A MODEL ON DETERIORATING ITEMS WITH PRICE DEPENDENT DEMAND RATE

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ABSTRACT
The objective of this model is to discuss the inventory system for deteriorating items with price dependent demand pattern where instantaneous rate of deterioration is considered. The Economic order quantity is determined for maximizing the profit per unit time. The result is illustrated with numerical example.

Keywords: *Deteriorating items, Inventory system, Price dependent demand.*

1. INTRODUCTION

Deteriorating items generally refer to some perishable products which are more likely to decay, damage, evaporation, expired, depreciation over time and these items widely exist in our daily life, such as fresh products, fruits, vegetables, seafood, etc, which decrease in quantity or utility during their delivery and storage stage periods. Accompany by the increasing of the variety and quantity of the deteriorating and fashion goods, consumers' appetite for high quality perishable items are continually upgrading rapidly. So the topic of deteriorating inventory system management has become more popular in the field of research and business. In the study of inventory policy, deteriorating inventory model are continuously modified to accommodate more and more practical situation.

Aggarwal(1978)[1], Ghare and Schrader(1963)[7], Shah and Jaiswal(1977)[14], Whitin(1957)[15], and others discussed inventory models with constant demand, constant deterioration and instantaneous replenishment. Dave and Patel(1981)[6] discussed an inventory model with time-proportional demand and instantaneous replenishment. Roy Chowdhury and Chaudhuri (1983)[13] developed an inventory model considering a finite rate of replenishment and allowing shortages in inventory. Dave(1986)[5] extended the model of Dave and Patel(1981)[6] to include shortages in inventory. Bahari-Kashani(1989)[2] presented a heuristic model with time-proportional demand. Giri, Goswami and Chaudhuri (1996)[9] developed an inventory model with a time-varying demand taking the deterioration rate, holding cost and ordering cost to be linearly time-dependent. In another paper, Giri and Chaudhri(1997)[8] discussed two heuristic models in which the demand rate, deterioration rate, ordering cost, holding cost and shortage cost were all assumed to be linearly increasing functions of time. Lin, Tan and Lee (2000)[12] developed an EOQ model with time-varying demand, partial back ordering and a linearly time-dependent deterioration rate.

Contrary to the abundance of inventory models with time-dependent demand, models with price-dependent demand are rare. Kunreuther and Richard (1971)[10] discussed the joint pricing an ordering decisions for nonseasonal products and this model was was extended by Kunreuther and Schrage (1972) [11] to the case of seasonal products. Cohen (1977) [4] discussed the joint pricing and ordering policy for an item deteriorating over time at a constant rate. Cheng(1990) [3] studied a similar problem for a non-deteriorating item with limitations on storage space and inventory investment.

In this paper an inventory model with price dependent demand rate and instantaneous deterioration rate have been developed . The price-dependence of the demand function is assumed to be nonlinear. The model is developed and solved analytically. The optimal order by maximizing the profit function has been established. A numerical example is discussed to illustrate the procedure of solving the model.

2. ASSUMPTIONS AND NOTATIONS

Following assumptions are made for the proposed model:

i. Single inventory is used.
ii. Lead time is assumed to be zero.

iii. The model is studied when shortages are not allowed.
iv. Nonlinear demand pattern is assumed.
v. Instantaneous rate of deterioration is assumed.
vi. Replenishment rate is infinite.

Following notations are made for the given model:

$I(t)$ = On hand inventory level at any time t, $t \geq 0$.

T =The length of cycle time.

A = The ordering cost per unit time

C = The purchasing cost per unit.

h = The inventory holding cost per unit per unit time.

p = The selling price per unit item.

$d(p)$ = The price-dependent demand rate where $d(p) = \gamma\ p^{-\beta}$ $\gamma > 0$, (γ is the scale parameter) and $\beta > 0$ (β is the shape parameter) of the demand curve.

$\theta = \alpha\ t, 0 < \alpha < 1$ = Instantaneous rate of deterioration of the on-hand inventory .

$L(t)$ = The loss in the stock due to decay in time $[0,t]$.

O_t =The quantity ordered in each cycle.

U =The total average cost of the system.

V = The total profit incurred in the system.

3. FORMULATION

Let $I(t)$ be the on-hand inventory level at any time $t \geq 0$.The demand rate is assumed to be positive in its entire domain. The amount of stock depletes in the period $[0,T]$ due to the combined effect of demand and deterioration. By this process, the stock reaches zero at time T . Hence, the inventory level at any instant of time t is described as follows.

At time $t + \Delta t$, the on-hand inventory in the interval $[0,T]$ will be

$$I(t+\Delta t) = I(t) - \theta\ I(t)\ \Delta t - d(p)\ \Delta t$$

Dividing by Δt and then taking as $\Delta t \to 0$ we get

$$\frac{dI(t)}{dt} + \alpha\ t\ I(t) = -d(p)\ ; \quad 0 \leq t \leq T \qquad \textbf{(3.1)}$$

With the condition

$$I(T) = 0, \qquad \textbf{(3.2)}$$

and as α is very small, ignoring the second and higher powers of α , the solution of the differential equation **(3.1)** is given by,

$$I(t) = I(0)\left(1 - \frac{\alpha\ t^2}{2}\right) - t\ d(p)\left(1 - \frac{\alpha\ t^2}{3}\right) \qquad \textbf{(3.3)}$$

Now $I(0)$ is the initial stock when there is decay. Let $L(t)$ be the loss in stock due to decay in time $[0,t]$ which will be the difference between the inventory levels without decay and with decay at any time t .

Now from **(3.3)**

$$I(0) = \left\{I(t) + t\ d(p)\left(1 - \frac{\alpha\ t^2}{3}\right)\right\}\left(1 + \frac{\alpha\ t^2}{2}\right) \qquad \textbf{(3.4)}$$

If $I^{'}(t)$ be the instantaneous inventory level at any time $t \in [0,t]$, when there is no decay, we have

$$I^{'}(t) = \lim_{\alpha \to 0} I(t) = I(0) - t\, d(p) \qquad \textbf{(3.5)}$$

The stock loss due to decay in time $[0,t]$ will be the difference $I^{'}(t) - I(t)$.

i.e. $$L(t) = I^{'}(t) - I(t) = \frac{\alpha\, t^2}{6}\{3I(0) - 2\, t\, d(p)\}. \qquad \textbf{(3.6)}$$

Now using **(3.4)** in **(3.6)** neglecting higher powers of α

$$L(t) = \frac{\alpha\, t^2}{6}\{3(I(t) + d(p)) - 2\, t\, d(p)\}. \qquad \textbf{(3.7)}$$

Again using **(3.2)** in **(3.7)**

$$L(T) = \frac{\alpha\, T^2 (3 - 2\,T)}{6}\, d(p). \qquad \textbf{(3.8)}$$

As the only loss to the inventory is due to either decay or demand, the quantity ordered in each cycle can be in the following form.

$$O_t = L(t) + T\, d(p) = T\, d(p) \left\{ \frac{\alpha\, T(3 - 2\,T) + 6}{6} \right\}. \qquad \textbf{(3.9)}$$

Since $I(0) = O_t$ now using **(3.3)** and **(3.9)** we have

$$I(t) = T\, d(p) \left\{ \frac{\alpha\, T(3-2\,T)+6}{6} \right\} \left\{ 1 - \frac{\alpha\, t^2}{2} \right\} - t\, d(p) \left\{ 1 - \frac{\alpha\, t^3}{3} \right\} \quad , \quad 0 \le t \le T \qquad \textbf{(3.10)}$$

Now the average total cost per cycle is given by

$$U(T,p) = \frac{1}{T} \text{[Ordering Cost + Purchasing Cost + Holding Cost]}$$

$$= \frac{1}{T}\left[A + C\, O_t + h \int_0^T I(t)\, dt \right]$$

$$= \frac{A}{T} + d(p) \left\{ \frac{\alpha\, T(3-2\,T)+6}{6} \right\} \left\{ C + h \left\{ T - \frac{\alpha\, T^3}{6} \right\} \right\} - h\, d(p) \left\{ \frac{T}{2} - \frac{\alpha\, T^3}{4} \right\}. \qquad \textbf{(3.11)}$$

Now we can write the profit rate as a function of the cycle length and price as,

$$V(T,p) = p\, d(p) - .U(T,p)$$

$$= p\, d(p) - \frac{A}{T} - d(p) \left\{ \frac{\alpha\, T(3-2\,T)+6}{6} \right\} \left\{ C + h \left\{ T - \frac{\alpha\, T^3}{6} \right\} \right\} + h\, d(p) \left\{ \frac{T}{2} - \frac{\alpha\, T^3}{4} \right\}. \qquad \textbf{(3.12)}$$

Our aim is to determine the values T and p which maximizes the profit $V(T,p)$. We have

$d(p) = \gamma\, p^{-\beta}$ for $\gamma > 0,\ \beta > 0$

Now from **(3.11) and (3.12),** we have

$$U(T,p) = \frac{A}{T} + \gamma\, p^{-\beta} \left\{ \frac{\alpha\, T(3-2\,T)+6}{6} \right\} \left\{ C + h \left\{ T - \frac{\alpha\, T^3}{6} \right\} \right\} - h\, \gamma\, p^{-\beta} \left\{ \frac{T}{2} - \frac{\alpha\, T^3}{4} \right\}. \qquad \textbf{(3.13)}$$

and

$$V(T,p) = \gamma\, p^{1-\beta} - \frac{A}{T} - \gamma\, p^{-\beta} \left\{ \frac{\alpha\, T(3-2\,T)+6}{6} \right\} \left\{ C + h \left\{ T - \frac{\alpha\, T^3}{6} \right\} \right\} + h\, \gamma\, p^{-\beta} \left\{ \frac{T}{2} - \frac{\alpha\, T^3}{4} \right\}. \qquad \textbf{(3.14)}$$

The necessary conditions for maximization of $V(T,p)$ are,

$$\frac{\partial V(T,p)}{\partial T}=0 \ \ and \ \ \frac{\partial V(T,p)}{\partial p}=0 \tag{3.15}$$

The sufficient condition for maximization of $V(T,p)$ requires that it must be a concave function for $T>0,\ \ p>0$.

We have

$$\frac{\partial V(T,p)}{\partial T}=\frac{A}{T^2}-\gamma\ p^{-\beta}\left\{\frac{3\alpha-4\alpha T}{6}\right\}-\gamma\ p^{-\beta}\left\{\frac{\alpha\,T(3-2T)+6}{6}\right\}h\left\{1-\frac{\alpha\,T^2}{2}\right\}+h\,\gamma\ p^{-\beta}\left\{\frac{1}{2}-\frac{3\alpha\,T^2}{4}\right\}. \tag{3.16}$$

$$\frac{\partial^2 V(T,p)}{\partial T^2}=-\frac{2A}{T^3}+\frac{2\,\alpha\,\gamma\ p^{-\beta}}{3}-\frac{\gamma\ h\ p^{-\beta}}{6}\left\{(3\alpha-4\,\alpha\,T)\left\{1-\frac{\alpha\,T^2}{2}\right\}+\alpha\,T\{\alpha\,T(3-2T)+6\}\right\}$$
$$-\frac{3\alpha\,h\,\gamma\,T\ p^{-\beta}}{2}. \tag{3.17}$$

$$\frac{\partial V(T,p)}{\partial p}=\gamma(1-\beta)\ p^{-\beta}+\gamma\ \beta\ p^{-\beta-1}\left\{\frac{\alpha\,T(3-2\,T)+6}{6}\right\}\left\{C+h\left\{T-\frac{\alpha\,T^3}{6}\right\}\right\}$$
$$-h\,\beta\,\gamma\ p^{-\beta-1}\left\{\frac{T}{2}-\frac{\alpha\,T^3}{4}\right\} \tag{3.18}$$

$$\frac{\partial^2 V(T,p)}{\partial p^2}=-\gamma\ \beta(1-\beta)\ p^{-\beta}-\gamma(\beta+1)\ \beta\ p^{-\beta-1}\left\{\frac{\alpha\,T(3-2\,T)+6}{6}\right\}\left\{C+h\left\{T-\frac{\alpha\,T^3}{6}\right\}\right\}$$
$$+h\ \beta\ \gamma(\beta+1)\ p^{-\beta-1}\left\{\frac{T}{2}-\frac{\alpha\,T^3}{4}\right\}. \tag{3.19}$$

$$\frac{\partial^2 V(T,p)}{\partial T\ \partial p}=\gamma\ \beta\ p^{-\beta-1}\left\{\left\{\frac{3\alpha-4\alpha\,T}{6}\right\}\left\{C+h\left\{T-\frac{\alpha\,T^3}{6}\right\}\right\}+\left\{\frac{\alpha\,T(3-2\,T)+6}{6}\right\}\left\{h\left\{1-\frac{\alpha\,T^2}{2}\right\}\right\}\right\}$$
$$-h\ \beta\ \gamma\ p^{-\beta-1}\left\{\frac{1}{2}-\frac{3\alpha\,T^2}{4}\right\}. \tag{3.20}$$

Now the function $V(T,p)$ will be concave if

$$\begin{vmatrix} \dfrac{\partial^2 V}{\partial T^2} & \dfrac{\partial^2 V}{\partial T\partial p} \\ \dfrac{\partial^2 V}{\partial p\partial T} & \dfrac{\partial^2 V}{\partial p^2} \end{vmatrix}>0 \tag{3.21}$$

And

$$\frac{\partial^2 V}{\partial T^2}<0. \tag{3.22}$$

We have

$$H = \begin{vmatrix} \dfrac{\partial^2 V}{\partial p^2} & \dfrac{\partial^2 V}{\partial p \partial T} \\ \dfrac{\partial^2 V}{\partial T \partial p} & \dfrac{\partial^2 V}{\partial T^2} \end{vmatrix} =$$

$$\left[-\frac{2A}{T^3} + \frac{2\,\alpha\,\gamma\,p^{-\beta}}{3} - \frac{\gamma\,h\,p^{-\beta}}{6}\left\{(3\alpha - 4\,\alpha\,T)\left\{1 - \frac{\alpha\,T^2}{2}\right\} + \alpha\,T\{\alpha\,T(3-2T)+6\}\right\} - \frac{3\alpha\,h\,\gamma\,T\,p^{-\beta}}{2}\right].$$

$$\left[-\gamma\,\beta(1-\beta)\,p^{-\beta} - \gamma(\beta+1)\,\beta\,p^{-\beta-1}\left\{\frac{\alpha\,T(3-2\,T)+6}{6}\right\}\left\{C + h\left\{T - \frac{\alpha\,T^3}{6}\right\}\right\}\right]$$

$$> \left[\gamma\,\beta\,p^{-\beta-1}\left\{\left\{\frac{3\alpha - 4\alpha\,T}{6}\right\}\left\{C + h\left\{T - \frac{\alpha\,T^3}{6}\right\}\right\} + \left\{\frac{\alpha\,T(3-2\,T)+6}{6}\right\}\left\{h\left\{1 - \frac{\alpha\,T^2}{2}\right\}\right\}\right\} - h\,\beta\,\gamma\,p^{-\beta-1}\left\{\frac{1}{2} - \frac{3\alpha\,T^2}{4}\right\}\right]^2 . \qquad \textbf{(3.23)}$$

From **(3.17)**, inequation **(3.22)** is obvious.
Now from **(3.15)** we have

$$\frac{A}{T^2} - \gamma\,p^{-\beta}\left\{\frac{3\alpha - 4\alpha T}{6}\right\} - \gamma\,p^{-\beta}\left\{\frac{\alpha\,T(3-2T)+6}{6}\right\}h\left\{1 - \frac{\alpha\,T^2}{2}\right\} + h\,\gamma\,p^{-\beta}\left\{\frac{1}{2} - \frac{3\alpha\,T^2}{4}\right\} = 0 \qquad \textbf{(3.24)}$$

and

$$\gamma(1-\beta)\,p^{-\beta} + \gamma\,\beta\,p^{-\beta-1}\left\{\frac{\alpha\,T(3-2\,T)+6}{6}\right\}\left\{C + h\left\{T - \frac{\alpha\,T^3}{6}\right\}\right\} - h\,\beta\,\gamma\,p^{-\beta-1}\left\{\frac{T}{2} - \frac{\alpha\,T^3}{4}\right\} = 0 \qquad \textbf{(3.25)}$$

In order to test that the profit is maximized, (3.24) and (3.25) have to be solved for T and p. These values must satisfy (3.22) and (3.23). Once the profit is maximized , cost is automatically minimized. As it is difficult to solve the problem by deriving a closed equation of the solution, Mat Lab Software has been used .

Thus we have used the following computational algorithm for our study.

4. COMPUTATIONAL ALGORITHMS

Step-1: Start.
Step-2: Initialize the value of the variables $A, h, \delta, \omega, \alpha, \beta, C_1, C_2, C_3, r, \gamma, t_1$.
Step-3: Evaluate $V(p,T)$
Step-4: Evaluate $\dfrac{\partial V(p,T)}{\partial p}, \dfrac{\partial V(p,T)}{\partial T}$.
Step-5: Solve the equation $\dfrac{\partial V(p,T)}{\partial p} = 0, \dfrac{\partial V(p,T)}{\partial T} = 0$.
Step-6: Choose the solution from Step-5.
Step-7: Evaluate $H \; and \, V_{pp}$.
Step-8: If the value of Step-7 i.e. $H > 0 \; and \, V_{pp} < 0$ then this solution is Optimal (maximum) and go to Step-10.
Step-9: Otherwise go to Step-6.
Step-10: End.

5. EXAMPLE

The values of the parameters are considered as follows:

$C = 50, A = 260, h = 2, \beta = 3.5, \gamma = 16 \times 10^7, \alpha = 0.02,$

Now using equation **(3.24), (3.25)** which can be solved to determine optimal $T^* = 1.4592$, $p^* = 65.4376$ and hence the optimal profit $V(T^*, p^*) = 825.563$.

6. OVERALL CONCLUSIONS

Here an EOQ model is derived for perishable items with nonlinear demand pattern Instantaneous rate of deterioration rate is used. The model is studied for Maximization of the profit per unit time. The result is illustrated with numerical example.

The model can further be studied for shortage state and for multiple items under identical conditions. This can also be extended for various deterioration conditions and also for discounted cash flow approach.

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