# THE $\left(\frac{G'}{G+G'}\right)$-EXPANSION METHOD TO WORK OUT EXACT SOLUTIONS OF THE (2+1) DIMENSIONAL ASYMMETRICAL NIZHNIK-NOVIKOV-VESELOV SYSTEM

**ZeJian Cui [1] & Jiao Zeng [2]**
[1]College of Mathematic and Information, China West Normal University, Nanchong, 637009, China
[2]College of Mathematic and Information, China West Normal University, Nanchong, 637009, China

**ABSTRACT**
Solving the nonlinear evolution equations has very important position, now there are many effective ways for explicit exact solutions of nonlinear evolution equations. This paper introduces the (2+1) dimensional asymmetrical Nizhnik-Novikov-Veselov system. Enrich the (2 + 1) -dimensional ANNV system solutions.

**Keywords:** *the* $\left(\frac{G'}{G+G'}\right)$*-expansion method; (2+1) dimensional ANNV system; exact solutions*.

## 1. INTRODUCTION

With the rapid development of China's automobile industry, the trading volume and the size of the used car Nonlinear evolution equations play an important part in the development of science. Therefore, seeking solution of nonlinear evolution equations is also important. So it gradually causes the attention of many scholars. Now there are many ways of solving nonlinear evolution equations, For instance, The hyperbolic function method [1],he homogeneous balance method [2], Jacobi elliptic function expansion method [3], Auxiliary equation method [4-5], Riccati function method [6-7], Extend the Riccati mapping method [8-9] , Symbolic computation algebra method [10] ,and so on. Typically established by Wang and others is the $\left(\frac{G'}{G+G'}\right)$-expansion method [11], and the improved $\left(\frac{G'}{G+G'}\right)$-expansion method [12], to work out exact solutions of the (2+1) dimensional asymmetrical Nizhnik-Novikov- Veselov system.

## 2. The $\left(\frac{G'}{G+G'}\right)$-expansion method

Suppose that (2 + 1) dimensional nonlinear partial differential equation (PDE) general form as follows:

$$F(u,u_t,u_x,u_y,u_{tt},u_{xt},u_{yt},u_{xy},u_{yy},\dots)=0 \quad (1)$$

Where $F$ is a polynomial in $u=u(x,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved， $u=u(x,t)$ is unknown function .we suppose that

$$u(x,t)=u(\xi),\quad \xi=x+y-ct. \quad (2)$$

By using (1) and (2) it is easily derived that:

$$F(u,u',u'',u''',\dots)=0. \quad (3)$$

Where $"'"=\frac{d}{d\xi}$.

Suppose that the solution of ODE (3) can be expressed by a polynomial in $\left(\frac{G'}{G+G'}\right)$ as follows:

$$u(\xi)=\sum_{i=0}^{n}a_i\left(\frac{G'}{G+G'}\right)^i. \quad (4)$$

Where $G = G(\xi)$ satisfies the second order LODE in the form：

$$GG'' - (\alpha+\beta)(G')^2 - \beta GG' = 0. \qquad (5)$$

$a_0, a_1, a_2, \dots, a_n$ and $\alpha, \beta$ are constants to be determined later, $a_n \neq 0, \alpha \neq 1, \beta \neq 0$.

The positive integer $n$ can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE (3).
By using（5）it is easily derived that：

$$\frac{G'}{G+G'} = -\frac{C_1\beta e^{\beta\xi}}{C_1\alpha e^{\beta\xi} - C_1 e^{\beta\xi} + C_2\alpha\beta + C_2\beta^2 - C_2\beta}. \qquad (6)$$

By substituting (4) and（6）into ODE(3), collecting all terms with the same order of $\left(\frac{G'}{G+G'}\right)$ together, the left-hand side of Eq.(3) is converted into another polynomial in $\left(\frac{G'}{G+G'}\right)$.Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $a_0, a_1, a_2, a_3, \dots\ a_n$, $\alpha$ and $\beta$
In the end, by substituting (5)-(6) into Eq. (4).we have more traveling wave solutions of the nonlinear partial differential equation(1).

**3. (2 + 1) -dimensional ANNV system of exact solution**
(2 + 1) -dimensional ANNV system:

$$\begin{cases} u_t + u_{xxx} - 3v_x u - 3vu_x = 0, \\ u_x = v_y. \end{cases} \qquad (7)$$

From the earliest Boiti [13] export, [14-16] is also made about the (2 + 1) -dimensional ANNV system, a series of discussion. [17] with the $\left(\frac{G'}{G+G'}\right)$ expansion method to solve the (2 + 1) -dimensional ANNV system solutions. Now, we use the $\left(\frac{G'}{G+G'}\right)$ expansion method (2 + 1) -dimensional ANNV system different forms of exact solution.

By using（2）into Eq.（7）：

$$\begin{cases} -cu' + u''' - 6uu' = 0, \\ u = v. \end{cases} \qquad (8)$$

According to the balance principle, we get $n = 2$ .Therefore, (8) has the solution in form of the following:

$$u(\xi) = a_0 + a_1\left(\frac{G'}{G+G'}\right) + a_2\left(\frac{G'}{G+G'}\right)^2 \qquad (9)$$

Where, $a_0, a_1$ and $a_2$ are constants, $a_2 \neq 0$ .

By using (6)、(8) and (9) it is derived that

$$u' = a_1\beta\left(\frac{G'}{G+G'}\right) + (a_1\alpha - a_1 + 2a_2\beta)\left(\frac{G'}{G+G'}\right)^2 + 2a_2(\alpha-1)\left(\frac{G'}{G+G'}\right)^3$$

$$u'' = a_1\beta^2\left(\frac{G'}{G+G'}\right) + (3a_1\alpha\beta - 3a_1\beta + 4a_2\beta^2)\left(\frac{G'}{G+G'}\right)^2 + \left(2a_1(\alpha-1)^2 + 10a_2\beta(\alpha-1)\right)\left(\frac{G'}{G+G'}\right)^3 + 6a_2(\alpha-1)^4$$

$$u''' = a_1\beta^3\left(\frac{G'}{G+G'}\right) + (7a_1\alpha\beta^2 - 7a_1\beta^2 + 8a_2\beta^3)\left(\frac{G'}{G+G'}\right)^2 + (12a_1\beta(\alpha-1)^2 + 38a_2\beta^2(\alpha-1))\left(\frac{G'}{G+G'}\right)^3$$
$$+(6a_1(\alpha-1)^3 + 54a_2\beta(\alpha-1)^2)\left(\frac{G'}{G+G'}\right)^4 + 24a_2(\alpha-1)^3\left(\frac{G'}{G+G'}\right)^5$$
$$uu' = (a_0a_1\beta)\left(\frac{G'}{G+G'}\right) + (a_1^2\beta + a_0(a_1\alpha - a_1 + 2a_2\beta))\left(\frac{G'}{G+G'}\right)^2 + (a_1a_2\beta + a_1(a_1\alpha - a_1 + 2a_2\beta) + 2a_0a_2(\alpha-1))\left(\frac{G'}{G+G'}\right)^3$$
$$(a_2(a_1\alpha - a_1 + 2a_2\beta) + 2a_1a_2(\alpha-1))\left(\frac{G'}{G+G'}\right)^4 + 2a_2^2(\alpha-1)\left(\frac{G'}{G+G'}\right)^5$$

Setting each coefficient of this polynomial to zero, we derive a set of over-determined differential equations for $a_0, a_1$ and $a_2$ as follows:

$$\left(\frac{G'}{G+G'}\right): -a_1\beta c + a_1\beta^3 - 6a_0a_1\beta = 0$$
$$\left(\frac{G'}{G+G'}\right)^2: -c(a_1\alpha - a_1 + 2a_2\beta) + (7a_1\alpha\beta^2 - 7a_1\beta^2 + 8a_2\beta^3) - 6(a_1^2\beta + a_0a_1\alpha - a_0a_1 + 2a_0a_2\beta) = 0$$
$$\left(\frac{G'}{G+G'}\right)^3: -2a_2c(\alpha-1) + (12a_1\beta(\alpha-1)^2 + 38a_2\beta^2(\alpha-1)) - 6(a_1a_2\beta + a_1(a_1\alpha - a_1 + 2a_2\beta) + 2a_0a_2(\alpha-1)) = 0$$
$$\left(\frac{G'}{G+G'}\right)^4: (6a_1(\alpha-1)^3 + 54a_2\beta(\alpha-1)^2) - 6(a_2(a_1\alpha - a_1 + 2a_2\beta) + 2a_1a_2(\alpha-1)) = 0$$
$$\left(\frac{G'}{G+G'}\right)^5: -12a_2^2(\alpha-1) + 24a_2(\alpha-1)^3 = 0$$

Solving the set of over-determined differential equations, we have:

$$a_0 = \frac{\beta^2 - c}{6} \qquad a_1 = 2\beta(\alpha-1) \qquad a_2 = 2(\alpha-1)^2 \tag{10}$$

Substituting (6) and (10) into (9), we have the fundamental solution of Eq. (7):

$$u(x,t) = v(x,t) = \frac{\beta^2 - c}{6} + 2\beta(\alpha-1)\left(-\frac{C_1\beta e^{\beta\xi}}{C_1\alpha e^{\beta\xi} - C_1e^{\beta\xi} + C_2\alpha\beta + C_2\beta^2 - C_2\beta}\right)$$
$$+2(\alpha-1)^2\left(-\frac{C_1\beta e^{\beta\xi}}{C_1\alpha e^{\beta\xi} - C_1e^{\beta\xi} + C_2\alpha\beta + C_2\beta^2 - C_2\beta}\right)^2 \tag{11}$$

Where $\xi = x + y - (\beta^2 - 6a_0)t$, $C_1, C_2$ are the integral constants . $\alpha$ and $\beta$ as an integer .

$\alpha \neq 1, \beta \neq 0$.

## 4. Conclusion

This chapter through the $\left(\frac{G'}{G+G'}\right)$-expansion method, we have (2 + 1) -dimensional ANNV systems of new exact solutions, so as to further enrich its solution system. Although this method of computation is large, from the point of solving the results of exact solutions, the solution has more obviously concise advantage than the original (2 + 1) -dimensional ANNV system and has obtained newer and various exact solution what the previous documents have not.

## 5. References

[1]. Li Z B, Zhang S Q 1997 Acta Math.-Phys.Sin. 17 81 (in Chinese)
[2]. Wang M L 1995 Phys.Lett.A 199 169
[3]. Liu S K,Liu S D,Fu Z T 2001 Acta Phys.Sin.50 2068(in Chinese)
[4]. Taogetusang, Sirendaoerji 2006 Acta Phys.Sin,55 13(in Chinese)
[5]. Li X Z,Li X Y,Zhao L Y,Zhang J L 2008 Acta Phys.Sin.57 22031(in Chinese)
[6]. Fang J P, Zheng C L, Zhu J M 2005 Commun.Theor.Phys.44 203
[7]. Fang J P, Zheng C L, Zhu J M 2005 Acta Phys.Sin.54 2990(in Chinese)
[8]. Ma S H, Fang J P 2006 Acta Phys.Sin.55 5611(in Chinese)
[9]. Ma S H, Fang J P, Zheng C L 2008 Chin. Phys. B 17 2767
[10]. Gao L, Xu W, Tang Y N et al 2007 Acta Phys. Sin. 56 1860(in Chinese)
[11]. Wang M L, Li X Z, Zhang J L 2008 Phys, Lett.A 372 417
[12]. Feng Q J, Journal of Shanxi Normal University Natural Science Edition
[13]. Boiti M, Leon J J P, Manan M, Penpinelli F 1986 Inver.Prob.2271
[14]. Chen Y, Wang Q 2004 Chin. Phys, 13 1796
[15]. Dai C Q, Zhou G Q 2007 Chin. Phys. 16 1201
[16]. Ma S H, Fang J P, Ren Q B 2007 Acta Phys.Sin. 56 6784(in Chinese)
[17]. Li Bang-Qing, MaYu-Lan $(G'/G)$ -expansion method and new exact solutions for (2+1) dimensional asymmetrical Nizhnik-Novikov-Veselov system

Project Name: The Natural Science Fund of Department of Education of Sichuan Province (09ZA119).
E-mail: cuizejian2007@163.com; zengjiao526@163.com.