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EVALUATION OF SOME RELIABILITY PARAMETERS OF A THREE STATE REPAIRABLE SYSTEM WITH ENVIRONMENTAL FAILURE

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ABSTRACT

This paper deals with a 3-state repairable complex system with three types of failure. In this paper a mathematical model has been developed for exponential failure and general repairs. Various states probabilities have been evaluated in the form of Laplace Transform. Expression for various Reliability parameters of the system are obtained by the inversion process and then computations are done for M.T.S.F. (Mean time to system failure) and Reliability of the system. All necessary graphical illustrations are given at the end so as to explain the practical utility of the model.

Key word: *Exponential failure, General repair, Environmental failure and Reliability*.

INTRODUCTION

Any system becomes unreliable due to various reasons. The principal causes of unreliability are design deficiency, unknown environmental conditions, lack of capacity of parts and equipments. It is observed that in the field of reliability, failures play a vital role. Many researchers [2,3, 5) define different types of failures. The causes of failure can be categorized into three basic types, i.e. wear out failure, random failure, and infant mortalities. But with increase in complexity of the present systems we cannot neglect the effect of various failures such as major failures, catastrophic failure, minor failure, critical human failure and Environmental failure, and so on.

The environmental failure mainly occurs due to the operation of the system in unusual conditions which were not considered at the designing stage of system.

Proctor and Singh [1], considered a repairable 3-state device in which no transition is possible between two types of failure, whereas Gupta and Agarwal S.C.[5],made the cost analysis of A-3 state repairable system with two types of failure, also a perusal of [2,3,4,6,7] reveals the fact that very little have been done in the direction of reliability parameter evaluation of complex systems incorporating the concept of environmental failure.

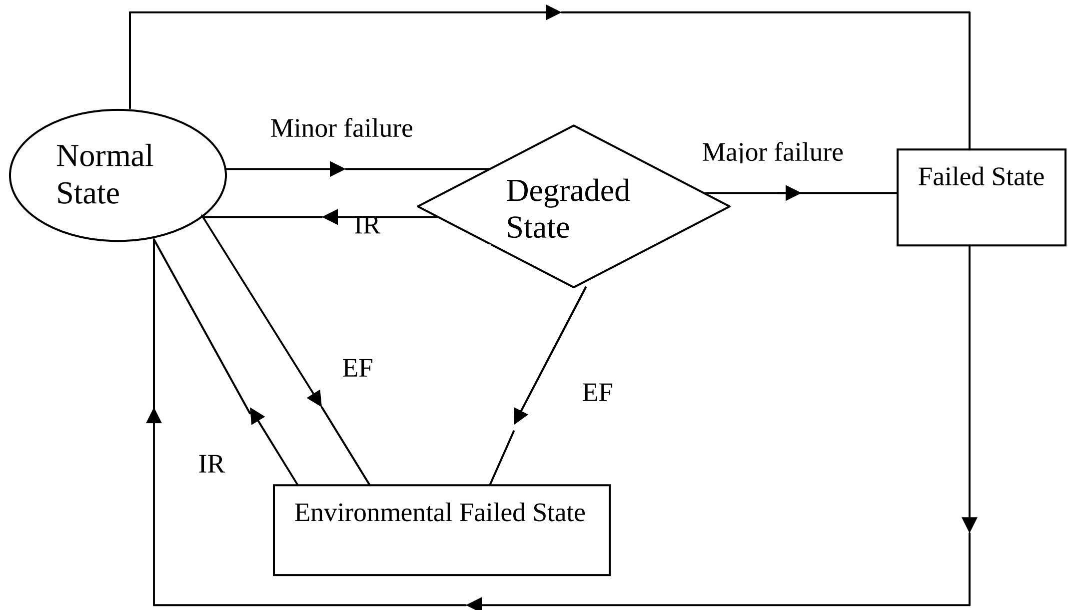
In view of the above, in this paper the author has tried to fill the gap by considering a 3-state repairable complex system with three types of failure. In this paper a mathematical model has been developed for exponential failure and general repairs. Various states probabilities have been evaluated in the form of Laplace Transform. Expression for various Reliability parameters of the system are obtained by the inversion process and then computations are done for M.T.S.F. (Mean time to system failure) and Reliability of the system. All necessary graphical illustrations are given at the end so as to explain the practical utility of the model.

ASSUMPTIONS:

1. Initially, the system is in operable state.
2. The system has 3-states normal (good), degraded and failed.
3. The system has only one repair facility which is always available to repair for every type of failure.

4. The failure and repair times for system follow exponential and general time distributions respectively.
5. If environment is disturbed, then whole system goes to failed state and nothing is operable. The failure rate γ and repair rate δ both being constant for environmental conditions.
6. The system can fail from degraded state too.
7. Repair is undertaken both in degraded and in failed state. After repair system works in normal state.

System Configuration Diagram:



IR: Instantaneous Repair, EF: Environmental Failure

NOTATION:

λ, μ	Constant failure rate of minor failure and major failure respectively
$\phi(x), \psi(y),$ and $S_i(r) \equiv$ where $i = \phi, \psi$	Transition rates and probability density functions, general repair in degraded state, failed state elapsed repair time x and y respectively.
$P_i(t)$	Probability for i=0,1,2,3 $0 \rightarrow$ Operable (Normal) state $1 \rightarrow$ Degraded state due to minor failure $2 \rightarrow$ Failed state due to major failure $3 \rightarrow$ Environmental failed state
$\overline{S}_k(s)$	Laplace transform of pdf of repair time $S_k(t)$
$\overline{F}(s)$	Laplace transform of F(t)
$S_i(s) = i(s)\exp\{-\int_0^x i(x)dx\}$	(Devis formula where $i = \phi, \mu$)

$$\int_0^\infty \exp\{-sx - \int_0^x \mu(x)d\}dx = \frac{1-\overline{S}_\mu(s)}{s} = F_\mu(s)$$

MATHEMATICAL FORMULATION OF THIS MODEL:

By probability and continuity arguments, the difference-differential equations for stochastic process which is continuous in time discrete in space, are as:

$$\left[\frac{d}{dt}+\lambda+\mu+\gamma\right]P_0(t)=\int_0^\infty P_1(x,t)\phi(x)dx+\int_0^\infty P_2(y,t)\psi(y)dy+\delta P_3(t) \qquad \dots (1)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\phi(x)+\mu+\gamma\right]P_1(x,t)=0 \qquad \dots (2)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\psi(y)\right]P_2(y,t)=0 \qquad \dots (3)$$

$$\left(\frac{d}{dt}+\delta\right)P_3(t)=\gamma.\left[P_0(t)+\int_0^\infty P_1(x,t)dx\right] \qquad \dots (4)$$

BOUNDARY CONDITIONS:

$$P_1(0,t)=\lambda.P_0(t) \qquad \dots 5)$$

$$P_2(0,t)=\mu.P_0(t)+\mu P_1(t) \qquad \dots (6)$$

INITIAL CONDITIONS:

$$P_i(0)=\begin{cases}1, \text{ where } \quad \mathrm{i}=0\\ 0, \text{ otherwise}\end{cases} \qquad \dots (7)$$

Taking Laplace Transform of equations (1) to (6) and using (7) one may obtain

$$\overline{P}_0(s)=\frac{1}{A(s)} \qquad \dots (8)$$

where

$$A(s)=(s+\lambda+\mu+\gamma)-\left\{\lambda.\overline{S}_\phi(s+\mu+\gamma)+\mu.(1+\lambda.F_\phi(s+\mu+\gamma)S_\psi(s)\right\}$$
$$+\frac{\delta\gamma}{(s+\delta)}\{1+\lambda.F_\phi(s+\mu+\gamma))\}$$

Now we have

$$\overline{P}_1(s)=\frac{\lambda}{A(s)}F_\phi(s+\mu+\gamma) \qquad \dots (9)$$

$$\overline{P}_2(s)=\frac{\mu}{A(s)}\{1+\lambda.F_\phi(s+\mu+\gamma)\}F_\psi(s) \qquad \dots (10)$$

$$\overline{P}_3(s)=\left(\frac{\gamma}{s+\delta}\right).\overline{P}_0(s)[1+\lambda.F_\phi(s+\mu+\gamma)] \qquad \dots (11)$$

EVALUATION OF UP AND DOWN STATE PROBABILITIES IN L.T. :

The Laplace transform of the probabilities that the system is in up and down state at time *t* are as follows:

$$\overline{P}_{up}(s) = \overline{P}_0(s) + \overline{P}_1(s)$$

$$\overline{P}_{up}(s) = \frac{1}{A(s)}\left[1 + \lambda F_{\phi}(s+\mu+\gamma)\right] \quad \dots (12)$$

$$\overline{P}_{down}(s) = \overline{P}_2(s) + \overline{P}_3(s)$$

$$\overline{P}_{down}(s) = \frac{\left[1 + \lambda . F_{\phi}(s+\mu+\gamma)\right]}{A(s)}\left\{\mu . F_{\psi}(s) + \frac{\gamma}{s+\delta}\right\} \quad \dots (13)$$

ASYMPTOTIC BEHAVIOR :

Using Able's Lemma in Laplace Transform

$\lim\limits_{s\to 0} s.\overline{F}(s) = F = \lim\limits_{t\to\infty} F(t)$ (say),

provided the limit on the right hand side exists, the following time independent state probabilities are obtained

$$P_{up} = \lim_{s\to 0} s \ \overline{P}_{up}(s)$$

$$P_{up} = \lim_{s\to 0} \frac{s\{(1 + \lambda . \lambda F_{\phi}(s+\mu+\gamma))\}}{A(s)}$$

$$P_{up} = \frac{1}{A'(0)} \quad \dots (14)$$

$$P_{down} = \lim_{s\to 0} s \ \overline{P}_{down}(s)$$

$$P_{down} = \frac{\gamma / \delta}{A'(0)} \quad \dots (15)$$

where

$$A'(s) = 1 - \left[\left\{\lambda . \overline{S}'_{\phi}(s+\mu+\gamma) + \mu . \left\{(1 + \lambda . F_{\phi}(s+\mu+\gamma))\overline{S}'_{\psi}(s) + \overline{S}_{\psi}(s)\lambda . F'_{\phi}(s+\mu+\gamma)\right\}\right.\right.$$
$$\left.\left. + \delta . \gamma\left\{\frac{(s+\delta)\lambda . F'_{\phi}(s+\mu+r)}{(s+\delta)^2} - (1 - \lambda . F_{\phi}(s+\mu+\gamma))\right\}\right]$$

$$A'(0) = \frac{\delta + \gamma}{\delta}$$

Note that

$$P_{up} + P_{down} = 1$$

PARTICULAR CASES:

Case I:

When repairs follow exponential time distribution,
setting

$$\overline{S}_{\phi}(s) = \frac{\phi}{s+\phi} \quad \text{and} \quad \overline{S}_{\psi}(s) = \frac{\psi}{s+\psi}$$

in P_{up} and then taking inverse of Laplace transform, we get

$$P_{up}(t) = \frac{\psi}{\psi + \mu} + \frac{\mu}{\psi + \mu} \exp\{-(\psi + \mu + \gamma)t\} \quad \text{… (16)}$$

Note that $P_{up} = \frac{\psi + \mu}{\psi + \mu} = 1$ at t=0

Case II:
For non-repairable system, all repair rates are zero. Then reliability and M.T.S.F. are obtained as follows:

$$R(s) = \frac{1}{(s + \lambda + \mu + \gamma)}$$

$$R(t) = \exp\{-(\lambda + \mu + \gamma)t\} \quad \text{… (17)}$$

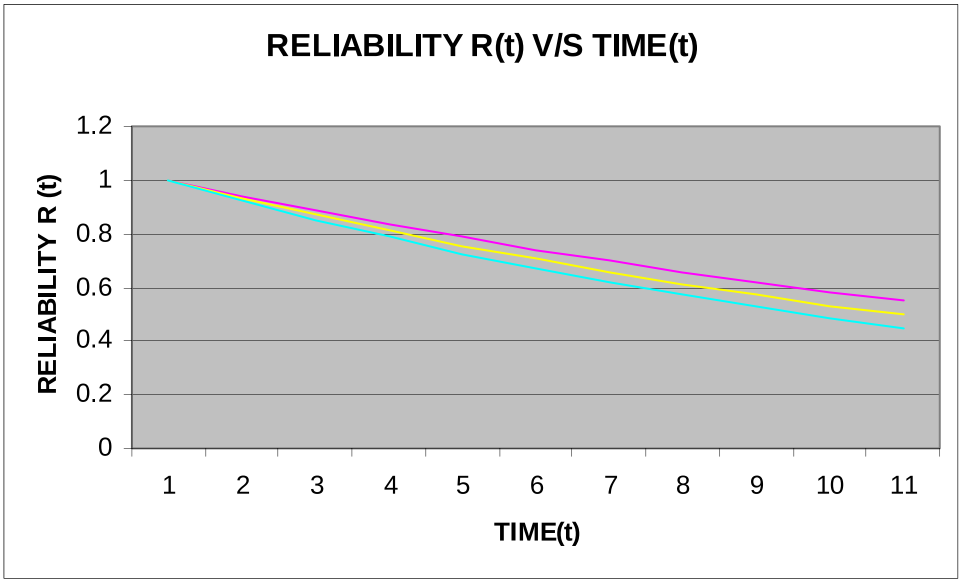
M.T.S.F. = $\lim_{s \to 0} R(s)$

M.T.S.F. = $\frac{1}{\lambda + \mu + \gamma}$ … (18)

NUMERICAL ILLUSTRATIONS:
ILLUSTRATION 1:
Setting $\mu = 0.02, \gamma = 0.03.$ in equation (17), the reliability of the system is calculated for different value of t, λ and μ .

t	R(t) at λ =0.01	R(t) at λ =0.02	R(t) at λ =0.03
0	1	1	1
1	0.941764534	0.93239382	0.923116346
2	0.886920437	0.869358235	0.852143789
3	0.835270211	0.810584246	0.78662786
4	0.786627811	0.755783742	0.726149037
5	0.740818221	0.70468809	0.670320046
6	0.697676326	0.65704682	0.618783392
7	0.65704682	0.612626394	0.571209064
8	0.618783392	0.571209064	0.527292424
9	0.582274825	0.532591801	0.486752256
10	0.548811636	0.495853038	0.449328964

Table 1.1



Graph 1.1

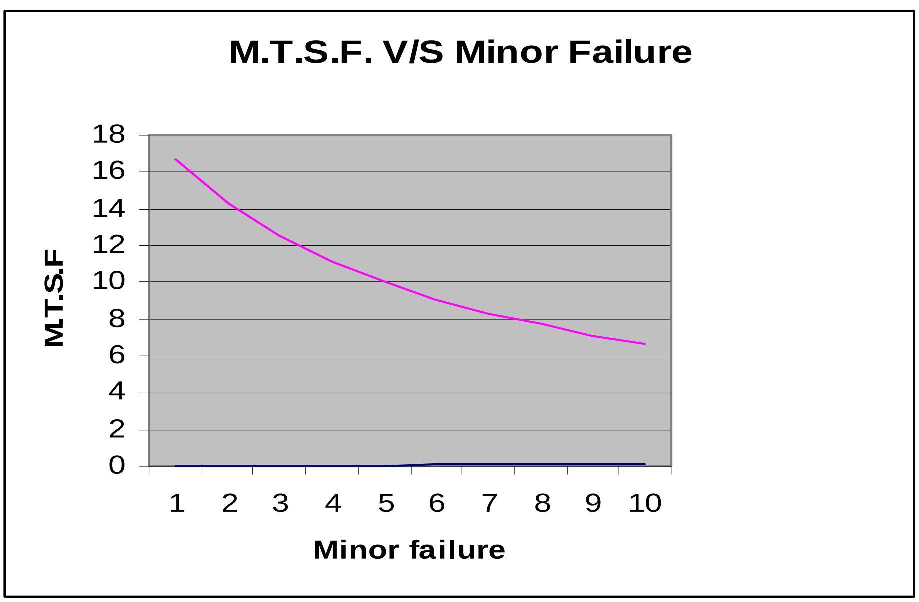
Series 1 λ =0.1, Series 2 λ =0.2, Series 3 λ =0.3

ILLUSTRATION 2 (a):

Setting $\mu = 0.02, \gamma = 0.03$ in equation (18) then M.T.S.F is calculated for different value of λ

Minor Failure (λ)	M.T.S.F.
0.01	16.666670
0.02	14.285714
0.03	12.500000
0.04	11.111111
0.05	10.000000
0.06	9.0909090
0.07	8.3333330
0.08	7.6923076
0.09	7.1428571
0.1	6.6666667

Table 1.2

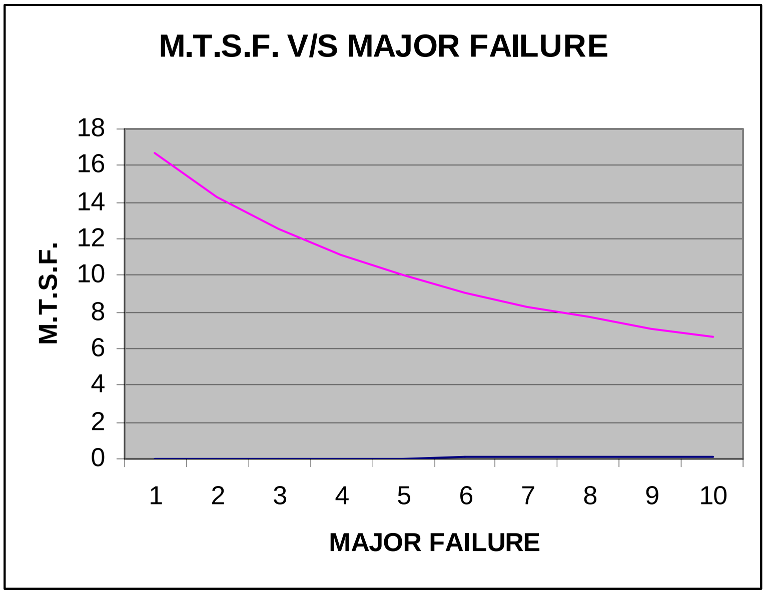


Graph 1.2

ILLUSTRATION 2(b):
Setting $\lambda = 0.01, \gamma = 0.03$ in equation (18) then M.T.S.F is calculated for different value of μ

Major Failure(μ)	M.T.S.F.
0.01	16.66667
0.02	14.28571
0.03	12.50000
0.04	11.11111
0.05	10.00000
0.06	9.090909
0.07	8.333333
0.08	7.692307
0.09	7.142857
0.1	6.666667

Table1.3



Graph 1.3

INTERPRETATION / CONCLUSIONS:

1. An inspection of Table 1.1 and the graph 1.1 (Reliability V/S Time) indicates that the reliability of the system decreases with the increase in time.
2. From the table 1.3 and graph 1.3, the behavior of this 3-state repairable system can be easily judged for M.T.S.F.
   M.T.S.F. decreases as λ, μ increase and after some time M.T.S.F. becomes almost constant.

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