RELIABILITY OF REPAIRABLE k–out–of–n: F SYSTEM HAVING DISCRETE REPAIR AND FAILURE TIMES DISTRIBUTIONS

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ABSTRACT
In this paper, repairable k – out – of – n:F system with single repairman is studied. The failure and repair times of components are assumed as geometric distribution which is one of the discrete distributions. Both the generalized and steady state transition probabilities of the states for the related system are obtained. The recursive relations for the reliability of the system are generated. Also, using geometric transformation technique, the MTSF (Mean Time System Failure) of the system is obtained. As an example of this system, 3 – out – of – 5:F system is considered. The transition probabilities, reliability and MTSF of this system are obtained and some graphs for the reliability and MTSF are drawn for different values of failure and repair time parameters.

Keywords: *Repairable k – out – of – n:F system, geometric distribution, reliability, MTSF*

1. INTRODUCTION

The k – out – of – n:F system is one of the mostly interested engineering system in reliability theory. A k – out – of – n:F system consists of a sequence of n ordered components such that the system is failed if and only if at least k components in the system are failed. Parallel and series systems are special cases of the k – out – of – n system. A series system is equivalent to a 1 – out – of – n:F system and a parallel system is equivalent to an n – out – of – n:F system. Therefore, parallel and series systems form special cases of the k – out – of – n:F system. For some examples and references, Optimal Reliability Modeling (Kuo and Zou [1]) can be reviewed.

Although various research studies have been performed for the different system structures such as k – out – of – n system, consecutive k – out – of – n system, recently some attention has been paid on for repairable systems. Zhang and Wang [2] and Zhang et al. [3] studied a linear and a circular consecutive –2 – out – of – n:F repairable system, respectively. Lam and Zhang ([4], [5]) considered a consecutive –2 – out – of – n:F repairable system with Markov dependence. Similar works considered different assumptions (fuzzy state assumption) and structure (consecutive –k – out – of – n:G repairable system) are appeared in literature (Guan and Wu [6], Zhang and Lam [7]). Eryilmaz [8] computed some reliability indices (availability, rate of occurrence of failure, mean time to the first failure) of repairable systems using signature based analysis. He attributed the limitation of his work to the working and repair times of components are assumed to be exponentially distributed.

In the aforementioned studies have been made under the assumption that the failure and repair time of the components follow an exponential distribution. However, in this study, we are interested in the reliability of a k – out – of – n:F repairable system when failure and repair times of components are geometrically distributed.

Recently, some studies considering the failure and repair time of the components have geometric distribution have been made. However, some small system structures have been assumed in these studies to simply the results. Eryilmaz [9] studied on cold standby repairable system when both working and repair time of the components have geometric distribution. Bhatti et al. ([10], [11], [12], [13]) and the references given in these studies considered some basic system structures such as single unit redundant system, two identical unit cold standby systems, two identical / non-identical parallel systems using different inspection policies and calculated various measures of reliability i.e MTSF, steady state availability, busy period of repairman and inspector, profit function assuming discrete working and repair time distributions.

In this study, repairable k – out – of – n:F system with single repairman is considered. The failure and repair times of components are taken as discrete random variables having geometric distribution. Under given assumptions in this paper, the generalized transition probabilities of the states for repairable k – out – of – n:F system are derived. 3 – out – of – 5:F system is considered as an example of repairable k – out – of – n:F system. The transition probabilities, reliability end MTSF (Mean Time System Failure) of this system are obtained. For the general case k – out – of – n:F system, the reliability is also derived.

2. NOTATIONS

- n: Number of components in the system
- k: Minimum number of working components required for the system to work

$N(t)$: The state of the system at time t
S_i: i th state of the system
$q_{ij}(t)/Q_{ij}(t)$: p.m.f / c.d.f. of direct transition time from state S_i to state S_j during time interval $(0,t)$
P_{ij}: Steady state transition probability from state S_i to state S_j during time interval $(0,t)$
pq^t: p.m.f. of failure time of any component $(p+q=1)$
rs^t: p.m.f. of repair time of any component $(r+s=1)$

3. MODEL DESCRIPTION AND ASSUMPTIONS

In this paper we consider a repairable k-out-of- n: F system. The failure and repair times of components are taken as discrete random variables having geometric distributions with different parameters. The following assumptions according to the model are as follows:

I. The system is a k-out-of- n: F system with identical components. Each component has only two states: working or failed. The system fails if and only if at least k components in the system fail.
II. At time $t=0$, all components are new and working.
III. There is a single repairman in the system. Each component after being repaired is "as good as new".
IV. When the system is in the failed state, the non-failed components will not fail any longer.
V. More than one failure may occur at the same time period.

Considering the model assumptions given above, we can define the state of the system by time t according to the number of failed components in the system. It is denoted by $N(t)$ and can be defined by the following way:

$$N(t)=\begin{cases} 0, & \text{if at time } t\text{, all components work, the system works,}\\ -1, & \text{if at time } t\text{, one component fails, the system works,}\\ \vdots & \\ -(k-1), & \text{if at time } t\text{, (k-1) components fail, the system works,}\\ k, & \text{if at time } t\text{, k components fail, the system fails.}\\ \vdots & \\ n, & \text{if at time } t\text{, all components fail, the system fails.}\end{cases}$$

Then, $N(t)$ forms a discrete time homogeneous Markov chain with the state space $\Omega=\{-(k-1),-1,0,k,\dots,n\}$. The set of working states is $W=\{-(k-1),\dots,0\}$ and the set of failure states $F=\{k,\dots,n\}$. Let's consider 3–out–of–5:F system as an example of repairable k – out – of – n:F system. Considering the possible values of $N(t)$ and possible states of the 3–out–of–5:F system, the transition diagram of the system can be shown with the following figure.
(◯ working state, ▭: failed state)

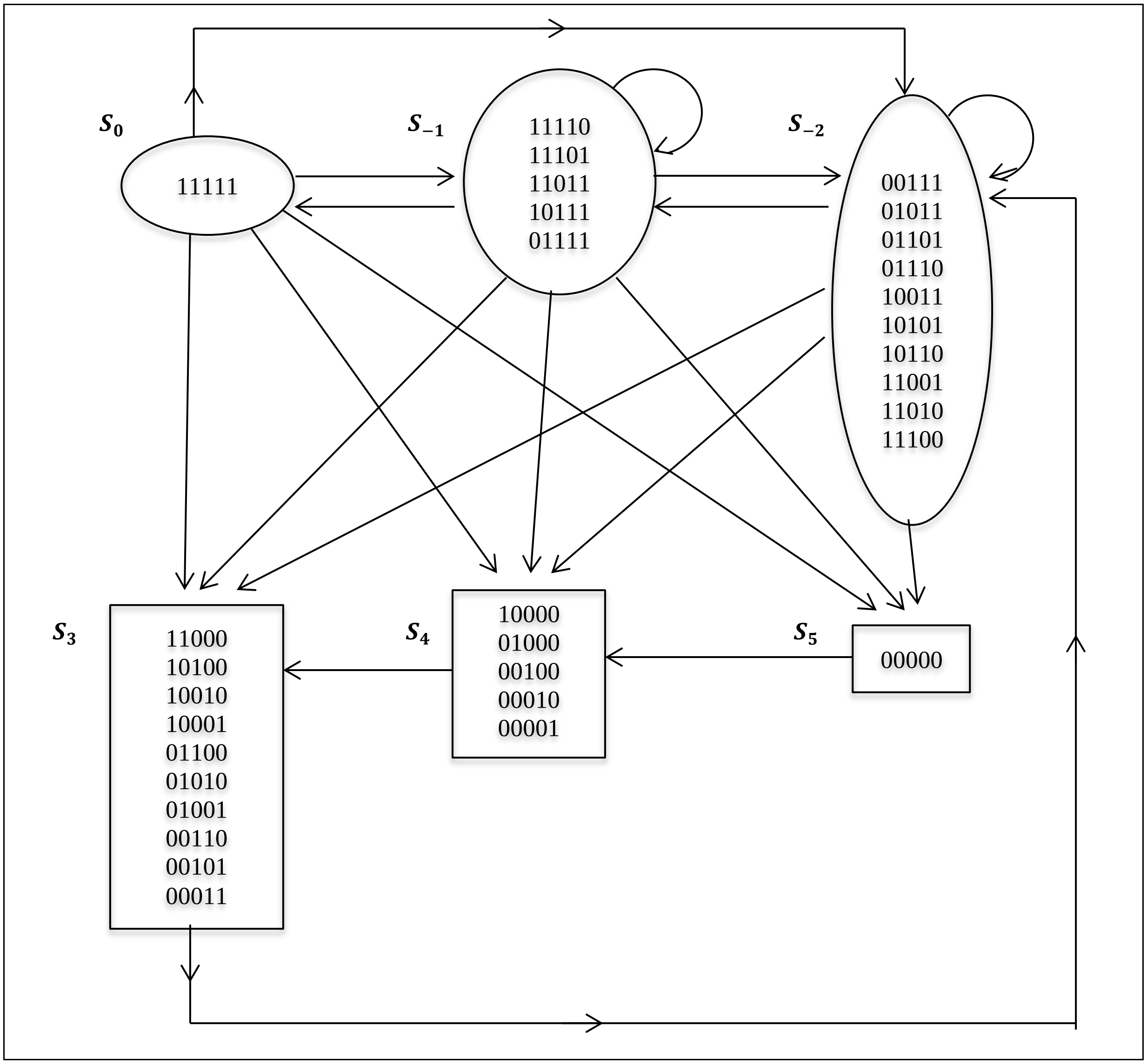

***Figure 1**: The transition diagram of 3–out–of–5:F system*

Similar diagram given in Figure 1 also can be drawn for the general case of k – out – of – n:F system.

4. TRANSITION PROBABILITIES

Let $Q_{ij}(t)$ be the probability that the system transits from S_i to S_j during time interval $(0, t)$, i.e., if T_{ij} is the transition time from S_i to S_j then,

$$Q_{ij}(t) = P(T_{ij} \leq t).$$

Using this definition and simple probabilistic arguments, transition probabilities of k – out – of – n:F system is obtained as

$$Q_{0\,j}(t) = P\left(T_{0\,j} \leq t\right) = \binom{n}{j} p^j q^{n-j} \frac{1-(q^n)^{t+1}}{1-q^n}, \quad j = -1, -2, \ldots, -(k-1), k, \ldots, n$$

$$Q_{-i\,j}(t) = P\left(T_{-i\,j} \le t\right) = \begin{cases} rq^{n-i}\,\dfrac{1-(q^{n-i}s)^{t+1}}{1-q^{n-i}s}, & j=-(i-1) \\ \dbinom{n-i}{1} rpq^{n-i-1}\,\dfrac{1-(q^{n-i}s)^{t+1}}{1-q^{n-i}s}, & j=-i \\ \left[\dbinom{n-i}{j-i} p^{j-i}q^{n-j+i-1}s + \dbinom{n-i}{j-i+1} p^{j-i+1}q^{n-j+i-2}r\right]\dfrac{1-(q^{n-i}s)^{t+1}}{1-q^{n-i}s}, & j=-(i+1),\dots,-(k-1),k,\dots,n-1 \\ p^{n-i}s\,\dfrac{1-(q^{n-i}s)^{t+1}}{1-q^{n-i}s}, & j=n \end{cases}$$

for $i = 1, 2, \dots, (k-1)$ and

$$Q_{i\,j}(t) = 1-(s)^{t+1} \text{ for } i=k,\; j=-(k-1).$$

The steady state transition probabilities from state S_i to state S_j can be obtained by taking t$\to\infty$. That is $P_{ij} = \lim_{t\to\infty} Q_{ij}(t)$.

$$P_{0\,j} = \frac{\binom{n}{j}p^jq^{n-j}}{1-q^n}, \quad j=-1,-2,\dots,-(k-1),k,\dots,n$$

$$P_{-i\,j} = \begin{cases} \dfrac{rq^{n-i}}{1-q^{n-i}s}, & j=-(i-1) \\ \dfrac{\binom{n-i}{1} rpq^{n-i-1}}{1-q^{n-i}s}, & j=-i \\ \dfrac{\left[\binom{n-i}{j-i} p^{j-i}q^{n-j+i-1}s + \binom{n-i}{j-i+1} p^{j-i+1}q^{n-j+i-2}r\right]}{1-q^{n-i}s}, & j=-(i+1),\dots,-(k-1),k,\dots,n-1 \\ \dfrac{p^{n-i}s}{1-q^{n-i}s}, & j=n \end{cases}$$

for $i = 1, 2, \dots, (k-1)$ and

$$P_{i\,j} = 1 \text{ for } i=k,\; j=-(k-1).$$

As an illustration, the steady state transition probabilities of 3–out–of–5:F system are obtained and given below.

$$P_{0-1} = \frac{5pq^4}{1-q^5}, \quad P_{0-2} = \frac{10p^2q^3}{1-q^5}, \quad P_{03} = \frac{10p^3q^2}{1-q^5}, \quad P_{04} = \frac{5p^4q}{1-q^5}, \quad P_{05} = \frac{p^5}{1-q^5}$$

$$P_{-10} = \frac{rq^4}{1-q^4s}, \quad P_{-1-1} = \frac{4pq^3r}{1-q^4s}, \quad P_{-1-2} = \frac{4pq^3s+6p^2q^2r}{1-q^4s}, \quad P_{-13} = \frac{6p^2q^2s+4p^3qr}{1-q^4s}, \quad P_{-14} = \frac{4p^3qs+p^4r}{1-q^4s}, \quad P_{-15} = \frac{p^4s}{1-q^4s}$$

$$P_{-2-1} = \frac{rq^3}{1-q^3s}, \quad P_{-2-2} = \frac{3pq^2r}{1-q^3s}, \quad P_{-23} = \frac{3pq^2s+3p^2qr}{1-q^3s}, \quad P_{-24} = \frac{3p^2qs+p^3r}{1-q^3s}, \quad P_{-25} = \frac{p^3s}{1-q^3s}$$

$$P_{3-2} = P_{43} = P_{54} = 1 \qquad (1)$$

We can observe the following relations:

$$P_{0-1}+P_{0-2}+P_{03}+P_{04}+P_{05}=1\,, \qquad P_{-10}+P_{-1-1}+P_{-1-2}+P_{-13}+P_{-14}+P_{-15}=1$$

$$P_{-2-1}+P_{-2-2}+P_{-23}+P_{-24}+P_{-25}=1, \qquad P_{3-2}=P_{43}=P_{54}=1$$

Let T_i be the sojourn time in state S_i, then mean sojourn time in state S_i is defined by

$$\mu_i=\sum_{t=0}^{\infty}P(T_i>t)$$

so that for 3–out–of–5:F system μ_i's $(i=0,-1,-2,3,4,5)$ can be obtained as

$$\mu_0=\frac{1}{1-q^5}\,,\quad \mu_{-1}=\frac{1}{1-q^4s}\,,\quad \mu_{-2}=\frac{1}{1-q^3s}\,,\quad \mu_3=\mu_4=\mu_5=\frac{1}{1-s} \tag{2}$$

5. RELIABILITY EQUATIONS OF THE SYSTEM

In here, $R_i(t)$ denotes the probability that the system does not fail up to epochs $0,1,2,\dots(t-1)$ when it is initially started from up state S_i. To determine the reliability of the system, we regard the failed states $S_k,S_{k+1},\dots,S_n$ as absorbing states. The expressions for $R_i(t)$; $i=0,-1,-2,\dots,-(k-1)$; we have the following set of equations:

$$R_0(t)=(q^n)^t+\sum_{u=0}^{t-1}q_{0-1}(u)R_{-1}(t-1-u)+\sum_{u=0}^{t-1}q_{0-2}(u)R_{-2}(t-1-u)+\cdots+\sum_{u=0}^{t-1}q_{0-(k-1)}(u)R_{-(k-1)}(t-1-u) \tag{3}$$

$$R_{-1}(t)=(q^{n-1}s)^t+\sum_{u=0}^{t-1}q_{-10}(u)R_0(t-1-u)+\sum_{u=0}^{t-1}q_{-1-1}(u)R_{-1}(t-1-u)+\cdots+\sum_{u=0}^{t-1}q_{-1-(k-1)}(u)R_{-(k-1)}(t-1-u) \tag{4}$$

$$R_{-2}(t)=(q^{n-2}s)^t+\sum_{u=0}^{t-1}q_{-2-1}(u)R_{-1}(t-1-u)+\sum_{u=0}^{t-1}q_{-2-2}(u)R_{-2}(t-1-u)+\cdots+\sum_{u=0}^{t-1}q_{-2-(k-1)}(u)R_{-(k-1)}(t-1-u) \tag{5}$$

$$\vdots$$

$$R_{-(k-1)}(t)=\left(q^{n-(k-1)}s\right)^t+\sum_{u=0}^{t-1}q_{-(k-1)-(k-2)}(u)R_{-(k-2)}(t-1-u)+\sum_{u=0}^{t-1}q_{-(k-1)-(k-1)}(u)R_{-(k-1)}(-1-u) \tag{6}$$

Using the probability mass function of one step or direct transition time from S_i to S_j in these equations, we can get the compact form of the above equations:

$$R_0(t)=(q^n)^t+\sum_{i=1}^{k-1}\sum_{u=0}^{t-1}\binom{n}{i}p^iq^{n-i}(q^n)^uR_{-i}(t-1-u)$$

$$R_{-1}(t) = (q^{n-1}s)^t + \sum_{i=0}^{k-1}\sum_{u=0}^{t-1}\binom{n-1}{i} rp^i q^{n-i-1}(q^{n-1}s)^u R_{-i}(t-1-u)$$
$$+\sum_{i=1}^{k-2}\sum_{u=0}^{t-1}\binom{n-1}{i} p^i q^{n-i-1} s (q^{n-1}s)^u R_{-(i+1)}(t-1-u)$$

$$R_{-2}(t) = (q^{n-2}s)^t + \sum_{i=0}^{k-2}\sum_{u=0}^{t-1}\binom{n-2}{i} rp^i q^{n-i-2}(q^{n-2}s)^u R_{-(i+1)}(t-1-u)$$
$$+\sum_{i=1}^{k-3}\sum_{u=0}^{t-1}\binom{n-2}{i} p^i q^{n-i-2} s (q^{n-2}s)^u R_{-(i+2)}(t-1-u)$$

$$\vdots$$

$$R_{-(k-1)}(t) = \left(q^{n-(k-1)}s\right)^t + \sum_{i=0}^{1}\sum_{u=0}^{t-1}\binom{n-(k-1)}{i} rp^i q^{n-(k-1)-i}(q^{n-(k-1)}s)^u R_{-(i+k-2)}(t-1-u)$$

For the 3–out–of–5:F system, we can write the following equations using above equations:

$$R_0(t) = (q^5)^t + \sum_{u=0}^{t-1} 5pq^4(q^5)^u R_{-1}(t-1-u) + \sum_{u=0}^{t-1} 10p^2q^3(q^5)^u R_{-2}(t-1-u) \qquad (7)$$

$$R_{-1}(t) = (q^4s)^t + \sum_{u=0}^{t-1} rq^4(q^4s)^u R_0(t-1-u) + \sum_{u=0}^{t-1} 4rpq^3(q^4s)^u R_{-1}(t-1-u) + \sum_{u=0}^{t-1}(6rp^2q^2(q^4s)^u + 4pq^3s(q^4s)^u) R_{-2}(t-1-u) \qquad (8)$$

$$R_{-2}(t) = (q^3s)^t + \sum_{u=0}^{t-1} rq^3(q^3s)^u R_{-1}(t-1-u) + \sum_{u=0}^{t-1} 3rpq^2(q^3s)^u R_{-2}(t-1-u) \qquad (9)$$

In here, it is clear that $R_0(0) = R_{-1}(0) = R_{-2}(0) = 1$.

Using the equations (7), (8) and (9), the curves for the reliability of 3–out–of–5:F system have been drawn for different values of parameters with respect to time and given below.

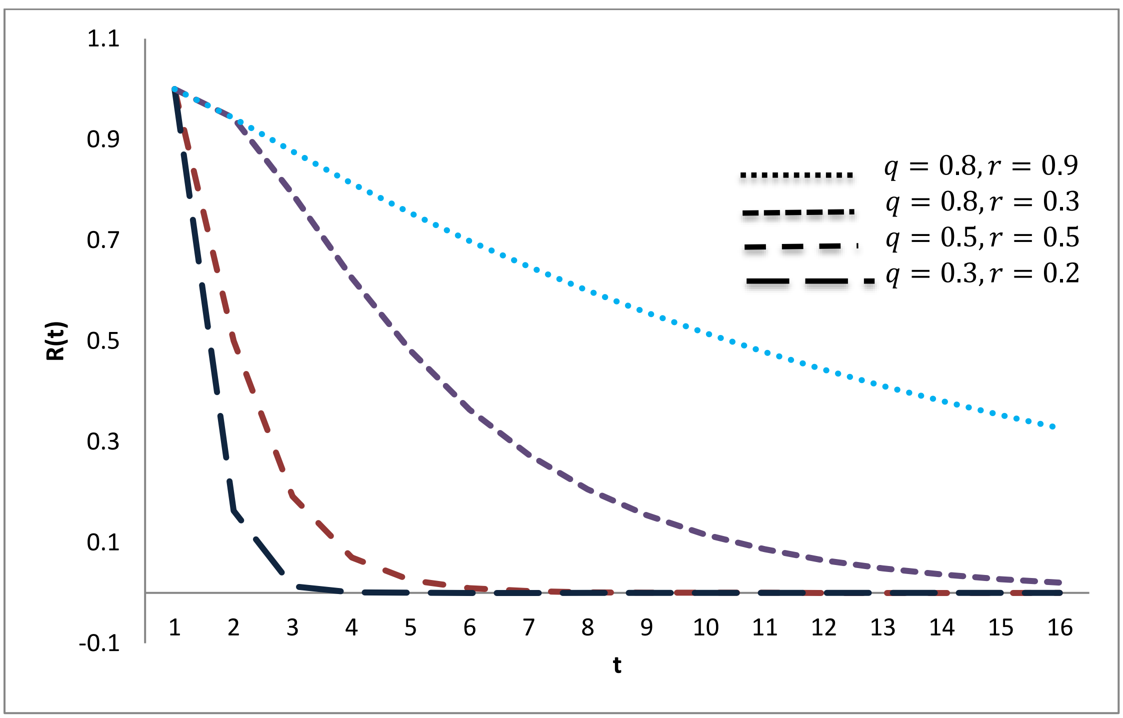

Figure 2: *Curves for reliability of 3–out–of–5:F system for different values of parameters with respect to time*

From the curves we observed that the reliability of the system decreases if the values of q (working rate of a component) and r (repair rate of a component) decrease. Also, as a result of calculations, for $q = r = 0.5$ we observed that the values of $R_0(t)$ and $R_{-1}(t)$ are equal.

As a special case, if $r = 0, s = 1$, then the system is nonrepairable. Thus, (7), (8), and (9) will become, respectively,

$$R_0(t) = (q^5)^t + \sum_{u=0}^{t-1} 5pq^4(q^5)^u R_{-1}(t-1-u) + \sum_{u=0}^{t-1} 10p^2q^3(q^5)^u R_{-2}(t-1-u)$$

$$R_{-1}(t) = (q^4)^t + \sum_{u=0}^{t-1} 4pq^3(q^4)^u R_{-2}(t-1-u)$$

$$R_{-2}(t) = (q^3)^t.$$

For $t = 1$,

$$R_0(1) = (q^5) + 5pq^4 + 10p^2q^3 = 6(q)^5 - 15q^4 + 10q^3$$

$$R_{-1}(1) = (q^4) + 4pq^3 \qquad R_{-2}(1) = (q^3)$$

and for $t = 2$,

$$\begin{aligned} R_0(2) &= (q^5)^2 + 5pq^4 R_{-1}(1) + 5pq^4(q^5)R_{-1}(0) + 10p^2q^3R_{-2}(1) + 10p^2q^3(q^5)R_{-2}(0) \\ &= (q^5)^2 + 5pq^4(q^4 + 4pq^3) + 5pq^4(q^5) + 10p^2q^3(q^3) + 10p^2q^3(q^5) \\ &= 6(q^2)^5 - 15(q^2)^4 + 10(q^2)^3 \end{aligned} \qquad (10)$$

From these calculations, it is clear that the results of $R_0(1)$ and $R_0(2)$ have similar structure with different exponent of q.

Remark: To confirm the results of (10), we consider the reliability of a k–out–of–n:F system with iid components, the reliability of this system can be computed by the following equation:

$$R(t) = \sum_{i=k}^{n} \binom{n}{i} (p(t))^i (1 - p(t))^{n-i}$$

In here, $p(t)$ denotes the reliability of a component. According to the geometric distribution, if we take reliability and unreliability of every component respectively as q^{t+1}, $1 - q^{t+1}$, the reliability of a 3–out–of–5:F system is

$$R(t) = (q^{t+1})^5 + 5(q^{t+1})^4(1 - q^{t+1}) + 10(q^{t+1})^3(1 - q^{t+1})^2 \qquad (11)$$

Using equation (11), it is easy to see that $R(0) = R_0(1)$ and the result of $R(1)$ obtained by the classical reliability is equal the result of $R_0(2)$ which is given by equation (10) and obtained as a special case of repairable system.

6. MEAN TIME TO SYSTEM FAILURE (MTSF)

Letting $Z_0(t) = (q^n)^t$ and $Z_{-i}(t) = (q^{n-i}s)^t$, $i = 1,2, \ldots, (k-1)$ and taking geometric transforms of equations (3) – (6) using

$$q_{ij}^*(h) = \sum_{t=0}^{\infty} h^t q_{ij}(t),$$

the obtained equations are written in the matrix form which is given below:

$$\begin{bmatrix} 1 & -hq^*_{0-1}(h) & -hq^*_{0-2}(h) & \dots & -hq^*_{0-(k-1)}(h) \\ -hq^*_{-10}(h) & 1-hq^*_{-1-1}(h) & -hq^*_{-1-2}(h) & \dots & -hq^*_{-1-(k-1)}(h) \\ 0 & -hq^*_{-2-1}(h) & 1-hq^*_{-2-2}(h) & \cdots & -hq^*_{-2-(k-1)}(h) \\ 0 & 0 & -hq^*_{-3-2}(h) & \dots & -hq^*_{-3-(k-1)}(h) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1-hq^*_{-(k-1)-(k-1)}(h) \end{bmatrix} \begin{bmatrix} R^*_0(h) \\ R^*_{-1}(h) \\ R^*_{-2}(h) \\ R^*_{-3}(h) \\ \vdots \\ R^*_{-(k-1)}(h) \end{bmatrix} = \begin{bmatrix} Z^*_0(h) \\ Z^*_{-1}(h) \\ Z^*_{-2}(h) \\ Z^*_{-3}(h) \\ \vdots \\ Z^*_{-(k-1)}(h) \end{bmatrix}$$

Cramer's rule is used for solving systems of linear equation and after employing determinants procedures, we obtained

$$R^*_0(h) = \frac{N_1(h)}{D_1(h)},$$

where

$$D_1(h) = 1 \times (R_2C_2)_D \times (R_3C_3)_D \times \dots \times (R_kC_k)_D$$

$$(R_2C_2)_D = -h^2 q^*_{-10}(h) q^*_{0-1}(h) + 1 - hq^*_{-1-1}(h)$$

$$(R_2C_i)_D = -h^2 q^*_{-10}(h) q^*_{0-(i-1)}(h) - hq^*_{-1-(i-1)}(h)\,, \quad i = 3,4,\dots,k$$

$$(R_3C_3)_D = \frac{R_2C_3}{R_2C_2} hq^*_{-2-1}(h) + 1 - hq^*_{-2-2}(h)$$

$$(R_3C_i)_D = \frac{R_2C_i}{R_2C_2} hq^*_{-2-1}(h) - hq^*_{-2-(i-1)}(h)\,, \quad i = 4,5,\dots,k$$

$$\vdots$$

$$(R_kC_k)_D = \frac{R_{k-1}C_k}{R_{k-1}C_{k-1}} hq^*_{-(k-1)-(k-2)}(h) + 1 - hq^*_{-(k-1)-(k-1)}(h)$$

and

$$N_1(h) = (-1)^{k+1}(-hq^*_{0-1)}(h)) \times (R_2C_2)_N \times (R_3C_3)'_N \times (R_4C_4)'_N \times \dots \times (R_kC_k)'_N$$

$$(R_2C_i)_N = -hq^*_{0-i}(h) \frac{1-hq^*_{-1-1}(h)}{hq^*_{0-1}(h)} - hq^*_{-1-i}(h)\,, \quad i = 2,3,\dots,k-1$$

$$(R_2C_k)_N = Z^*_0(h) \frac{1-hq^*_{-1-1}(h)}{hq^*_{0-1}(h)} + Z^*_1(h)$$

$$(R_3C_2)_N = hq^*_{0-2}(h) \frac{hq^*_{-2-1}(h)}{hq^*_{0-1}(h)} + 1 - hq^*_{-2-2}(h)\,,$$

$$(R_3C_i)_N = hq^*_{0-i}(h) \frac{hq^*_{-2-1}(h)}{hq^*_{0-1}(h)} - hq^*_{-2-i}(h)\,, \quad i = 3,4,\dots,k-1$$

$$(R_3C_k)_N = Z^*_0(h) \frac{hq^*_{-2-1}(h)}{-hq^*_{0-1}(h)} + Z^*_2(h)$$

$$(R_3C_i)'_N = (R_2C_i)_N \frac{-(R_3C_2)_N}{(R_2C_2)_N} + (R_3C_i)_N \qquad i = 3,4,\dots,k$$

$$(R_4C_3)_N = (R_2C_3)_N \frac{hq^*_{-3-2}(h)}{(R_2C_2)_N} + 1 - hq^*_{-3-3}(h)\,,$$

$$(R_4C_i)_N = (R_2C_i)_N \frac{hq^*_{-3-2}(h)}{(R_2C_2)_N} - hq^*_{-3-i}(h)\,, \quad i = 4,5,\dots,k-1$$

$$(R_4C_k)_N = (R_2C_k)_N \frac{hq^*_{-3-2}(h)}{(R_2C_2)_N} + Z^*_3(h)$$

$$(R_4C_i)'_N = (R_3C_i)'_N \frac{-(R_4C_3)_N}{(R_3C_3)'_N} + (R_4C_i)_N \qquad i = 4,5,\dots,k$$

$$\vdots$$

$$(R_kC_{k-1})_N = (R_{k-2}C_{k-1})'_N \frac{hq^*_{-(k-1)-(k-2)}(h)}{(R_{k-2}C_{k-2})'_N} + 1 - hq^*_{-(k-1)-(k-1)}(h)$$

$$(R_kC_k)_N = (R_{k-2}C_{k-1})'_N \frac{hq^*_{-(k-1)-(k-2)}(h)}{(R_{k-2}C_{k-2})'_N} + Z^*_{k-1}(h)$$

$$(R_kC_k)'_N = (R_{k-1}C_k)'_N \frac{-(R_kC_{k-1})_N}{(R_{k-1}C_{k-1})'_N} + (R_kC_k)_N$$

The above procedure may be easier and more understandable for 3–out–of–5:F system. Taking geometric transforms of equations (7) – (9), the matrix form of the systems of linear equation will be obtained as:

$$\begin{bmatrix} 1 & -hq^*_{0-1}(h) & -hq^*_{0-2}(h) \\ -hq^*_{-10}(h) & 1 - hq^*_{-1-1}(h) & -hq^*_{-1-2}(h) \\ 0 & -hq^*_{-2-1}(h) & 1 - hq^*_{-2-2}(h) \end{bmatrix} \begin{bmatrix} R^*_0(h) \\ R^*_{-1}(h) \\ R^*_{-2}(h) \end{bmatrix} = \begin{bmatrix} Z^*_0(h) \\ Z^*_{-1}(h) \\ Z^*_{-2}(h) \end{bmatrix}$$

With the help of determinants procedures, $R^*_0(h)$ can be obtained using Cramer's rule as

$$R^*_0(h) = \frac{N(h)}{D(h)} = \frac{Z^*_0(h)\times(R_2C_2)_N\times(R_3C_3)'_N}{1\times(R_2C_2)_D\times(R_3C_3)_D} \tag{12}$$

where

$$(R_2C_2)_N = \frac{Z^*_{-1}(h)}{Z^*_0(h)} hq^*_{0-1}(h) + 1 - hq^*_{-1-1}(h) \qquad (R_2C_2)_D = -h^2q^*_{-10}(h)q^*_{0-1}(h) + 1 - hq^*_{-1-1}(h)$$

$$(R_2C_3)_N = \frac{Z^*_{-1}(h)}{Z^*_0(h)} hq^*_{0-2}(h) - hq^*_{-1-2}(h) \qquad (R_2C_3)_D = -h^2q^*_{-10}(h)q^*_{0-2}(h) - hq^*_{-1-2}(h)$$

$$(R_3C_2)_N = \frac{Z^*_{-2}(h)}{Z^*_0(h)} hq^*_{0-1}(h) - hq^*_{-2-1}(h) \qquad (R_3C_3)_D = \frac{R_2C_3}{R_2C_2} hq^*_{-2-1}(h) + 1 - hq^*_{-2-2}(h)$$

$$(R_3C_3)_N = \frac{Z^*_{-2}(h)}{Z^*_0(h)} hq^*_{0-2}(h) + 1 - hq^*_{-2-2}(h)$$

$$(R_3C_3)'_N = \frac{-(R_3C_2)_N}{(R_2C_2)_N}(R_2C_3)_N + (R_3C_3)_N$$

In more explicit form, $N(h)$ and $D(h)$ are as follows:

$$\begin{aligned} N(h) = Z^*_0(h)[1 - hq^*_{-2-2}(h) - hq^*_{-1-1}(h) + h^2q^*_{-1-1}(h)q^*_{-2-2}(h) \; - h^2q^*_{-2-1}(h)q^*_{-1-2}(h)] \\ + hq^*_{0-1}(h)[Z^*_{-1}(h) - Z^*_{-1}(h)hq^*_{-2-2}(h) \; + Z^*_{-2}(h)hq^*_{-1-2}(h)] \\ - hq^*_{0-2}(h)[-Z^*_{-1}(h)hq^*_{-2-1}(h) - Z^*_{-2}(h) + Z^*_{-2}(h)hq^*_{-1-1}(h)] \end{aligned}$$

$$\begin{aligned} D(h) =[1- hq^*_{-2-2}(h) - hq^*_{-1-1}(h) + h^2q^*_{-1-1}(h)q^*_{-2-2}(h) - \; h^2q^*_{-2-1}(h)q^*_{-1-2}(h)] + \\ hq^*_{0-1}(h)[-hq^*_{-10}(h) + h^2q^*_{-10}(h)q^*_{-2-2}(h)] - \; hq^*_{0-2}(h)[h^2q^*_{-10}(h)q^*_{-2-1}(h)] \end{aligned}$$

By using probabilistic argument and taking geometric transformation on both sides of the above equations 3–out–of–5:F system, we get $R^*_0(h)$ in given by (12). Then mean time system failure (MTSF) is given by

$$MTSF = \lim_{h\to 1}\frac{N(h)}{D(h)} - 1$$

where $\lim_{h\to 1} q_{ij}^{*}(h) = P_{ij}$ and μ_i are given by (1) and (2), respectively. Then,

$$N(1) = \mu_0[1 - P_{-2-2} - P_{-1-1} + P_{-2-2}P_{-1-1} - P_{-2-1}P_{-1-2}] + \mu_{-1}[P_{0-1} - P_{0-1}P_{-2-2} + P_{0-2}P_{-2-1}] + \mu_{-2}[P_{0-1}P_{-1-2} + P_{0-2} - P_{0-2}P_{-1-1}]$$

$$D(1) = [1 - P_{-2-2} - P_{-1-1} + P_{-2-2}P_{-1-1} - P_{-2-1}P_{-1-2} - P_{0-1}P_{-10} + P_{0-1}P_{-10}P_{-2-2} - P_{0-2}P_{-10}P_{-2-1}]$$

The behavior of MTSF has been drawn with respect to different repair and failure rates and given as follows:

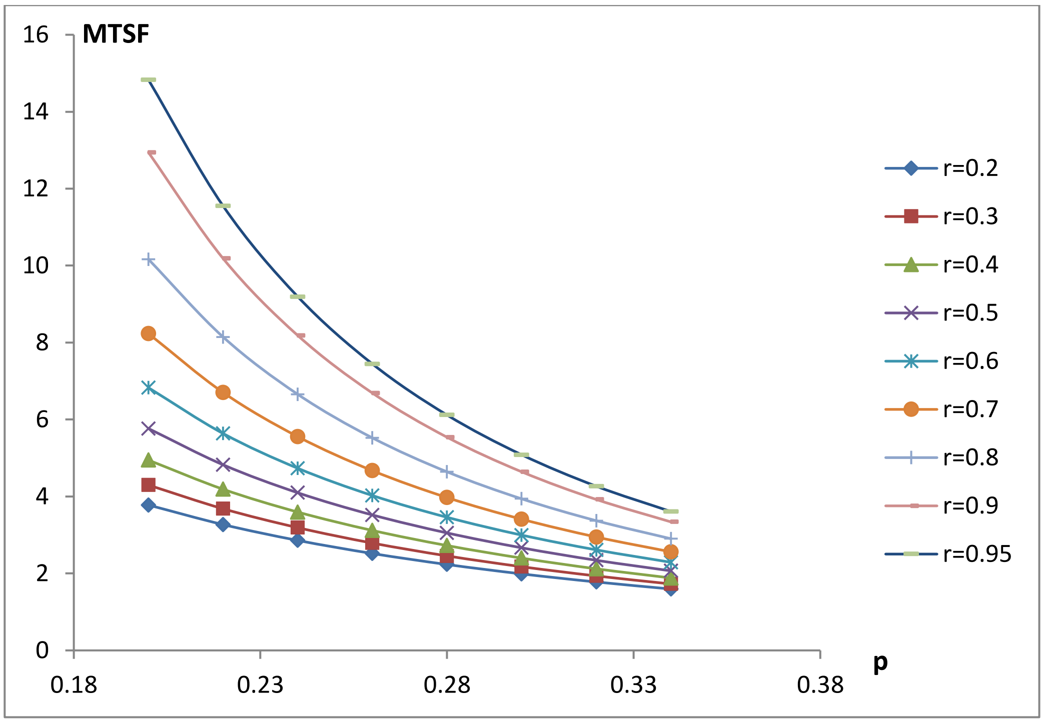

Figure 3: *The behavior of MTSF with respect to different repair and failure rates*

Figure 3 shows the variations in MTSF with respect to failure rate (p) and repair rate (r). From the graph we observed that MTSF decreases as the value of p increase and increases with the increase of r.

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