EVALUATION OF SOME RELIABILITY PARAMETERS FOR TELE-COMMUNICATION SYSTEM BY BOOLEAN FUNCTION TECHNIQUE

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ABSTRACT

The author in this paper have tried to evaluate the reliability and M.T.T.F. of telecommunication system by using Boolean function technique. Reliability of telecommunication system has been evaluated by considering that failure for various components of the telecommunication system follow arbitrary distribution. In particular, the symbolic reliability; viz., M.T.T.F. (mean time to failure) has also been calculated, in case, all the failure follow exponential time distribution. A numerical example with graphical illustrations has been appended at the end to highlight the practical utility of the model. Therefore, for a given set of parametric values of failure rates, one can obtained the values of the reliability and M.T.T.F. well in advance to forecast the operable behavior of such a complex system.

KEYWORDS: Boolean function technique, Perfect switching, Mean time to failure, Standby redundancy.

INTRODUCTION:

Past researchers in the field of reliability have used various methods to obtain symbolic reliability expressions for different complex systems. But most of the methods lead to cumbersome and tedious calculations. Keeping above facts in view, the author, in this chapter, has evaluated some reliability parameters of telecommunication system by using Boolean function technique. The whole system is divided into three parts, e.g., telephone instrument, main exchange and radar. The work of telephone instrument is to convert the human voice into radio waves and vice-versa. The main exchange receives these waves and transfers to their respective destination (dialed by instrument) through the function of transferring machine and radar. In this model, we assume that, there are two identical telephone instruments in parallel redundancy and the two identical transferring machines, in an exchange, in standby redundancy. On failure of one transferring machine, the standby unit followed on line through a perfect switching device. Reliability of telecommunication system has been evaluated by considering that failure times for various components of the telecommunication system follow arbitrary distribution. In particular, the symbolic reliability expression has also been calculated for Exponential and Weibull distributions. Moreover, an important parameter of reliability; viz., M.T.T.F. (mean time to failure) has also been calculated, in case, all the failures follow exponential time distribution. A numerical example with graphical illustrations has been appended at the end to highlight the practical utility of the model.

ASSUMPTIONS:

The following assumptions have been associated with this model:

i. Initially, all components are in operable condition.
ii. The state of each component and of the whole system is either good or fail.
iii. There is no repair facility.
iv. The states of all components of the system are statistically independent.
v. The reliability of all components of the system is known in advance.
vi. The failure times of all components are arbitrary.
vii. Supply between any two components of the system is fully reliable.
viii. On failure of one transferring machine of the main exchange the second machine followed on line through a perfect switching device and it consume no time.

Fig. 1 System configuration

NOTATIONS:
The following notations have been used throughout this model:

- $x_1, x_2, x_8$: States of instruments $I_1, I_2, I_3$ respectively;
- $x_3, x_4$: The states of transferring machines of first main exchange;
- $x_6, x_7$: The states of transferring machines of second main exchange;
- $x_5$: State of radar;
- $x'_i (i = 1, 2, \ldots, 8)$: Negation of $x_i$;
- $x_i (i = 1, 2, \ldots, 8)$: $\begin{cases} 0, & \text{in bad state} \\ 1, & \text{in good state} \end{cases}$;
- $\wedge, \vee$: Conjunction, disjunction;
- $\|$: This notation has used to represent logical matrix.
- $R_i (i = 1, 2, \ldots, 8)$: Reliability of $i^{th}$ component of the system.

FORMULATION OF MATHEMATICAL MODEL:

By using Boolean Function Technique, the condition of capability of the successful operation of this telecommunication system in terms of logical matrix is expressed as under:
\[ F(x_1, x_2, \ldots, x_8) = \begin{bmatrix} x_1 & x_3 & x_5 & x_6 & x_8 \\ x_1 & x_3 & x_5 & x_7 & x_8 \\ x_1 & x_4 & x_5 & x_6 & x_8 \\ x_2 & x_3 & x_5 & x_6 & x_8 \\ x_2 & x_3 & x_5 & x_7 & x_8 \\ x_2 & x_4 & x_5 & x_6 & x_8 \\ x_2 & x_4 & x_5 & x_7 & x_8 \end{bmatrix} \]  

... (1)

**SOLUTION OF THE MODEL:**

By the application of algebra of logics, equation (1) may be written as:

\[ F(x_1, x_2, \ldots, x_8) = \left[ x_5 \land x_8 \land f(x_1, x_2, \ldots, x_7) \right] \]  

where,

\[ f(x_1, x_2, \ldots, x_7) = \begin{bmatrix} x_1 & x_3 & x_6 \\ x_1 & x_3 & x_7 \\ x_1 & x_4 & x_6 \\ x_2 & x_3 & x_6 \\ x_2 & x_3 & x_7 \\ x_2 & x_4 & x_6 \\ x_2 & x_4 & x_7 \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \\ M_7 \\ M_8 \end{bmatrix} \]  

... (2)

\[ M_1 = \begin{bmatrix} x_1 \\ x_3 \\ x_6 \end{bmatrix} \]  

... (4)

\[ M_2 = \begin{bmatrix} x_1 \\ x_3 \\ x_7 \end{bmatrix} \]  

... (5)

\[ M_3 = \begin{bmatrix} x_1 \\ x_4 \\ x_6 \end{bmatrix} \]  

... (6)

\[ M_4 = \begin{bmatrix} x_1 \\ x_4 \\ x_7 \end{bmatrix} \]  

... (7)

\[ M_5 = \begin{bmatrix} x_2 \\ x_3 \\ x_6 \end{bmatrix} \]  

... (8)

\[ M_6 = \begin{bmatrix} x_2 \\ x_3 \\ x_7 \end{bmatrix} \]  

... (9)

\[ M_7 = \begin{bmatrix} x_2 \\ x_4 \\ x_6 \end{bmatrix} \]  

... (10)

and \[ M_8 = \begin{bmatrix} x_2 \\ x_4 \\ x_7 \end{bmatrix} \]  

... (11)

Using orthogonalisation algorithm, equation (3) may be written as:
Using algebra of logics, one may obtain the following results:

\[
M_1' M_2 = \begin{vmatrix}
    x_1' \\
    x_6
\end{vmatrix}
\begin{vmatrix}
    x_1 & x_3 & x_6 & x_7
\end{vmatrix}
\]

... (13)

Similarly,

\[
M_1' M_2' M_3 = \begin{vmatrix}
    x_1 & x_3 & x_4 & x_6
\end{vmatrix}
\]

... (14)

\[
M_1' M_2' M_3' M_4 = \begin{vmatrix}
    x_1 & x_3' & x_4 & x_6 & x_7
\end{vmatrix}
\]

... (15)

\[
M_1' M_2' M_3' M_4' M_5 = \begin{vmatrix}
    x_1 & x_2 & x_3
\end{vmatrix}
\]

... (16)

\[
M_1' M_2' M_3' M_4' M_5' M_6 = \begin{vmatrix}
    x_1' & x_2 & x_3 & x_6 & x_7
\end{vmatrix}
\]

... (17)

\[
M_1' M_2' M_3' M_4' M_5' M_6' M_7 = \begin{vmatrix}
    x_1' & x_2 & x_3' & x_4 & x_6
\end{vmatrix}
\]

... (18)

and

\[
M_1' M_2' M_3' M_4' M_5' M_6' M_7' M_8 = \begin{vmatrix}
    x_1' & x_2 & x_3' & x_4 & x_6 & x_7
\end{vmatrix}
\]

... (19)

Making use of equations (4) and (13) through (19) in equation (12), one can obtain:

\[
f(x_1, x_2, \ldots, x_7) = \begin{vmatrix}
    x_1 & x_3 & x_6 & x_7 \\
    x_1 & x_3' & x_4 & x_6 \\
    x_1 & x_3' & x_4 & x_6 \\
    x_1 & x_3 & x_6 & x_7 \\
    x_1' & x_2 & x_3' & x_4 & x_6 \\
    x_1' & x_2 & x_3' & x_4 & x_6 \\
    x_1' & x_2 & x_3' & x_4 & x_6 \\
    x_1' & x_2 & x_3' & x_4 & x_6 \\
\end{vmatrix}
\]

... (20)
In view of equation (20), equation (2) implies that:

\[
F(x_1, x_2, \ldots, x_8) = \begin{vmatrix}
  x_1 & x_3 & x_5 & x_6 & x_8 \\
  x_1 & x_3 & x_5 & x_6' & x_8 \\
  x_1 & x_3' & x_4 & x_5 & x_8 \\
  x_1 & x_3' & x_4 & x_5 & x_8 \\
  x_1' & x_2 & x_3 & x_5 & x_8 \\
  x_1' & x_2 & x_3 & x_5 & x_8 \\
  x_1' & x_2 & x_3' & x_4 & x_5 & x_8 \\
  x_1' & x_2 & x_3' & x_4 & x_5 & x_8 \\
\end{vmatrix}
\]  \ldots \text{(21)}

Since R.H.S. of equation (21) is the disjunction of pairwise disjoint conjunctions, therefore the reliability of considered telecommunication system is given by

\[
R_s = P_r \{ F(x_1, x_2, \ldots, x_8) = 1 \}
\]

\[
= R_1 R_2 \left[ R_3 R_5 + R_3 R_6 \left( -R_6 \frac{\beta_7}{\alpha} + R_1 \frac{\beta_7}{\alpha} \right) + (1 - R_1) R_4 (1 - R_6) R_7 + R_2 R_6 (1 - R_1) \right] + (1 - R_1) R_2 R_3 (1 - R_6) \frac{\beta_7}{\alpha} + (1 - R_1)(1 - R_3) R_2 R_4 R_6 \phantom{2}
\]
\[ + (1 - R_1) R_2 \frac{\beta_7}{\alpha} \frac{\beta_7}{\alpha} \frac{\beta_7}{\alpha} \phantom{2} \]  \ldots \text{(22)}

**PARTICULAR CASES:**

**Case I:** If the reliability of each component of the system is \( R \); then equation (22) yields

\[
R_s = 8R^5 - 12R^6 + 6R^7 - R^8 \]  \ldots \text{(23)}

**Case II:** When failure rates follow Weibull distribution

Let failure rate of each component of the complex system be \( a \), then the reliability of the complex system at an instant \( t \) is given by

\[
R_{sw}(t) = 8\exp\{-5at^p\} - 12\exp\{-6at^p\} + 6\exp\{-7at^p\} - \exp\{-8at^p\} \]  \ldots \text{(24)}

where, \( p \) is a positive parameter.

**Case III:** When failure rates follow exponential distribution

Exponential distribution is nothing but a particular case of Weibull distribution for \( p=1 \) and is very useful in numerous practical problems. Therefore the reliability of this telecommunication system at an instant \( t \) is given by:

\[
R_{se}(t) = 8e^{-5at} - 12e^{-6at} + 6e^{-7at} - e^{-8at} \]  \ldots \text{(25)}

The expression for M.T.T.F. in this case is given by M.T.T.F. = \( \int_0^\infty R_{se}(t)dt \)

\[
= \frac{1}{a} \left[ \frac{8}{5} - 2 + \frac{6}{7} - \frac{1}{8} \right] \phantom{2}
\]

\[
= \frac{0.332143}{a} \]  \ldots \text{(26)}
NUMERICAL COMPUTATION:

Setting $a=0.1$, $p=2$, $t=0, 1, 2…$ in equations (24) and (25) and $a=0.01, 0.02, 0.03,…$ in equation (26), one can sketch the graphs as given in figs 2,3 respectively.

RESULTS AND DISCUSSION:

Fig. 2 represents the reliability of the whole system at any time $t$, when failure rates follow exponential or Weibull distribution. A critical examination of the graph, reliability Vs time, indicates that the reliability of this complex system decreases approximately at a uniform rate in case of exponential distribution, whereas it decreases very rapidly when failure follows Weibull distribution. Further, fig. 3 computes the mean time to failure of the system for different values of failure rates. An inspection of the graph, M.T.T.F. Vs failure rate, shows that in the beginning M.T.T.F. decreases catastrophically but latter on it decreases approximately at a uniform rate.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$R_{SE}(t)$</th>
<th>$R_{SW}(t)$</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0.994086</td>
<td>0.994086</td>
</tr>
<tr>
<td>2</td>
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<td>3</td>
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</tr>
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<td>5</td>
<td>0.270046</td>
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</tr>
<tr>
<td>6</td>
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<td>7.578</td>
</tr>
<tr>
<td>7</td>
<td>0.113652</td>
<td>1.141</td>
</tr>
<tr>
<td>8</td>
<td>0.072255</td>
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</tr>
<tr>
<td>9</td>
<td>0.045519</td>
<td>1.2917</td>
</tr>
<tr>
<td>10</td>
<td>0.028475</td>
<td>9.6422</td>
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Table 1

![Reliability Vs Time](image)
<table>
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<th>a</th>
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<tr>
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<td>11.07143</td>
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<tr>
<td>0.1</td>
<td>3.32143</td>
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</tbody>
</table>

Table-2

Fig-3
REFERENCES

1. Dillon, B.S., Reliability Quality and Safety Engineers (Book style). Taylor Francis, U.K., 2004