International Journal of Research and Reviews in Applied Sciences
ISSN: 2076-734X, EISSN: 2076-7366
Volume 2, Issue 1(January 2010)

AN ORTHOTROPIC ADAPTIVE SHALLOW CYLINDRICAL SHELL ON ELASTIC FOUNDATION

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ABSTRACT

With growth and emergence in the field of adaptive materials, the need arises to study their applications in the field of structural, aerodynamic, aerospace and other fields. These materials can be used as sensors, transducers, and actuators. Although their basic constitutive relations are already developed, but there is still a great deal of scope left in the field of applications. With this aim, a nonlinear static analysis of orthotropic piezoelectric shallow cylindrical shell on Pasternak foundation is investigated in the present work. Basic formulation of the problem is based on strain energy concept, and the governing differential equations are obtained by using Euler's variational principle. Galerkin error minimization technique has been used to solve the governing differential equations. The results are presented for simply supported immovable edge boundary condition. Influences of shell geometry, foundation parameter, and piezoelectric properties on load–deflection characteristics for different radius-to-thickness ratios are studied. Numerical results have been obtained for different values of geometrical parameters in terms of load, displacement, and electric potential. Geometrical parameters are represented through non-dimensional entities $\eta = a^2/Rh$, $\lambda = Ka^4/D_{11}$, and $\bar{\mu} = Ga^2/D_{11}$.The results are compared with nonlinear static analysis of an orthotropic shallow cylindrical shell without piezoelectric layer on Pasternak foundation. It is observed that an increase in the value of piezoelectric constant decreases the deflection of the shallow cylindrical shell under the identical values.

Keywords: *Orthotropic; piezoelectric; shallow cylindrical shell; elastic foundation; nonlinear static analysis.*

1. INTRODUCTION

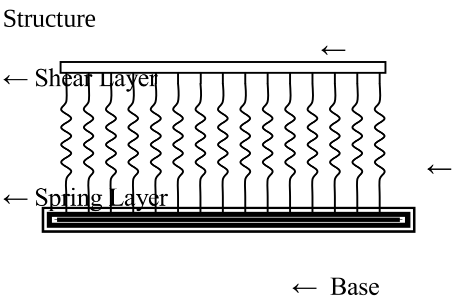
Structural engineering and material science have entered into a new age bought about by the development of adaptive materials and their applications in the intelligent structures. Piezoelectric materials provide a transformation between electric and mechanical energy, and can thus be used for mechanical and acoustic input and output devices, Owing to this ability to provide sensor (input) and actuator (output) functions at the same time, piezoelectric belong to the materials family that is often referred to as 'intelligent materials'. Present study is concentrated on studies of shallow shells .A shell is usually regarded as being shallow if $\frac{l_{\min}}{f} \geq 5$ and $\frac{R_{\min}}{h} \geq 5$, where lmin is minimum dimension of the shell in plan and 'f' is sag, the highest elevation above the base.

Foundation is a load-carrying member. Successful design of structures involving foundation- structure interaction depends to a large extent upon the use of appropriate foundation model. There are various elastic foundation models (Fig.1a-b-c), out of which the present study is concentrated on Pasternak foundation. Pasternak model is a biparametric elastic foundation model characterized by two independent elastic constants. It is suitable for homogeneous soil deposits and rocky soils.

The strain energy equation of the Pasternak foundation can be expressed in the terms foundation modulii as.

$$U = \frac{1}{2}\int \left\{ k_1 w^2 + G\left[\left(\frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] \right\} da$$

where k_1= modulus of sub grade reaction for the foundation and G = shear modulus of the sub grade. If it is viewed as a constant tension, it becomes the Pasternak modulus, and if G = 0, the modulus reduces to the Winker Pasternak foundation



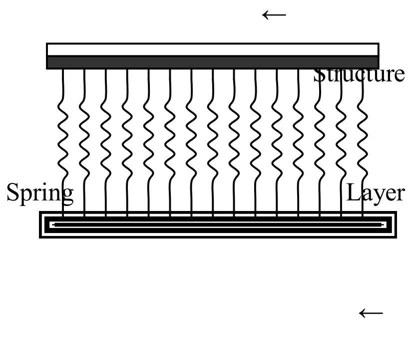


Fig.1 a Winkler foundation *Fig.1 b Pasternak foundation*

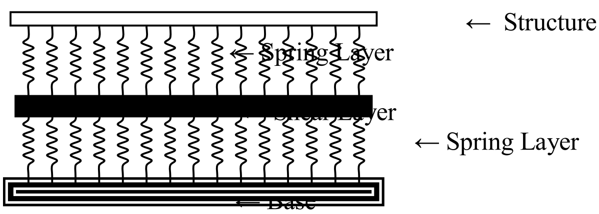


Fig.1 c Kerr foundation

2. LITERATURE REVIEW AND OBJECTIVE OF PRESENT WORK

Investigations carried out in the past [1, 2] were limited to the cases of non-piezoelectric shallow cylindrical shells. The works presented in [3-5] were restricted to cylindrical shells only and no consideration was made for shallow cylindrical shells. Based on the literature survey, it can be inferred that a lot of work has been done in the field of shells on elastic foundation as well as on piezoelectric shells, but not on shallow cylindrical shell on elastic foundation. Piezoelectric shell in cylindrical bending has also been extensively investigated, but only for the case of axi-symmetric loading.

In case of shells on elastic foundation, the loading and deformation are not symmetrical along the circumferential direction. The objective of this study is to investigate the nonlinear static behavior of orthotropic shallow cylindrical shells on a Pasternak type elastic foundation and study the influence of various shell and foundation parameters on load-deflection as well as load-electric potential characteristics.

3. FORMULATION OF THE PROBLEM

We consider the large symmetric deflection of a shallow cylindrical shell subjected to uniform normal pressure 'q' on its concave surface and resting on elastic foundation. Coordinate system employed is shown in Fig. 1. For circular cylindrical shell the distance of the middle surface of the shell is given by

$$z = \frac{a^2}{2.R}\left(1 - \frac{x^2}{a^2}\right) \tag{1}$$

The assumptions made in this analysis are (1) The shell is orthotropic shallow cylindrical without initial imperfections (2) It is assumed that the piezoelectric material is incorporated into the matrix. (3) Thickness of the shell is much less than other dimensions of the shell.(4) Material of the shell is assumed to be homogeneous and orthotropic.(5) The shell is initially subjected to axisymmetric external pressure.(6) Electric field is considered varying linearly in radial direction.(7) Body forces are assumed to be zero.

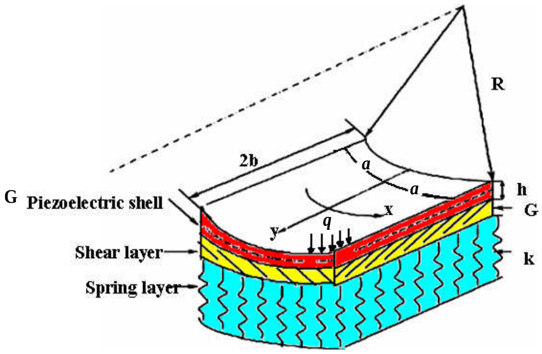


Fig.2. Piezoelectric shell on Pasternak foundation

Constitutive equations for the piezoelectric material are given by

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{23}\\ \sigma_{31}\end{bmatrix}=\begin{bmatrix}c_{11}&c_{12}&c_{13}&c_{14}&c_{15}&c_{16}\\ c_{21}&c_{22}&c_{23}&c_{24}&c_{25}&c_{26}\\ c_{31}&c_{32}&c_{33}&c_{34}&c_{35}&c_{36}\\ c_{41}&c_{42}&c_{41}&c_{44}&c_{45}&c_{46}\\ c_{51}&c_{52}&c_{53}&c_{54}&c_{55}&c_{56}\\ c_{61}&c_{62}&c_{63}&c_{64}&c_{65}&c_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ \varepsilon_{12}\\ \varepsilon_{23}\\ \varepsilon_{31}\end{bmatrix}+\begin{bmatrix}e_{11}&e_{12}&e_{13}\\ e_{21}&e_{22}&e_{23}\\ e_{31}&e_{32}&e_{33}\\ e_{41}&e_{42}&e_{43}\\ e_{51}&e_{52}&e_{53}\\ e_{61}&e_{62}&e_{63}\end{bmatrix}\begin{bmatrix}E_1\\ E_2\\ E_3\end{bmatrix} \tag{2}$$

where σ_{ij} are the stresses, c_{ij} are elastic coefficients , ϵ_{ij} are strains, e_{ij} are piezoelectric stress coefficients and E_s are electric field vector.

The relations between electric displacement and electric field are given by

$$\begin{bmatrix}D_1\\ D_2\\ D_3\end{bmatrix}=\begin{bmatrix}e_{11}&e_{21}&e_{31}&e_{41}&e_{51}&e_{61}\\ e_{12}&e_{22}&e_{32}&e_{42}&e_{52}&e_{62}\\ e_{13}&e_{23}&e_{33}&e_{43}&e_{53}&e_{63}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ \varepsilon_{12}\\ \varepsilon_{23}\\ \varepsilon_{31}\end{bmatrix}+\begin{bmatrix}\kappa_{11}&\kappa_{12}&\kappa_{13}\\ \kappa_{21}&\kappa_{22}&\kappa_{23}\\ \kappa_{31}&\kappa_{32}&\kappa_{33}\end{bmatrix}\begin{bmatrix}E_1\\ E_2\\ E_3\end{bmatrix} \tag{3}$$

where D_s are electric displacement vector and k_{ij} are dielectric coefficients.

3.1 SIMPLIFICATION TO 2-DIMENSIONAL PLANE STRESS CONDITION

As per the conditions of the problem, the formulation is reduced to a 2-dimensional plane stress condition. Hence

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{bmatrix}=\begin{bmatrix}c_{11}&c_{12}&0\\ c_{21}&c_{22}&0\\ 0&0&c_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{12}\end{bmatrix}+\begin{bmatrix}0&0&e_{31}\\ 0&0&e_{32}\\ 0&0&e_{36}\end{bmatrix}\begin{bmatrix}E_1\\ E_2\\ E_3\end{bmatrix} \tag{4}$$

and

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ e_{31} & e_{32} & e_{36} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{bmatrix} + \begin{bmatrix} \kappa_{11} & 0 & 0 \\ 0 & \kappa_{22} & 0 \\ 0 & 0 & \kappa_{33} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \tag{5}$$

The normal forces N and moment M per unit length are obtained by

$$N = \int \sigma . dz \quad \text{and} \quad M = \int z . \sigma . dz \tag{6}$$

If the directions '1' and '2' are represented by coordinate axes 'x'and 'y' respectively, then [N] and [M] are given by

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 \\ A_{12} & A_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{11} & D_{12} & 0 \\ 0 & 0 & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \\ \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{bmatrix} + h \begin{bmatrix} k_{11} & 0 & 0 \\ 0 & k_{22} & 0 \\ 0 & 0 & k_{33} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \tag{7}$$

and

$$\begin{bmatrix} D'_{xx} \\ D'_{yy} \\ D'_{xy} \end{bmatrix} = h \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ e_{31} & e_{32} & e_{36} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{bmatrix} + h \begin{bmatrix} \kappa_{11} & 0 & 0 \\ 0 & \kappa_{22} & 0 \\ 0 & 0 & \kappa_{33} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \tag{8}$$

The relation between strain and mechanical displacements are written as

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\partial w}{\partial x}\frac{\partial z}{\partial x} \tag{9a}$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{\partial w}{\partial y}\frac{\partial z}{\partial y} \tag{9b}$$

$$\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \tag{9c}$$

The curvature displacement relation are expressed as

$$k_{xx} = \frac{\partial^2 w}{\partial x^2}, k_{yy} = \frac{\partial^2 w}{\partial y^2}, k_{xy} = \frac{\partial^2 w}{\partial x . \partial y} \tag{10a-c}$$

As the total energy of the system is equal to the sum of strain energies due to extension, bending, piezoelectric effect and deformation of the foundation; hence it is expressed as

$$U = \frac{1}{2}\Big(N_x\varepsilon_{xx} + N_y\varepsilon_{yy} + N_{xy}\varepsilon_{xy} + M_x k_{xx} + M_y k_{yy} + M_{xy}k_{xy} + D'_x E_x + D'_y E_y + D'_{xy} E_z +$$

$$k.w^2 - 2qw + G\left\{\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right\}\Big\}.dx.dy \tag{11}$$

$$=\frac{1}{2}\Big\{\left(A_{11}\varepsilon_{xx}+A_{12}\varepsilon_{yy}+h.e_{31}.E_{3}\right)\varepsilon_{xx}+\left(A_{12}\varepsilon_{xx}+A_{22}\varepsilon_{yy}+h.e_{32}.E_{3}\right)\varepsilon_{yy}+\left(A_{66}\varepsilon_{xy}+h.e_{36}.E_{3}\right)\varepsilon_{xy}$$

$$+\left(D_{11}k_{xx}+D_{12}k_{yy}\right)k_{xx}+\left(D_{12}k_{xx}+D_{22}k_{yy}\right)k_{yy}+\left(D_{66}k_{xy}\right)k_{xy}+h.\kappa_{11}.E_{x}^{2}+h.\kappa_{22}.E_{y}^{2}+$$

$$h(e_{31}.\varepsilon_{xx}+e_{32}.\varepsilon_{yy}+e_{36}.\varepsilon_{xy})E_{z}k.w^{2}-2qw+G\left\{\left(\frac{\partial w}{\partial x}\right)^{2}+\left(\frac{\partial w}{\partial y}\right)^{2}\right\}\Big\}dx.dy$$

With the help of Eq. (7), the Eq. (11) becomes

$$U=\Bigg[\frac{1}{2}\Big\{\left(A_{11}\varepsilon_{xx}+A_{12}\varepsilon_{yy}\right)\varepsilon_{xx}+\left(A_{12}\varepsilon_{xx}+A_{22}\varepsilon_{yy}\right)\varepsilon_{yy}+\left(A_{66}\varepsilon_{xy}\right)\varepsilon_{xy}+\left(D_{11}k_{xx}+D_{12}k_{yy}\right)k_{xx}+$$

$$\left(D_{12}k_{xx}+D_{22}k_{yy}\right)k_{yy}+\left(D_{66}k_{xy}\right)k_{xy}\Big\}+\Big\{h.e_{31}.\varepsilon_{xx}.E_{z}+h.e_{32}.\varepsilon_{yy}.E_{z}+h.e_{36}.\varepsilon_{xy}.E_{z}+$$

$$\frac{1}{2}\left(h\kappa_{11}.E_{x}^{2}+h\kappa_{22}.E_{y}^{2}+h\kappa_{33}.E_{z}^{2}\right)\Big\}+\frac{1}{2}\left(k.w^{2}-2qw+G\left\{\left(\frac{\partial w}{\partial x}\right)^{2}+\left(\frac{\partial w}{\partial y}\right)^{2}\right\}\right)\Bigg]dx.dy \qquad (12)$$

where $E_x=\frac{\partial\phi}{\partial x}$, $\quad E_y=\frac{\partial\phi}{\partial y}$, and $\quad E_z=\frac{\partial\phi}{\partial z}=\frac{\phi}{z}$

Since we know that for a shallow circular shell, the geometry is

$z=\frac{a^2}{2.R}\left(1-\frac{x^2}{a^2}\right)$, therefore $\frac{\partial z}{\partial x}=\frac{a^2}{2.R}\left(-\frac{2x}{a^2}\right)=-\frac{x}{R}$ and $\frac{\partial z}{\partial y}=0$

Using Euler's variation principle:

$$\frac{\partial F}{\partial u}-\frac{\partial}{\partial x}\left(\frac{\partial F}{\partial u_x}\right)-\frac{\partial}{\partial y}\left(\frac{\partial F}{\partial u_y}\right)+\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 F}{\partial u_{xx}}\right)+\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2 F}{\partial u_{yy}}\right)+\frac{\partial^2}{\partial x\partial y}\left(\frac{\partial^2 F}{\partial u_{xy}}\right)=0 \qquad (13)$$

$$\frac{\partial F}{\partial v}-\frac{\partial}{\partial x}\left(\frac{\partial F}{\partial v_x}\right)-\frac{\partial}{\partial y}\left(\frac{\partial F}{\partial v_y}\right)+\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 F}{\partial v_{xx}}\right)+\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2 F}{\partial v_{yy}}\right)+\frac{\partial^2}{\partial x\partial y}\left(\frac{\partial^2 F}{\partial v_{xy}}\right)=0 \qquad (14)$$

$$\frac{\partial F}{\partial w}-\frac{\partial}{\partial x}\left(\frac{\partial F}{\partial w_x}\right)-\frac{\partial}{\partial y}\left(\frac{\partial F}{\partial w_y}\right)+\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 F}{\partial w_{xx}}\right)+\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2 F}{\partial w_{yy}}\right)+\frac{\partial^2}{\partial x\partial y}\left(\frac{\partial^2 F}{\partial w_{xy}}\right)=0 \qquad (15)$$

$$\frac{\partial F}{\partial \phi}-\frac{\partial}{\partial x}\left(\frac{\partial F}{\partial \phi_x}\right)-\frac{\partial}{\partial y}\left(\frac{\partial F}{\partial \phi_y}\right)+\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 F}{\partial \phi_{xx}}\right)+\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2 F}{\partial \phi_{yy}}\right)+\frac{\partial^2}{\partial x\partial y}\left(\frac{\partial^2 F}{\partial \phi_{xy}}\right)=0 \qquad (16)$$

and applying Eq. (13) to integrand in Eq. (12) , we get

$$0-\frac{\partial}{\partial x}\left[\frac{12}{h^2}\left\{2.D_{11}.\varepsilon_{xx}+2.D_{12}.\varepsilon_{yy}\right\}\right]-\frac{\partial}{\partial y}\left(8.D_{66}.\varepsilon_{xy}\right)-\frac{\partial}{\partial x}\left(e_{31}.\phi\right)-\frac{\partial}{\partial y}\left(e_{36}.\phi\right)=0 \qquad (17)$$

On substituting the strain terms given by Eqs. (9a-b-c) in the Eq. (17), we get

$$-\frac{\partial}{\partial x}\left[\frac{12}{h^2}\left\{2.D_{11}\cdot\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial z}{\partial x}\right)+2.D_{12}\cdot\left(\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2+\frac{\partial w}{\partial y}\frac{\partial z}{\partial y}\right)\right\}\right]$$

$$-\frac{\partial}{\partial y}\left(8.D_{66}\cdot\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)\right)-\frac{\partial}{\partial x}\left(e_{31}.\phi\right)-\frac{\partial}{\partial y}\left(e_{36}.\phi\right)=0 \tag{18}$$

Similarly Eqs (14, 15, 16) are applied to integrand in Eq (12), which yield Eqs. (19a), (20a) and (21a). Further on substituting the strain terms given by Eqns. (9a-b-c) in Eqs. (19a), (20a), (21a); we get Eqs. (19b), (20b), and (21b). These are given below.

$$0-\frac{\partial}{\partial y}\left[\frac{12}{h^2}\left\{2.D_{12}.\varepsilon_{xx}+2.D_{22}.\varepsilon_{yy}\right\}\right]-\frac{\partial}{\partial x}\left(8.D_{66}.\varepsilon_{xy}\right)-\frac{\partial}{\partial y}\left(e_{32}.\phi\right)-\frac{\partial}{\partial x}\left(e_{36}.\phi\right)=0 \tag{19a}$$

$$-\frac{\partial}{\partial y}\left[\frac{12}{h^2}\left\{2.D_{12}\cdot\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial z}{\partial x}\right)+2.D_{22}\cdot\left(\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2+\frac{\partial w}{\partial y}\frac{\partial z}{\partial y}\right)\right\}\right]$$

$$-\frac{\partial}{\partial x}\left(8.D_{66}\cdot\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)\right)-\frac{\partial}{\partial y}\left(e_{32}.\phi\right)-\frac{\partial}{\partial x}\left(e_{36}.\phi\right)=0 \tag{19b}$$

$$2.k.w-2q-\frac{\partial}{\partial x}\left[\frac{12}{h^2}\left\{D_{11}.\varepsilon_{xx}\left(\frac{\partial w}{\partial x}+\frac{\partial z}{\partial x}\right)+2.D_{12}.\varepsilon_{yy}\cdot\left(\frac{\partial w}{\partial x}+\frac{\partial z}{\partial x}\right)+8.D_{66}.\varepsilon_{xy}\cdot\frac{\partial w}{\partial y}\right\}+\right.$$

$$\left.2G\frac{\partial w}{\partial x}\right]-\frac{\partial}{\partial y}\left[\frac{12}{h^2}\left\{2.D_{12}.\varepsilon_{xx}\left(\frac{\partial w}{\partial y}+\frac{\partial z}{\partial y}\right)+2D_{22}.\varepsilon_{yy}\left(\frac{\partial w}{\partial y}+\frac{\partial z}{\partial y}\right)+8.D_{66}\varepsilon_{xy}\cdot\frac{\partial w}{\partial x}\right\}+\right.$$

$$\left.2G\frac{\partial w}{\partial y}\right]+8.D_{66}\cdot\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^2}{\partial x^2}\left(2.D_{11}\frac{\partial^2 w}{\partial x^2}+2.D_{12}\frac{\partial^2 w}{\partial y^2}\right)+\frac{\partial^2}{\partial y^2}\left(2.D_{12}\frac{\partial^2 w}{\partial x^2}+\right.$$

$$\left.2.D_{22}\frac{\partial^2 w}{\partial y^2}\right)-\frac{\partial}{\partial x}\left\{e_{31}.\phi\left(\frac{\partial w}{\partial x}-\frac{x}{R}\right)+e_{36}.\phi\frac{\partial w}{\partial y}\right\}-\frac{\partial}{\partial y}\left\{e_{32}.\phi\left(\frac{\partial w}{\partial y}\right)+e_{36}.\phi\frac{\partial w}{\partial x}\right\}=0 \tag{20a}$$

$$2.k.w-2q-\frac{\partial}{\partial x}\left[\frac{12}{h^2}\left\{D_{11}\cdot\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial z}{\partial x}\right)\left(\frac{\partial w}{\partial x}+\frac{\partial z}{\partial x}\right)+2.D_{12}\cdot\left(\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right.\right.\right.$$

$$\left.\left.\left.+\frac{\partial w}{\partial y}\frac{\partial z}{\partial y}\right)\left(\frac{\partial w}{\partial x}+\frac{\partial z}{\partial x}\right)+8.D_{66}\cdot\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)\cdot\frac{\partial w}{\partial y}\right\}+2G\frac{\partial w}{\partial x}\right]-\frac{\partial}{\partial y}\left[\frac{12}{h^2}\right.$$

$$\left\{2.D_{12}\cdot\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial z}{\partial x}\right)\left(\frac{\partial w}{\partial y}+\frac{\partial z}{\partial y}\right)+2D_{22}\cdot\left(\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2+\frac{\partial w}{\partial y}\frac{\partial z}{\partial y}\right)\right.$$

$$\left.\left.\left(\frac{\partial w}{\partial y}+\frac{\partial z}{\partial y}\right)+8.D_{66}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)\cdot\frac{\partial w}{\partial x}\right\}+2G\frac{\partial w}{\partial y}\right]+8.D_{66}\cdot\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^2}{\partial x^2}$$

$$\left(2.D_{11}\frac{\partial^2 w}{\partial x^2}+2.D_{12}\frac{\partial^2 w}{\partial y^2}\right)+\frac{\partial^2}{\partial y^2}\left(2.D_{12}\frac{\partial^2 w}{\partial x^2}+2.D_{22}\frac{\partial^2 w}{\partial y^2}\right)-\frac{\partial}{\partial x}\left\{e_{31}.\phi\left(\frac{\partial w}{\partial x}-\frac{x}{R}\right)\right.$$

$$\left.+e_{36}.\phi\frac{\partial w}{\partial y}\right\}-\frac{\partial}{\partial y}\left\{e_{32}.\phi\left(\frac{\partial w}{\partial y}\right)+e_{36}.\phi\frac{\partial w}{\partial x}\right\}=0$$

$$e_{31}.\varepsilon_{xx}+e_{32}.\varepsilon_{yy}+e_{36}.\varepsilon_{xy}+\frac{\kappa_{33}}{h}\phi-\frac{\partial}{\partial x}\left(h.\kappa_{11}\frac{\partial\phi}{\partial x}\right)-\frac{\partial}{\partial y}\left(h.\kappa_{22}\frac{\partial\phi}{\partial y}\right)\frac{e_{31}}{R}=0 \tag{21a}$$

$$-\frac{e_{31}}{R}+e_{31}.\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial z}{\partial x}\right)+e_{32}.\left(\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2+\frac{\partial w}{\partial y}\frac{\partial z}{\partial y}\right)+$$
$$e_{36}.\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)+\frac{\kappa_{33}}{h}\phi-\frac{\partial}{\partial x}\left(h.\kappa_{11}\frac{\partial \phi}{\partial x}\right)-\frac{\partial}{\partial y}\left(h.\kappa_{22}\frac{\partial \phi}{\partial y}\right)=0 \quad (21b)$$

4. SOLUTION OF GOVERNING EQUATIONS

Thus, on applying the conditions of extremum, four equations in four variables are obtained. In order to solve these equations, Galerkin-error minimizing technique is used. The approximate solutions are assumed according to the boundary conditions present at the ends. For the simply supported immovable end conditions the u, *v*, *w* and ϕ are assumed to be the functions of x and y as follows.

$$u=u_0.Sin\left[\frac{\pi\times x}{2a}\right].Sin\left[\frac{\pi\times y}{2b}\right]$$
$$v=v_0.Sin\left[\frac{\pi\times x}{2a}\right].Sin\left[\frac{\pi\times y}{2b}\right]$$
$$w=w_0.Sin\left[\frac{\pi\times x}{2a}\right].Sin\left[\frac{\pi\times y}{2b}\right] \quad (22)$$
$$\phi=\phi_0.Sin\left[\frac{\pi\times x}{2a}\right].Sin\left[\frac{\pi\times y}{2b}\right]$$

On recalling Eq. (18) and expanding the derivation terms, we get coupled equations. Similar equations are obtained from Eqs. (19b), (20b) and (21b). These are given as follows.

APPENDIX I

Now recalling Eqn. (18), we write

$$-\frac{\partial}{\partial x}\left[\frac{12}{h^2}\left\{2.D_{11}.\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial z}{\partial x}\right)+2.D_{12}.\left(\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2+\frac{\partial w}{\partial y}\frac{\partial z}{\partial y}\right)\right\}\right]$$
$$-\frac{\partial}{\partial y}\left(8.D_{66}.\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)\right)-\frac{\partial}{\partial x}(e_{31}.\phi)-\frac{\partial}{\partial y}(e_{36}.\phi)=0$$

Further on expanding the derivative terms, we get

$$-\left[\frac{12}{h^2}\left\{2.D_{11}.\left(\frac{\partial^2 u}{\partial x^2}+\left(\frac{\partial w}{\partial x}\right)\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial x^2}\frac{\partial z}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial^2 z}{\partial x^2}\right)+2.D_{12}.\left(\frac{\partial^2 v}{\partial y\partial x}+\left(\frac{\partial w}{\partial y}\right)\frac{\partial^2 w}{\partial x\partial y}\right.\right.\right.$$
$$\left.\left.\left.+\frac{\partial^2 w}{\partial y\partial x}\frac{\partial z}{\partial y}+\frac{\partial w}{\partial y}\frac{\partial^2 z}{\partial y\partial x}\right)\right\}\right]-\left(8.D_{66}.\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 w}{\partial x\partial y}\frac{\partial w}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial y^2}\right)\right)-\frac{\partial}{\partial x}(e_{31}.\phi)$$
$$-\frac{\partial}{\partial y}(e_{36}.\phi)=0$$

(A1)

On putting the values of derivative terms in the above expression, we obtain

$$-\frac{12}{h^2}\left[2D_{11}\left\{-\frac{\pi^2 u_0.Sin\left[\frac{\pi\times x}{2a}\right].Sin\left[\frac{\pi\times y}{2b}\right]}{4a^2}+\left(\frac{\pi w_0.Cos\left[\frac{\pi\times x}{2a}\right].Sin\left[\frac{\pi\times y}{2b}\right]}{2a}\right)\right.\right.$$

$$\left(-\frac{\pi^2 w_0.Sin\left[\frac{\pi\times x}{2a}\right].Sin\left[\frac{\pi\times y}{2b}\right]}{4a^2}\right)+\left(-\frac{x}{R}\right)\left(-\frac{\pi^2 w_0.Sin\left[\frac{\pi\times x}{2a}\right].Sin\left[\frac{\pi\times y}{2b}\right]}{4a^2}\right)+$$

$$\left.\left(-\frac{1}{R}\right).\left(\frac{\pi w_0.Cos\left[\frac{\pi\times x}{2a}\right].Sin\left[\frac{\pi\times y}{2b}\right]}{2a}\right)\right\}+2D_{12}\left\{\left(\frac{\pi v_0.Cos\left[\frac{\pi\times x}{2a}\right].Cos\left[\frac{\pi\times y}{2b}\right]}{4ab}\right).\right.$$

$$\left.\left.+\left(\frac{\pi w_0.Cos\left[\frac{\pi\times y}{2a}\right].Sin\left[\frac{\pi\times x}{2b}\right]}{2b}\right)\left(\frac{\pi^2 w_0.Cos\left[\frac{\pi\times x}{2a}\right].Cos\left[\frac{\pi\times y}{2b}\right]}{4ab}\right)\right\}\right]-8.D_{66}.$$

$$\left\{\left(-\frac{\pi u_0.Sin\left[\frac{\pi\times x}{2a}\right].Sin\left[\frac{\pi\times y}{2b}\right]}{4b^2}\right)+\left(\frac{\pi v_0.Cos\left[\frac{\pi\times x}{2a}\right].Cos\left[\frac{\pi\times y}{2b}\right]}{4ab}\right)+\left(\frac{\pi w_0.Cos\left[\frac{\pi\times x}{2a}\right]\right.\right.$$

$$\left.\frac{.Sin\left[\frac{\pi\times y}{2b}\right]}{}\right)\left(-\frac{\pi^2 w_0.Sin\left[\frac{\pi\times x}{2a}\right].Sin\left[\frac{\pi\times y}{2b}\right]}{4a^2}\right)+\left(\frac{\pi w_0.Sin\left[\frac{\pi\times x}{2a}\right].Cos\left[\frac{\pi\times y}{2b}\right]}{2b}\right)$$

$$\left.\left(\frac{\pi w_0.Cos\left[\frac{\pi\times x}{2a}\right].Cos\left[\frac{\pi\times y}{2b}\right]}{4ab}\right)\right\}-\frac{\pi.e_{31}.\phi_0.Cos\left[\frac{\pi\times x}{2a}\right].Sin\left[\frac{\pi\times y}{2b}\right]}{2a}-\frac{\pi.e_{32}.\phi_0.Sin\left[\frac{\pi\times x}{2a}\right].}{2a}$$

$$\frac{Cos\left[\frac{\pi\times y}{2b}\right]}{}=0 \tag{A2}$$

Similarly, from Eqn. (19b) we get,

$$-\frac{12}{h^2}\left[2.D_{12}\left\{\frac{\pi^2 u_0.Cos\left[\frac{\pi\times x}{2a}\right].Cos\left[\frac{\pi\times y}{2b}\right]}{4ab}+\left(\frac{\pi w_0.Cos\left[\frac{\pi\times x}{2a}\right].Sin\left[\frac{\pi\times y}{2b}\right]}{2a}\right)\right.\right.$$

$$\left..\left(\frac{\pi^2 w_0.Cos\left[\frac{\pi\times x}{2a}\right].Cos\left[\frac{\pi\times y}{2b}\right]}{4ab}\right)+\left(-\frac{x}{R}\right)\left(\frac{\pi^2 w_0.Cos\left[\frac{\pi\times x}{2a}\right].Cos\left[\frac{\pi\times y}{2b}\right]}{4ab}\right)+0\right\}$$

$$
\begin{aligned}
&2.D_{22}.\left\{\left(\left(-\frac{\pi^2 v_0.Sin\left[\frac{\pi\times x}{2a}\right].Sin\left[\frac{\pi\times y}{2b}\right]}{4a^2}\right)+\left(\frac{\pi w_0.Sin\left[\frac{\pi\times x}{2a}\right].Cos\left[\frac{\pi\times y}{2b}\right]}{2b}\right)\times\right.\right.\\
&\left.\left.\left(-\frac{\pi^2 w_0.Sin\left[\frac{\pi\times x}{2a}\right].Sin\left[\frac{\pi\times y}{2b}\right]}{4a^2}\right)+0\right\}\right]\Bigg]-8D_{66}\left\{\left(\frac{\pi^2 u_0.Cos\left[\frac{\pi\times x}{2a}\right].Cos\left[\frac{\pi\times y}{2b}\right]}{4ab}\right)-\right.\\
&\frac{\pi^2 v_0.Sin\left[\frac{\pi\times x}{2a}\right].Sin\left[\frac{\pi\times y}{2b}\right]}{4a^2}+\left(\frac{\pi w_0.Sin\left[\frac{\pi\times x}{2a}\right].Cos\left[\frac{\pi\times y}{2b}\right]}{2b}\right)\Bigg)\left(-\frac{\pi^2 w_0.Sin\left[\frac{\pi\times x}{2a}\right].}{4a^2}\right.\\
&\left.\left.\frac{Sin\left[\frac{\pi\times y}{2b}\right]}{}\right)+\left(\frac{\pi^2 w_0.Cos\left[\frac{\pi\times x}{2a}\right].Cos\left[\frac{\pi\times y}{2b}\right]}{4a^2}\right)\left(\frac{\pi w_0.Cos\left[\frac{\pi\times x}{2a}\right].Sin\left[\frac{\pi\times y}{2b}\right]}{2a}\right)\right\}\\
&-\left(\frac{\pi.e_{36}.\phi_0.Cos\left[\frac{\pi\times x}{2a}\right].Sin\left[\frac{\pi\times y}{2b}\right]}{2a}\right)-\left(\frac{\pi.e_{32}.\phi_0.Sin\left[\frac{\pi\times x}{2a}\right].Cos\left[\frac{\pi\times y}{2b}\right]}{2b}\right)=0 \qquad \text{(A3)}
\end{aligned}
$$

Using Galerkin error minimizing technique the value of ϕ_0 for square panel (i.e. a =b), is found in terms of w_0 as

$$\phi_0 = \frac{2\left(9a^2.h.e_{31}.w_0 - 8.h.R.e_{31}w_0^2 - 8.h.R.e_{31}w_0^2 - 16.h.R.e_{36}w_0^2\right)}{9R\left(h^2\pi^2\kappa_{11} + h^2\pi^2\kappa_{22} + 4a^2\kappa_{33}\right)} \qquad (23)$$

and as $e_{31} = e_{32}$, therefore

$$\phi_0 = \frac{2\left(9a^2.h.e_{31}.w_0 - 16.h.R.e_{31}w_0^2 - 16.h.R.e_{36}w_0^2\right)}{9R\left(h^2\pi^2\kappa_{11} + h^2\pi^2\kappa_{22} + 4a^2\kappa_{33}\right)}$$

It can be seen that Eq (23) gives a direct relation between ϕ_0 and w_0. Thus for a deformed shape/profile the potential can be directly obtained.

5. RESULTS AND DISCUSSION

Numerical results have been obtained for different values of geometrical parameter in terms of load, displacement and electric potential. Geometrical parameters are represented through non-dimensional entities η =a^2/Rh, λ =ka^4/D$_{11}$ and $\overline{\mu}$ =Ga^2/D$_{11}$. Values of material properties are as follows.

- D_{11} = 1 Nm, D_{22} =0.785 Nm, D_{66} = 0.463 Nm ,
- ν_1 =0.089, ν_2 =0.07, a= $\sqrt{20}$ m,
- h=0.1m , κ_{11} =1.531$\times 10^{-8}$ F/m,
- κ_{22} =1.531$\times 10^{-8}$ F/m, κ_{33} =1.5045$\times 10^{-8}$ F/m

Following values of non-geometrical parameters are considered:

- η = 0.25, 1.25, 2.5, λ = 20, 50, 100, 150,

- $\bar{\mu}$ = 20,50,100,150
- $e_{31} = e_{32} = -171\text{x } 10^{-12}$m/V, $171\text{x } 10^{-8}$m/V
  $-171\text{x } 10^{-5}$m/V, $171\text{x } 10^{-4}$m/V

5.1 EFFECT OF GEOMETRICAL PARAMETER η

The effects of variation in η on load-deflection response are depicted in Table 1 and Fig.3.

Table 1. Load- deflection variations for simply supported *piezoelectric* shell for different values of η

Q	w_0/h for $\eta = 0.25$	w_0/h for $\eta = 1.25$	w_0/h for $\eta = 2.5$
0	0	0	0
100	0.831	0.705	0.530
200	1.260	1.103	0.880
300	1.552	1.383	1.143
400	1.778	1.604	1.355
500	1.966	1.789	1.535

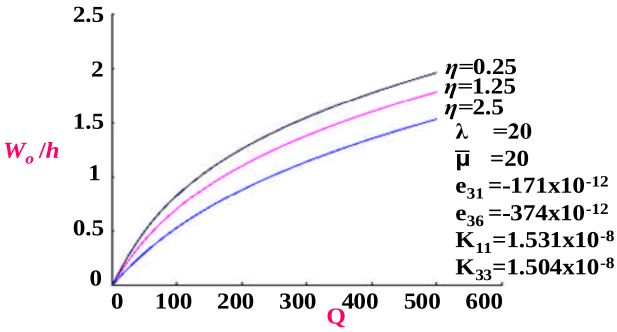


Fig 3. Load- deflection curves for simply supported piezoelectric shell for different values of η

The variation in the values of w_o/h is shown as a function of non-dimensional load Q for different values ofη. It shows a decreasing trend in non-dimensional displacement w_0/h with increase in the value of η. At Q = 500, there is 21.943% decrease in w_0/h when the value of η is increased from 0.25 to 2.5.

5.2 EFFECT OF PIEZOELECTRIC PROPERTIES.

Different values of displacement are plotted against Q for piezoelectric shell as well as for the shell without piezoelectric properties for η=0.25. As is evident from Table 2 and Fig. 4, there is a decrease in the value of w_0/h with the application of piezoelectric properties to the shell. The decrease is, however, very small. It is about 1.14% only at Q=500. This can be attributed to very small values of piezoelectric constants.

Table 2. Effect on deflection for shell with piezoelectric properties.

Q	w_0/h for simple orthotropic shell	w_0/h for piezoelectric shell
0	0	0
100	0.831	0.826
200	1.260	1.248
300	1.552	1.536
400	1.778	1.759
500	1.966	1.944

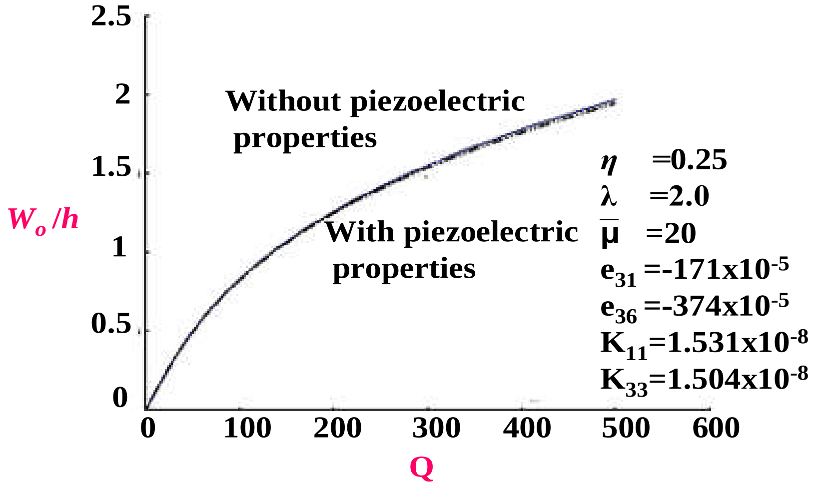


Fig 4. Load- deflection curves showing variation in deflection by applying piezoelectric properties to the shell

5.3 EFFECT OF CHANGE IN PIEZOELECTRIC CONSTANTS.

Table 3. Load- deflection variations for simply supported piezoelectric shell for different values of e_{31}.

Q	w_0/h for $e_{31}=-171\times10^{-12}$	w_0/h for $e_{31}=-171\times10^{-8}$	w_0/h for $e_{31}=-171\times10^{-5}$
0	0	0	0
100	0.831	0.831	0.826
200	1.260	1.260	1.248
300	1.552	1.552	1.536
400	1.778	1.778	1.759
500	1.966	1.966	1.944

ISSN: 2076-734X, EISSN: 2076-7366
Volume 2, Issue 1(January 2010)

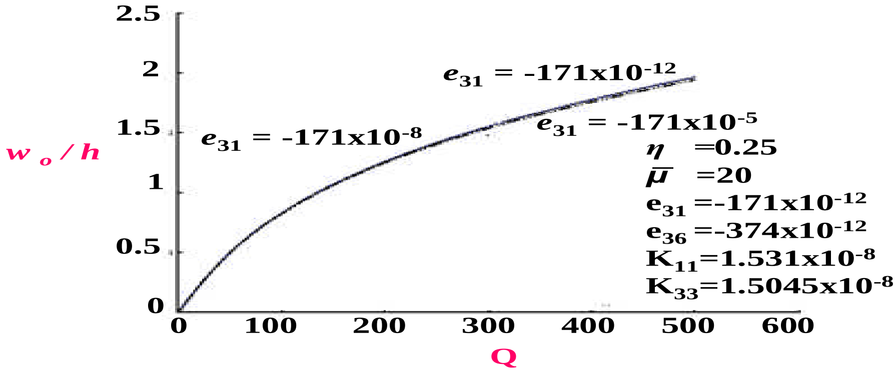


Fig.5. Load- deflection variations for simply supported piezoelectric shell for different values of e_{31}.

Variation in the piezoelectric constants e_{31} and e_{36} shows same behaviour as in case of λ as shown in fig. 4.3.3. By the application of piezoelectric constant $e_{31} = -171\times10^{-12}$ there is a negligible decrease in the values of w_0/h corresponding to Q, about 10^{-7} % at Q = 500. When the constant is increased to $e_{31} = -171\times10^{-8}$, the change is still negligible. The increase to the value of $e_{31} = -171\times10^{-5}$ shows the decrease of about 1.0919 %. The small decrease in the values of w_0/h may be attributed to the very small values of piezoelectric constants that are taken as compared to the values of PZT-5A..

5.4 EFFECT OF NON-DIMENSIONAL FOUNDATION PARAMETER λ

Effects of variation in λ on load-deflection response of the shallow cylindrical shell are presented in Table 4 and Fig .6

Table 4. Load- deflection variation for simply supported piezoelectric shell as a function of different values of λ.

Q	w_0/h for $\lambda =$ 20	w_0/h for $\lambda = 5$ 0	w_0/h for $\lambda = 10$ 0
0	0	0	0
100	0.831	0.750	0.546
200	1.260	1.182	0.952
300	1.552	1.480	1.257
400	1.778	1.711	1.499
500	1.966	1.903	1.701

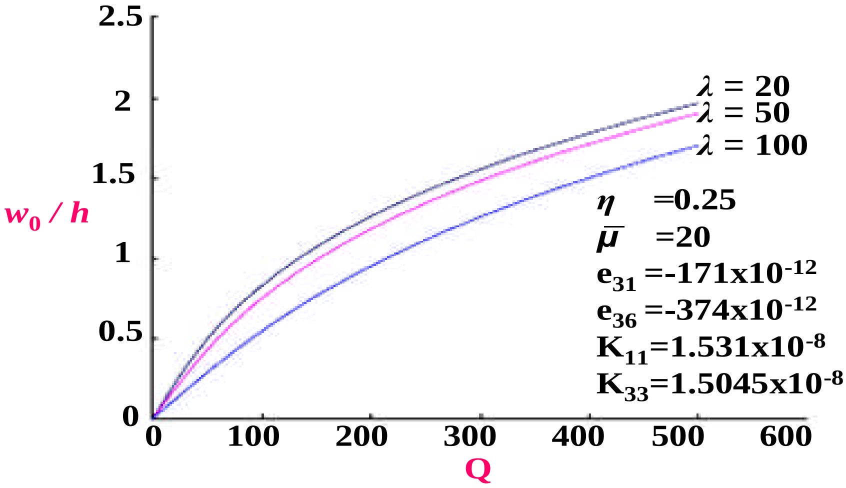


Fig 6. Load- deflection curves for simply supported piezoelectric shell for different values of λ.

The variation in the values of w_0/h is shown as a function of non- dimensional load Q for different values of λ. It is shown that with an increase in the value of λ from 20 to 50, the value of w_0/h decreases by 3.19 % at Q= 500 and 13.48 % at Q=1000.

5.5 EFFECT OF PIEZOELECTRIC PROPERTIES.

Similarly the effects of piezoelectric properties are shown in Table 5 and Fig 7. Different values of displacement are plotted against Q for piezoelectric shell as well as shell without piezoelectric properties, for constant $e_{31} = -171\times10^{4}$. It is evident from the Fig. 1 and Table 1 that there is a decrease in the value of w_0/h with the application of piezoelectric properties to the shell. The decrease is about 36.97 % at Q =500.

Table 5. Load- deflection variations for simply supported orthotropic shell and piezoelectric shell.

Q	w_0/h for non-piezoelectric shell	w_0/h for piezoelectric shell
0	0	0
100	0.750	0.571
200	1.182	0.811
300	1.480	0.971
400	1.711	1.096
500	1.903	1.200

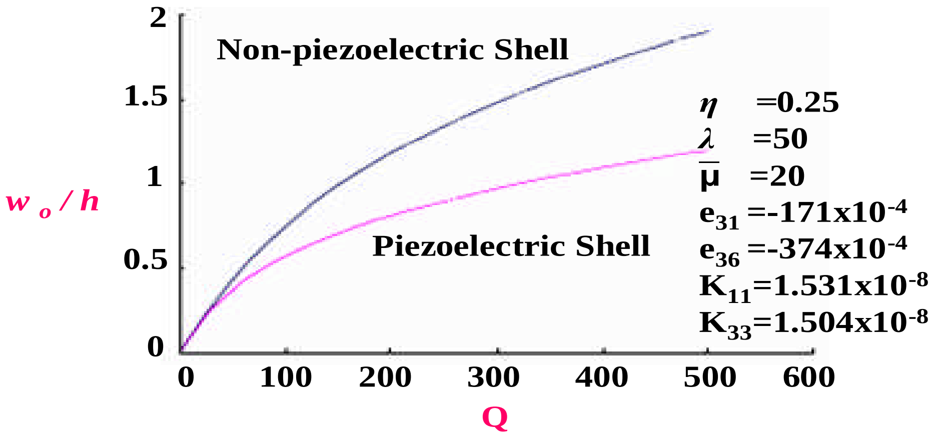


Fig 7. Load- deflection curves for simply supported orthotropic shell and piezoelectric shell

6 EFFECT OF CHANGE IN PIEZOELECTRIC CONSTANTS.

Variation in piezoelectric constant e_{31} presents very interesting results as shown in Fig. 8 and Table 6. When the piezoelectric constant $e_{31} = -171 \times 10^{-12}$, there is a negligible decrease in the value of w_0/h. When this constant is increased to $e_{31} = -171 \times 10^{-8}$, the change is still negligible. But, when the value of $e_{31} = -171 \times 10^{-4}$, there is an unexpected decrease in the values of w_0/h. It is about 36.97% at Q=500.

Table 6. Load- deflection variations for different values of e_{31}

Q	w_o /h for e_{31} =-171x10^{-12}	w_o /h for e_{31}=-171x10^{-8}	w_o /h for e_{31}=-171x10^{-5}	w_o /h for e_{31}=-171x10^{-4}
0	0	0	0	0
100	0.750	0.751	0.746	0.572
200	1.182	1.183	1.173	0.811
300	1.480	1.480	1.466	0.972
400	1.712	1.712	1.694	1.096
500	1.904	1.904	1.883	1.200

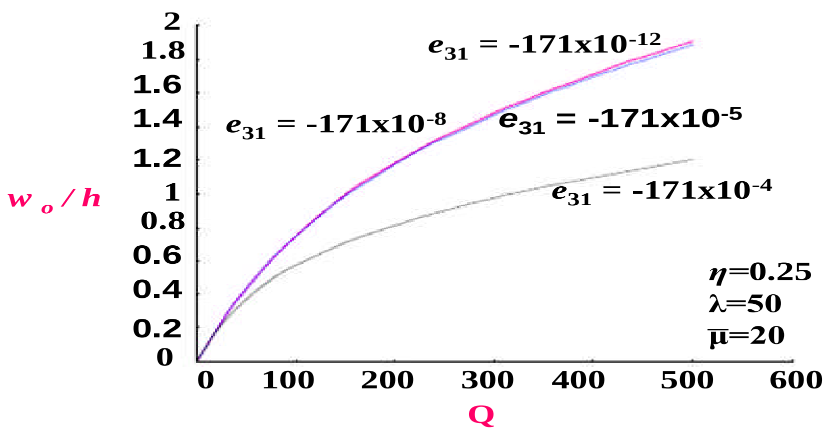


Fig 8. Load- deflection curves for simply supported piezoelectric shell for different values of e_{31}

6.1 EFFECT OF GEOMETRICAL PARAMETER $\bar{\mu}$

Variation in the values of $\bar{\mu}$ is shown in Table 7 and Fig 9. It shows that with increase in the value of $\bar{\mu}$ from 20 to 50, w_0/h decreases by 15.26% at Q = 500, by 36.57% at Q=100, and by 51.6% at Q=150.

Table 7. Load- deflection curves for simply supported piezoelectric shell for different values of $\bar{\mu}$

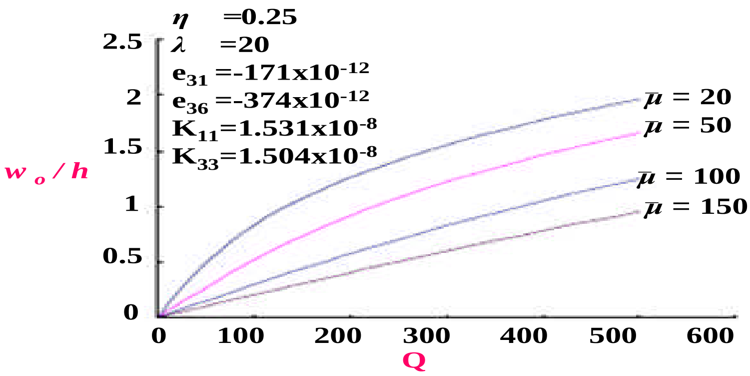


Fig 9. Load- deflection curves for simply supported piezoelectric shell for different values of $\bar{\mu}$.

7 EFFECT OF PIEZOELECTRIC PROPERTIES.

Different values of displacement are plotted against Q for piezoelectric shell as well as for the shell without piezoelectric properties for the constant $e_{31} = -171\times10^{-4}$. As is evident from Table 8 and Fig. 10, there is a decrease in the value of w_0/h with the application of piezoelectric properties to the shell. The decrease is about 0.88% at Q =500.

Table 8. Load- deflection curves for simply supported shell with and without piezoelectric properties.

Q	w_0/h for orthotropic shell	w_0/h for piezoelectric shell
0	0	0
100	0.518	0.517
200	0.917	0.912
300	1.220	1.212
400	1.463	1.451
500	1.666	1.651

Q	w_0/h for $\bar{\mu}=20$	w_0/h for $\bar{\mu}=50$	w_0/h for $\bar{\mu}=100$	w_0/h for $\bar{\mu}=150$
0	0	0	0	0
100	0.831	0.518	0.297	0.205
200	1.260	0.917	0.574	0.406
300	1.552	1.220	0.825	0.598
400	1.778	1.463	1.048	0.780
500	1.966	1.666	1.247	0.951

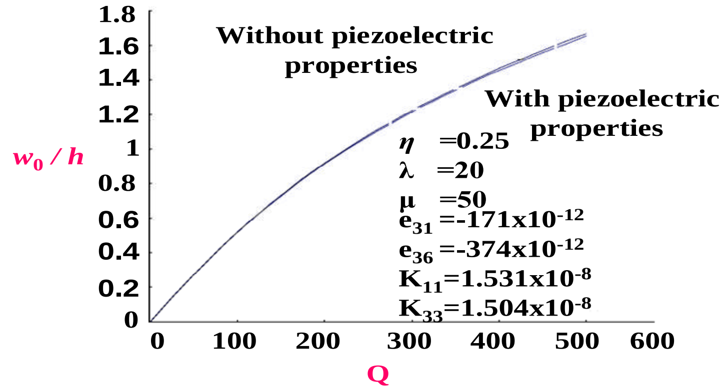


Fig 10. Load- deflection curves for simply supported shell with and without piezoelectric properties.

8 EFFECT OF CHANGE IN PIEZOELECTRIC CONSTANTS.

Variation in the value of piezoelectric constants e_{31} is shown in Table 9 and Fig 11. By taking $e_{31} = -171\times10^{-12}$, there is negligible decrease in the value of w_0/h. But if this value is taken as $e_{31} = -171\times10^{-4}$ there is an unexpected decrease in it. It is about 32.96% at Q=500. Thus, it can be inferred that the effect of increase in the value of piezoelectric constant is to reduce the value of w_0/h.

Table 9. Load- deflection variations for simply supported piezoelectric shell for different values of e_{31}.

Q	w_0/h for $e_{31} = -171\times10^{-12}$	w_0/h for $e_{31} = -171\times10^{-8}$	w_0/h for $e_{31} = -171\times10^{-4}$
100	0.518	0.518	0.451
200	0.917	0.917	0.703
300	1.220	1.220	0.875
400	1.463	1.463	1.007
500	1.666	1.666	1.117

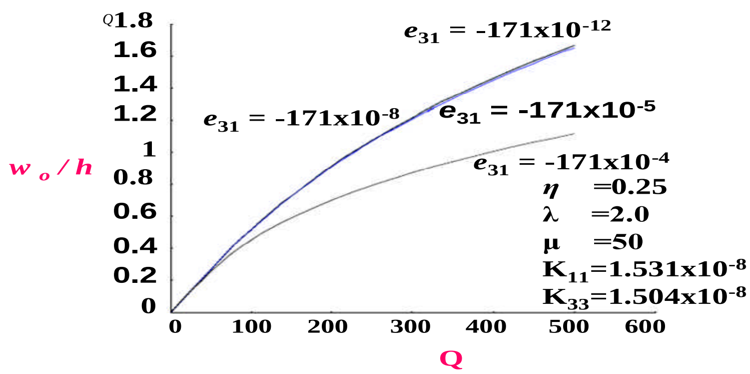


Fig 11. Load- deflection curves for simply supported piezoelectric shell for different values of e_{31}.

9. NON-DIMENSIONAL LOAD VERSUS ELECTRIC POTENTIAL CHARACTERISTICS

9.1 EFFECT OF VARIATION IN η .

The effect of variation in η is shown in Fig 12. and Table 10 clearly depicts that there is a decreasing trend in the value of ϕ_0 with increase in the value of η. For non-dimensional load Q = 500, the decrease is 25.77%

when η is increased from 0.25 to 1.25, a decrease of 55.58% when η is increased to 2.5. Thus it can be asserted that with increase in η, the value of electric potential ϕ_0 decreases.

Table 10. Electric potential non-dimensional load variations with different values of η

Q	$\phi_0 \times 10^6$ for $\eta = 0.25$	$\phi_0 \times 10^6$ for $\eta = 1.25$	$\phi_0 \times 10^6$ for $\eta = 2.5$
0	0	0	0
100	1.051	0.550	0.096
200	2.461	1.564	0.621
300	3.758	2.584	1.288
400	4.954	3.566	1.989
500	6.072	4.507	2.697

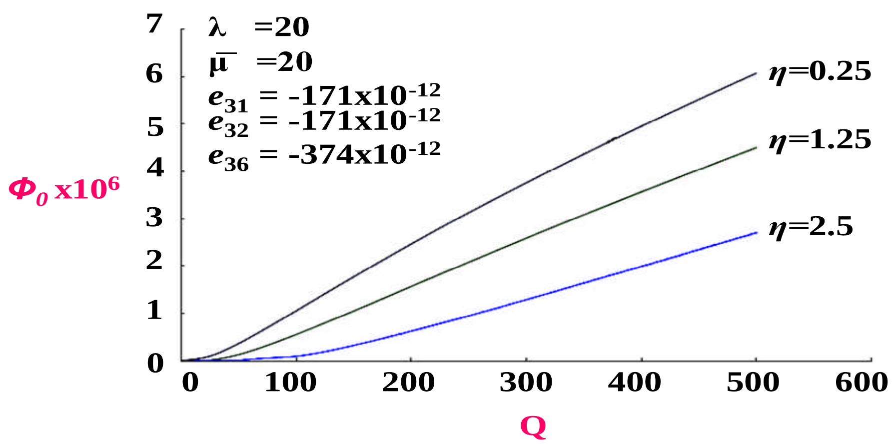


Fig 12. Electric potential non-dimensional load characteristic

10 EFFECT OF GEOMETRICAL PARAMETER λ.

The effect of variation in the values of λ on ϕ_0 is shown in table 11 and Fig.13. With increase in the value of λ from 20 to 50, the value of ϕ_0 increases by 6.35% at Q = 500, by 16.3% at Q=100, and by 25.41% at Q=150.

Table 11. Electric potential versus non-dimensional load variations for different values of λ

Q	ϕ_0 at $\lambda = 20$	ϕ_0 at $\lambda = 50$	ϕ_0 at $\lambda = 100$	ϕ_0 at $\lambda = 150$
0	0	0	0	0
100	1.051	0.851	0.605	0.440
200	2.461	2.161	1.734	1.390
300	3.758	3.414	2.897	2.450
400	4.954	4.585	4.015	3.505
500	6.072	5.686	5.082	4.529

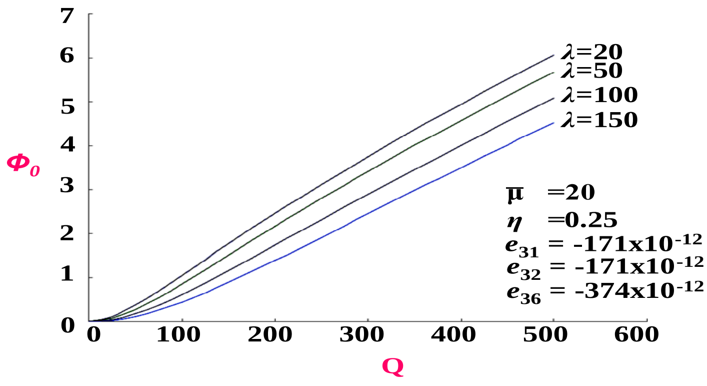


Fig 13. Electric potential non-dimensional load curve for different λ

11 EFFECT OF CHANGE IN PIEZOELECTRIC CONSTANTS

The effects of variation in piezoelectric constants e_{31} are shown in Table12 and Fig 14. With $e_{31} = -171\times10^{-12}$, there is negligible increase in the value of ϕ_0 corresponding to Q. It is about 5.68 x 10^{-6} at Q = 500. When the constant is increased to $e_{31} = -171\times10^{-8}$, the value of ϕ_0 increases by 82.82 % at Q = 500. The increase in the value of $e_{31} = -171\times10^{-5}$ shows an increase of about 10^7 %. And if this value is increased to $e_{31} = -171\times10^{-4}$, there is an unexpected increase in the value of ϕ_0. It is about 3.19 x 10^7 % at Q = 500. Thus, it can be inferred that the effect of increase in the value of piezoelectric constant is to increase the value of ϕ_0.

Table 12. Electric potential – non-dimensional load variations for different values of piezoelectric constants

Q	$\phi_0 \times10^6$ at $e_{31} = -171\times10^{-12}$	ϕ_0 at $e_{31} = -171\times10^{-8}$	ϕ_0 at $e_{31} = -171\times10^{-4}$
0	0	0	0
100	0.851	0.008	48.409
200	2.161	0.021	99.934
300	3.414	0.034	144.78
400	4.585	0.045	185.3
500	5.686	0.056	222.766

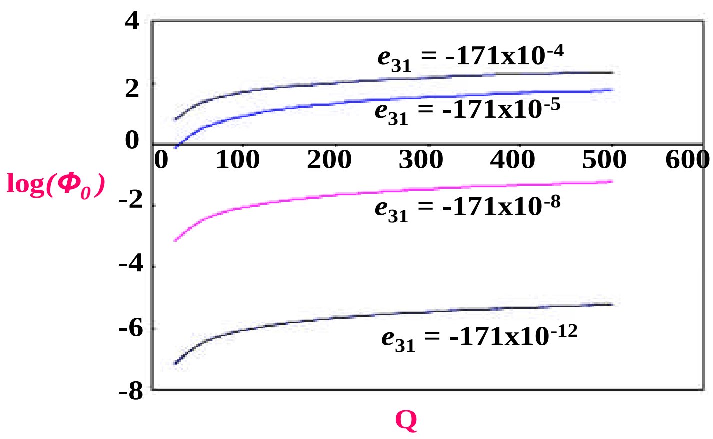


Fig 14. Logarithmic electric potential versus non-dimensional load curves for different values of piezoelectric constants.

11.1 EFFECT OF GEOMETRICAL PARAMETER $\bar{\mu}$.

The variation in the values of $\bar{\mu}$ shows the same trend towards change in the values of ϕ_0 as shown by η and λ. With an increase in the value of $\bar{\mu}$ from 20 to 50, the ϕ_0 increases by 28.49% at Q = 500, by 60.31% at Q=100, and by 77.15% at Q=150. The trend is shown in Table 12 and Fig 14.

Table 13. Electric potential – non-dimensional load variations for different values of $\bar{\mu}$

Q	$\phi_0 \times 10^6$ for $\bar{\mu}$ =20	$\phi_0 \times 10^6$ for $\bar{\mu}$ =50	$\phi_0 \times 10^6$ for $\bar{\mu}$ =100	$\phi_0 \times 10^6$ for $\bar{\mu}$ =150
0	0	0	0	0
100	1.051	0.394	0.120	0.053
200	2.461	1.285	0.489	0.236
300	3.758	2.305	1.036	0.532
400	4.954	3.335	1.691	0.923
500	6.072	4.341	2.410	1.387

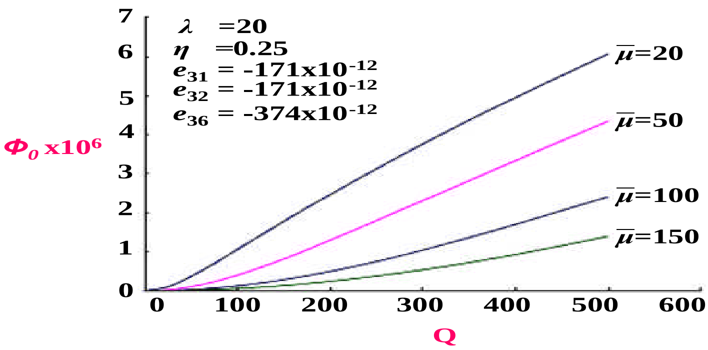


Fig 15. Electric potential versus non-dimensional load curves for different values of μ

CONCLUSIONS

Based on the results obtained, the following conclusions can be drawn.

(1). Geometric parameter η has an adverse effect on the displacement of the shell. It decreases with increase in the value of η. It is about 9.02 % when η increases from 0.25 to 1.25, and 21.94 % when η increased to 2.5. Thus it can also be inferred that with increase in the ratio of a/h or a/R, the deflection decreases.

(2). Geometric parameter/foundation parameter λ also has the same effect as η. When λ increases from 20 to 50, it decreases by 3.19 %, and when λ becomes 100, it decreases by 13.48 %. Thus with increase in spring constant of the foundation, the deflection decreases.

(3). Effect of $\bar{\mu}$ can be summarized as follow. It is 15.26 % decrease for increase of $\bar{\mu}$ from 20 to 50, 36.57% decrease for $\bar{\mu}$ = 100, and 51.60% decrease for $\bar{\mu}$ =150. Thus it can be said that an increase in the value of shear modulus of the foundation layer adversely affects the value of deflection.

(4). Increase in the value of geometric parameter η has positive effect on the electric potential. It increases by 25.77 % for an increase in the value of η from 0.25 to 1.25, and 55.58 % for increase in η from 0.25 to 1.25.

(5). Increase in the value of geometric parameter/foundation parameter λ also has a positive effect on electric potential. It increases by 6.35 % for an increase in λ from 20 to 50, 16.3 % for increase in λ from 20 to 100, and 25.41 % for λ increasing to 150.

(6). Increase in the value of geometric parameter $\overline{\mu}$ has same effect on electric potential as η and λ. It increases by 28.49 % for increase in $\overline{\mu}$ from 20 to 50, by 60.31 % for increase in $\overline{\mu}$ from 20 to 100, and 77.15 % for $\overline{\mu}$ increasing to 150.

(7). Increase in the values of piezoelectric constants decreases the value of deflection and increases electric potential.

(6). Increase in the value of geometric parameter $\overline{\mu}$ has same effect on electric potential as η and λ. It increases by 28.49 % for increase in $\overline{\mu}$ from 20 to 50, by 60.31 % for increase in $\overline{\mu}$ from 20 to 100, and 77.15 % for $\overline{\mu}$ increasing to 150.

(7). Increase in the values of piezoelectric constants decreases the value of deflection and increases electric potential.

NOMENCLATURE

A_{ij}	
a , b	Dimensions of cylindrical shell
$[C]$	Elastic constant matrix
c_{ij}	Elastic constants
$D_1, D_2, \& D_k$	Flexural rigidities of the orthotropic shell $\frac{E_1h^3}{12(1-\nu_1\nu_2)}$ and $\frac{E_2h^3}{12(1-\nu_1\nu_2)}$
respectively. $\{D\}$	Electrical displacement vector
D_x, D_y, D_z	Electrical displacement in x, y and z directions, respectively
E_1,E_2	Young's Modulii of elasticity in orthogonal directions
$\{E\}$	Electric field vector
E^o	Applied uniform electric field in radial direction
$[e]$	Piezoelectric stress coefficient matrix
e_{ij}	Piezoelectric stress coefficients
e_1	First invariant of middle surface strains
e_2	Second invariant of middle surface strains
F	Integrand
f	Sag, the highest elevation above the base
G	Shear modulus of the foundation
H	Thickness of the orthotropic shell
k	Foundation Modulus
M	Moments per unit length
N	Normal forces per unit length
$[\kappa]$	Dielectric matrix. i.e. $\kappa_{11,}\kappa_{22}$ and κ_{33}

l_{min}	Minimum dimension of the shell in the plan
Q	Non-dimensional load
q	Uniform normal pressure on the concave side of the shell
R	Radius of curvature of cylindrical *shell*
U	Potential energy
u, v, w	Displacement components in r, θ and z directions respectively
w_o	Maximum deflection at the center of the shell
r, θ, z	Cylindrical coordinates
x, y, z	Rectangular coordinates
z	Distance of the middle surface of the shell from the base plane
ϵ_{ij}	Strains
$\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$	Shear strains in xy, yz and zx planes, respectively
$\{\varepsilon\}$	Strain vector
η	Geometric parameter
λ	Non-dimensional foundation modulus
$\overline{\mu}$	Non-dimensional (foundation) shear modulus, Ga^2/D_{11}
$\nu_1 \& \nu_2$	Poisson's ratio in orthogonal directions.
$\overline{\nu}$	$\sqrt{\nu_1 \times \nu_2}$
σ_{ij}	Stress
$\{\sigma\}$	Stress vector
ϕ_0	Increment of the electric potential from an initial State.

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