the Shapley value, proposed by Lloyd Shapley. This paper estimates how each component has contributed to OEE at the a decided period, and estimates during which period the activity was best performed using the Shapley value.

Keywords: Shapley value, Overall Equipment Effectiveness, Contribution evaluation

1. INTRODUCTION

Normally, a goal is achieved by a total combination of contributions from several efforts. For example, in a soccer game, eleven players are working together as a team to win the game. Each player contributes their skills to the team and the team with the higher value of a combination of skills will win the game. However, the individual contribution of each player toward the target achieved is different. Therefore, it is necessary to consider the contribution of each player or section to obtain the results when cooperating with one another.

Since advanced technologies and customer demands are changing at a high speed in the current manufacturing industries, manufacturing processes have become more precise and required high technology machines. Therefore, the maintenance of the these machines is an indispensable factor influencing production costs. Overall Equipment Effectiveness(OEE) is a key performance indicator (KPI) that provides the overall performance of a single piece of machinery, or for an entire factory. OEE breaks the performance of a machine into three separate but measurable components: Performance Rate (PR), Quality Rate (QR) and Availability(A)

We could say that Performance Rate, Quality Rate and Availability are working together to achieve an OEE and their combinations in each period are always different. We define the components that are working together and that have their own contribution to achieve a common goal in a certain period as a contributor and a period as a player. Period is also considered as a player because in maintenance system, maintenance activities performed in a previous period continue to have an influential until today as well as in the future. If previously performed maintenance works efficiently, it will lead to a reduction of maintenance activities for the next periods.

From our survey, we found that Performance Rate, Quality Rate and Availability are handle by different sections in an organization. For example, the Production department is responsible for Performance Rate. The Quality department and Maintenance department are responsible for Quality rate and Availability, respectively. Knowing the contribution factors of each distribution helps with ease of managing activities. The best tool to evaluate the contribution of these components or contributors is the Shapley value, proposed by Lloyd Shapley in 1953.[1]

This paper estimates how each contributor has contributed to OEE at the a decided period, and estimates during which period the activity was best performed using the Shapley value. This paper is organized as follows; Section two describes the outline of the Shapley value. Section three provides an illustrative example and a case study of the Shapley value's application in evaluating the contributors of overall equipment effectiveness. Finally, we conclude our work in section four.

2. SHAPLEY VALUE

The Shapley value is a concept of the game theory aimed at proposing the fairest allocation of the profits collectively obtained between the players in a cooperation. A fundamental basis of this concept is to find out all the players's relative importance in a cooperative activity.

Assume that there are *n* players with *m* contributor and let *w* be the weight to the contributor. Any subset *S* of the player set *N*=(1,…,*n*) is called a coalition The record for the coalition *S* is defined by

$$x_i(S) = \sum_{j \in S} x_{ij} \qquad (i = 1,\ldots,m) \tag{1}$$

where x_{ij} is the record of player *j* to the contributor *i*.

This coalition aims at obtaining the maximal outcome c(*S*):

$$c(S) = \max \sum_{i=1}^{m} w_i x_i(S) \tag{2}$$
$$\text{s.t} \sum_{i=1}^{m} w_i = 1, \quad w_i \geq 0 \ (\forall i)$$

The $c(S)$, with $c(\varnothing) = 0$, defines a characteristic function of the coalition *S*. Thus, we have a game in coalition form with transferable utility, as represented by (*N*,*c*)[2]

The Shapley value of the game (*N*,*c*) for the player *k* is the average of its marginal contribution to all possible coalitions:

$$\varphi_k(c) = \sum_{\text{all } S} \gamma_n(S)\left[c(S) - c(S - \{k\})\right] \tag{3}$$

with weights of probability to enter into a coalition *S* defined as following:

$$\gamma_n(S) = \frac{(s-1)!(n-s)!}{n!} \tag{4}$$

In (3) and (4), *n* is the total number of all the participants, *s* is the number of members in the *S*th coalition, and $c(\cdot)$ is the characteristic function used for estimation of utility for each coalition. If a subset $S(\subset N)$ includes player *k*, *k*'s marginal contribution is obtained as $c(S) - c(S - \{k\})$.[3][4]

3. EVALUATION OF THE CONTRIBUTORS OF OVERALL EQUIPMENT EFFECTIVENESS USING SHAPLEY VALUE

3.1 AN ILLUSTRATIVE EXAMPLE

Assume that an activity with three contributors was conducted in four periods. The outcome of this activity is shown in Table 1. Let periods t_1, t_2, …, t_4 be the players and let Performance Rate, Quality Rate, and Availability be the contributors. We use the same value of OEE but with different values of Performance Rate, Quality Rate, and Availability to strengthen the influence of these contributors.

Table 1. Example of a Coalition

Player / Contributor	t_1	t_2	t_3	t_4	Sum
PR	0.9086	0.8435	0.8908	0.7802	3.4231
QR	0.9086	0.9086	0.8997	0.9786	3.6955
A	0.9086	0.9787	0.9359	0.9825	3.8057
OEE	0.7501	0.7501	0.7501	0.7501	

From Table 1, we divided each row by row-sum to normalized the sum to 1 as shown in Table 2.

Table 2. Normalized value for Table 1

Player / Contributor	t_1	t_2	t_3	t_4	Sum
PR	0.2654	0.2464	0.2602	0.2279	1.0000
QR	0.2459	0.2459	0.2435	0.2648	1.0000
A	0.2387	0.2572	0.2459	0.2582	1.0000

From Table 2, maximum outcome of t_1 is given by;

c(t_1)=max $0.2654\,w_1 + 0.2459\,w_2 + 0.2387\,w_3$

subject to; $w_1 + w_2 + w_3 = 1$

$$w_1, w_2, w_3 \geq 0$$

where *w* is the weight of the contributor.

The optimal solution, c(t_1) =0.2654 is obtained when w_1=1, w_2=0, and w_3=0. The underline in Table 2 indicates the maximum outcome of each column.

We enumerate all coalition's values for each contributor in Table 2. For example, the value of coalition $\{t_1, t_2\}$ for Performance Rate is given as 0.2654 + 0.2464 = 0.5118. Coalition $\{t_1, t_3\}$ and $\{t_2, t_3\}$ is calculated by 0. 2654 + 0. 2602 = 0. 5257, 0. 2464 + 0.2602 = 0. 5066, respectively. The maximum outcomes or characteristic functions for each column are underlined as shown in Table 3.

Table 3. Coalition and Characteristic Function

Coalition / Contributor	$\{t_1,t_2\}$	$\{t_1,t_3\}$	$\{t_1,t_4\}$	$\{t_2,t_3\}$	$\{t_2,t_4\}$	$\{t_3,t_4\}$
PR	0.5118	0.5257	0.4934	0.5066	0.4743	0.4882
QR	0.4917	0.4893	0.5107	0.4893	0.5107	0.5083
A	0.4959	0.4847	0.4969	0.5031	0.5153	0.5041

Coalition / Contributor	$\{t_1,t_2, t_3\}$	$\{t_1,t_2, t_4\}$	$\{t_1,t_3, t_4\}$	$\{t_2,t_3, t_4\}$
PR	0.7721	0.7398	0.7536	0.7346
QR	0.7352	0.7565	0.7541	0.7541
A	0.7418	0.7541	0.7428	0.7613

Player t_1, t_2, t_3 and t_4 created 24 permutation as per Table 7. In permutation $t_1\,t_2\,t_3\,t_4$, player t_1 is the first comer to the coalition, follows by player t_2, t_3 and finally player t_4. Thus, from Table 2 and Table 3, marginal contribution of each player to coalition can be evaluated as below;

t_4's marginal contribution is;
c($\{t_1, t_2, t_3, t_4\}$) - c($\{t_1, t_2, t_3\}$) = 1 - 0. 7721 = 0. 2279

t_3's marginal contribution is;
c($\{t_1, t_2, t_3\}$) - c($\{t_1, t_2\}$) = 0.7721 - 0. 5118 = 0. 2603

t_2's marginal contribution is;
c($\{t_1, t_2\}$) - c($\{t_1\}$) = 0. 5118 - 0. 2654 = 0. 2464

Lastly, t_1's marginal contribution is

c({ t_1}) - c({ $\varnothing$}) = 0. 2654 - 0 = 0. 2654

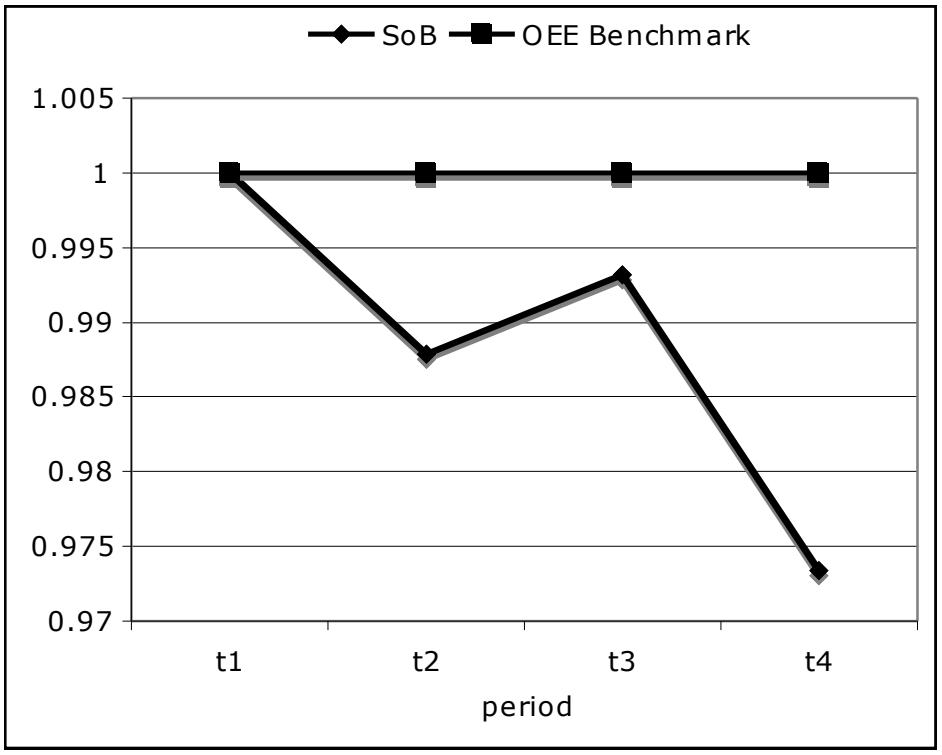
The same calculation then was repeated for every permutation. The average of marginal contribution of the player was respectively taken from Appendix 1, and this average is described as the Shapley value (see Table 4). Furthermore, each player's Shapley value was divided by the highest value of the Shapley value to obtain a score for each player. We refer to this score as the " Scale of Balance"(SoB). The SoB gives the position of each player against the best-performing player. A high contribution with a good combination balance from the contributor will lead to higher allocation for the player. We defined this phenomenon as SoB=1. As a result, we can say that t_2, t_3 and t_4 are 98.79% , 99.32% and 97.34% of t_1, respectively. The same exercise was conducted for OEE to benchmark against each other. Note that the value of Performance Rate, Quality Rate, and Availability in t_1 is equal.

Table 4. Comparison between the Shapley value and OEE

	t_1	t_2	t_3	t_4
Shapley Value	0.2529	0.2498	0.2512	0.2462
SoB	1	0.9879	0.9932	0.9734
OEE	0.7501	0.7501	0.7501	0.7501
OEE Benchmark	1	1	1	1

From Table 4, althrough the value of OEE is constant, since the contribution values among contributors are different,the Shapley value also shows the differences as illustrated in Figure 1.

Figure 1. The differences between the Shapley value and OEE



3.2 CASE STUDY

The OEE and it components for every 6 months over three years of a company are shown in Table 5. The Shapley value can be calculated by the same method as the illustrative example.The acquired Shapley value is shown in Table 6.

Table 5. Coalition of the case study

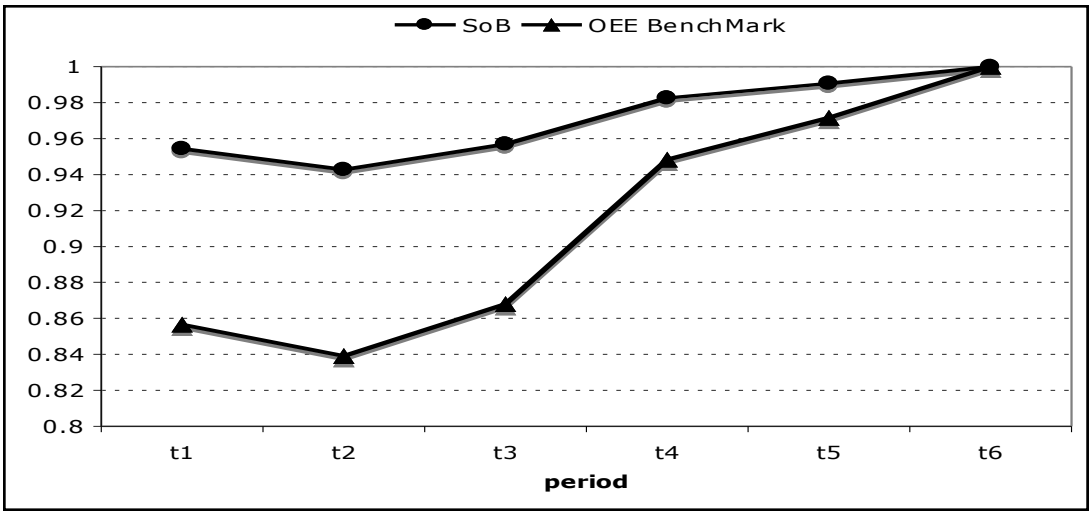
	t_1	t_2	t_3	t_4	t_5	t_6	Sum
QR	89.79	90.63	91.48	94.85	95.61	96.82	559.18
A	90.52	88.73	89.63	88.53	89.17	93.09	539.67
PR	91.37	90.47	91.78	97.94	98.83	96.21	566.6
OEE	74.26	72.75	75.25	82.24	84.26	86.71	

Table 6. Comparison between the Shapley value and OEE for Table 5

Player / Contributor	t_1	t_2	t_3	t_4	t_5	t_6
Shapley Value	0.1638	0.1618	0.1642	0.1686	0.1700	0.1716
SoB	0.9545	0.9429	0.9569	0.9825	0.9907	1.0000
OEE	74.26	72.75	75.25	82.24	84.26	86.71
OEE BenchMark	0.8564	0.8390	0.8678	0.9484	0.9717	1.0000

Figure 2 presents a plotted graph of the result from Table 6.

Figure 2. The differences between the Shapley value and OEE for Table 5



Next, we transpose the position of player and contributor where QR,A and PR play a role as a player and period as the contributor as shown in Table 7. Since Availability, Quality Rate, and Performance Rate are managed by the maintenance department, production department and quality control department, respectively, the highest-performing department can be estimated using the Shapley value.

Table 7. Tranpose Coalition of Table 5

Player / Contributor	QR	A	PR	Sum
t_1	89.79	90.52	91.37	271.68
t_2	90.63	88.73	90.47	269.83
t_3	91.48	89.63	91.78	272.89
t_4	94.85	88.53	97.94	281.32
t_5	95.61	89.17	98.83	283.61
t_6	96.82	93.09	96.21	286.12

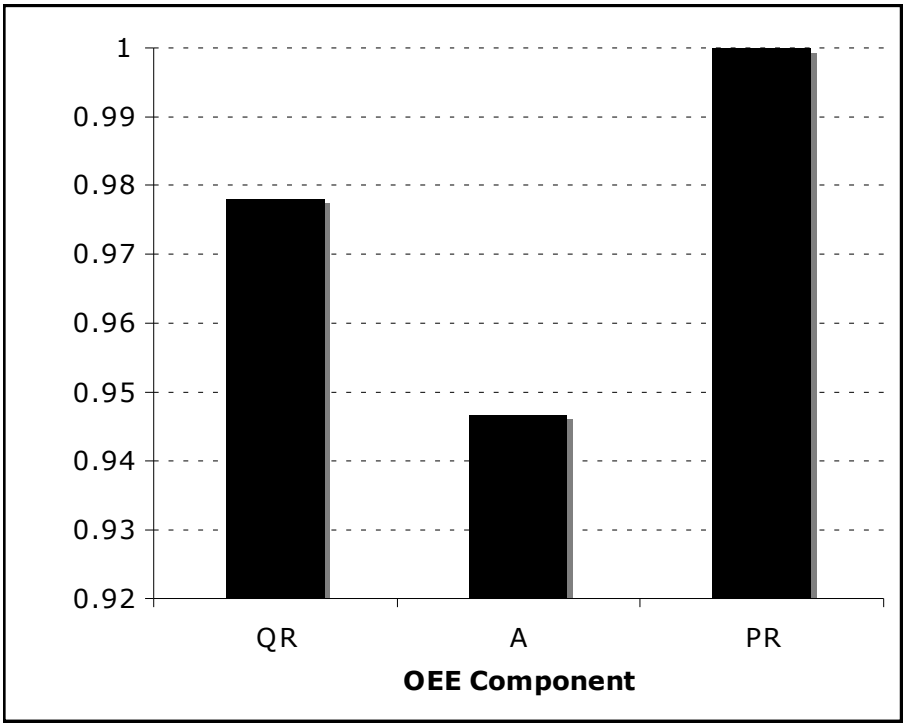
Table 8 shows the calculation result of Table 7.

Table 8. The Shapley value and Sob for Table 7

	QR	A	PR
Shapley Value	0.3344	0.3237	0.3419
SoB	0.9781	0.9468	1.0000

Figure 3 plots the graph of Table 8. From this graph.we able to compare the value of contribution of each component of OEE where PR is the best-performed section during the period where the activity was conducted. This information is important because we wanted to avoid any sections or departments in the same organization from being left behind. The section or department that leads ahead can share their knowledge or experience to another sections that are left behind. Managers could also allocate the resources from a higher contributor to a lower contributor to maintain a good balance in activity.

Figure 3. SoB for OEE component.



4. CONCLUSION

From the case study, the contribution of components of OEE by the Shapley value was able to carry out two kinds of evaluations by changing the position of the player and contributor.That is,components of OEE as the contributor and period as the player,and vice versa.The former is to measure during which period the activity was best performed.The latter measures how each cooperator has contributed to OEE at the determined period

The evaluation performed in this paper is for a static evaluation and the time sequence is not considered. However, a performance trend shown over time sequence cannot be disregarded for maintenance activities since the maintenance activities performed in a previous period continue to have an influence in the next period. Therefore, in our future work, we would like to investigate how each contributor's contribution can be dynamically evaluated.One of the possibilities for doing this is to use a the time dependent version of the Shapley value.

The Shapley value is applicable in the various maintenance situation.For example, the six tangible effects of Total Productive Maintenance(TPM) ,which are production,quality, cost, morale, safety,hygiene and environment and delivery or the eight TPM pillars could be used as the contributor or the player to estimate contributions to the achievement of TPM activities.By knowing the value of a contribution, it becomes easy to identify on which section

needs further improvement.Additionally, new viewpoint for evalution can also be devised with a combination of the Shapley value and other techniques such as Data Envelopment Analysis.

5. REFERENCES:

[1] Roth,E.A.,1988,The Shapley Value,Cambridge University Press,New York, USA.

[2] Cooper,W.W., Seiford M.L., Tone,K., 2007, Data Envelopment Analysis Second Edition, Spinger, New York, USA

[3] Nakabayashi,K., Tone,K.,2006,"Egoist's dilemma: a DEA game" ,Omega 34, pp.135-148

[4] Michael Conklin, Ken Powaga, Stan Lipovetsky, Customer satisfaction analysis: Identification of key drivers, European Journal of Operational Research 154,2004,pp:819-827

[5] Nakajima,S.,Shirase,K., (1992), New TPM Development Program for Assemby Process (in Japanese), JIPM Solution,Tokyo,Japan

Appendix 1: Marginal contribution of the players in every permutations

Player / Permutation	t_1	t_2	t_3	t_4
$t_1\ t_2\ t_3\ t_4$	0.2654	0.2464	0.2603	0.2279
$t_1\ t_2\ t_4\ t_3$	0.2654	0.2464	0.2435	0.2447
$t_1\ t_3\ t_2\ t_4$	0.2654	0.2464	0.2603	0.2279
$t_1\ t_3\ t_4\ t_2$	0.2654	0.2459	0.2603	0.2285
$t_1\ t_4\ t_3\ t_2$	0.2654	0.2459	0.2435	0.2452
$t_1\ t_4\ t_2\ t_3$	0.2654	0.2459	0.2435	0.2452
$t_2\ t_1\ t_3\ t_4$	0.2547	0.2572	0.2603	0.2279
$t_2\ t_1\ t_4\ t_3$	0.2547	0.2572	0.2435	0.2447
$t_2\ t_3\ t_1\ t_4$	0.2654	0.2572	0.2495	0.2279
$t_2\ t_3\ t_4\ t_1$	0.2459	0.2572	0.2495	0.2475
$t_2\ t_4\ t_1\ t_3$	0.2412	0.2572	0.2435	0.2582
$t_2\ t_4\ t_3\ t_1$	0.2387	0.2572	0.2459	0.2582
$t_3\ t_1\ t_2\ t_4$	0.2654	0.2464	0.2603	0.2279
$t_3\ t_1\ t_4\ t_2$	0.2654	0.2459	0.2603	0.2285
$t_3\ t_2\ t_1\ t_4$	0.2654	0.2464	0.2603	0.2279
$t_3\ t_2\ t_4\ t_1$	0.2387	0.2464	0.2603	0.2546
$t_3\ t_4\ t_1\ t_2$	0.2459	0.2459	0.2603	0.2480
$t_3\ t_4\ t_2\ t_1$	0.2387	0.2530	0.2603	0.2480
$t_4\ t_1\ t_2\ t_3$	0.2459	0.2459	0.2435	0.2648
$t_4\ t_1\ t_3\ t_2$	0.2459	0.2459	0.2435	0.2648
$t_4\ t_2\ t_1\ t_3$	0.2412	0.2505	0.2435	0.2648
$t_4\ t_2\ t_3\ t_1$	0.2387	0.2505	0.2459	0.2648
$t_4\ t_3\ t_1\ t_2$	0.2459	0.2459	0.2435	0.2648
$t_4\ t_3\ t_2\ t_1$	0.2387	0.2530	0.2435	0.2648