HALL CURRENT EFFECTS ON A FLOW IN A VARIABLE MAGNETIC FIELD PAST AN INFINITE VERTICAL, POROUS FLAT PLATE

Mark O. Okongo, Gichohi P. Nderitu & James M. Mugambi
Physical Sciences Department, Chuka University, Kenya

ABSTRACT

In this study, natural convection flow of an incompressible fluid past an infinite vertical flat plate in a variable magnetic field has been investigated. The research is done to find the effects of Hall current on primary and secondary velocities of a fluid. In this case the flow is considered as steady and restricted to laminar domain. The equations governing the flow are solved numerically using finite difference method. The difference equations are then solved iteratively for various values of Hall current parameter ranging from 0.0 to 1.0. This study shows that an increase in the Hall current affects the velocity of the fluid.

Keywords: *Electron pressure gradient, Hall currents, Magnetohydrodynamics, Magnetic induction vector, Magnetic field intensity.*

1. INTRODUCTION

Hydro magnetic is the science of the motion or flow of electrically conducting fluid in the presence of a magnetic field. The situation is one of mutual interaction between the fluid velocity field and the electromagnetic field. When an electrically conducting material (whether solid or denser fluid) moves in a magnetic field, it experiences a force that tends to move it perpendicularly to the electric field. In the case of a fluid, this happens due to the magnetic field acting on both electrons and ionized atoms. The motion of the electrically conducting fluid past a magnetic field induces current, a phenomenon known as Hall Effect. This induced current in turn produces induced magnetic field. Let $\vec{B}$ be the magnetic field and $\vec{E}$ be the electric field. If the magnetic field $\vec{B}$ is perpendicular to $\vec{E}$, there will be an electromagnetic force. The resulting current and magnetic field combine to produce a force that resists the fluid motion. The induced current also generates its own magnetic field, which distorts the original magnetic field. If the electromagnetic force generated is of the same order of magnitude as the hydro dynamical and inertia forces, the electromagnetic forces have to be taken into account in the flow field.

The first research in magneto hydrodynamics (MHD) flows was done by Faraday [1] in 1839. He performed an experiment with mercury flowing in a glass tube between the poles of a magnet and proposed the use of tidal currents in the terrestrial magnetic field for power generation.

Prasada *et al* [2] solved the problem of MHD flow with wavy porous boundary, the influence of the heat source, parameters, suction velocity and waviness of the boundary on the flow field was numerically analyzed.

Kumar *et al* [3] presented their study on compressible magneto hydrodynamic boundary layer in the stagnation region of a sphere. The effects of the induced magnetic field, mass transfer and viscous dissipation were taken into account.

Rao *et al* [4] studied the heat transfer in a porous medium in the presence of transverse magnetic field. The effects of the heat source parameter and Nusselt number were analyzed. They discovered that the effect of increasing porous parameter is to increase the Nusselt number.

Ram [5] used the finite difference method to analyze the MHD stokes problem for a vertical plate and ion-slip currents.

Dash and Asha [6] presented magneto hydrodynamic unsteady free convection effect on the flow past an exponentially accelerated vertical plate. They observed that exponential acceleration of the plate has no significant contribution over the impulsive motion.

Dash and Das [7] considered heat transfer in viscous flow a long a plane wall with periodic suction and heat source. The effects of various parameters on the heat transfer in a three-dimensional laminar boundary layer past a flat plate in the presence of a heat source when a sinusoidal transverse suction velocity is applied to the walls were studied.

Ram *et al* [8] analyzed the effects of Hall current and wall temperature oscillation on convective flow in a rotation fluid through porous medium bounded by an infinite vertical limiting surface. The effect of various parameters on the velocity and shear stress were determined.

Hong-Sen and Huang [9] investigated some transformations for natural convection on a vertical flat plate embedded in porous media with prescribed wall temperature. They analyzed transformation for boundary layer equation for two dimensional steady natural convection a long a vertical flat plate, embedded in a porous media with prescribed wall temperature.

Kinyanjui *et al* [10] presented their work on MHD free convection heat and mass transfer of a heat generating fluid past an impulsively stated infinite vertical porous plate with Hall current and radiation absorption. An analysis of the effects of the parameters on skin friction, rates of mass and heat transfer was reported.

Kwanza *et al* [11] presented their work on MHD strokes free convection flow past an infinite vertical porous plate subjected to a constant heat flux with ion-slip and radiation absorption. The concentration, velocity and temperature distributions were discussed and result were presented in tables and graphs.

Adel *et al* [12] investigated heat and mass transfer along a semi-infinite vertical flat plate under the combined buoyancy force effects of thermal and species diffusion in the presence of a strong non-uniform magnetic field. The similarity equations were solved numerically by using a forth-order Runge-Kutta scheme with the shooting method.

Emad *et al* [13] studied Hall current effect on magneto hydrodynamics free-convection flow past a semi-infinite vertical plate with mass transfer. They discussed the effects of magnetic parameter, Hall parameter and the relative buoyancy force effect between species and thermal diffusion on the velocity, temperature and concentration.

Youn J. [14] investigated the unsteady two-dimensional laminar flow of a viscous incompressible electrically conducting fluid in the vicinity of semi-infinite vertical porous moving plate in the presence of a transverse magnetic field. The plate moves with constant velocity in the direction of fluid flow, and the free stream velocity follows the exponentially increasing small permutation law, the effect of increasing values of the suction velocity parameter results into a slight increase in surface skin friction for lower values of plate moving velocity. It was also observed that for several values of Prandtl number, the surface heat transfer decreases by increasing the magnitude of suction velocity.

Emad *et al* [15] studied the effects of viscous dissipation and Joule heating on MHD free convection flow past a semi-infinite vertical flat plate in the presence of the combined effect of Hall and ion-slip current for the case of the power-law variation of the wall temperature. They found the magnetic field acts as a retarding force on the tangential flow but have a propelling effect on the induced lateral flow. The skin-friction factor for the lateral flow increases as the magnetic field increases. The skin- friction factor for the tangential and lateral flows are increased while the Nusselt number is decreased if the effect of viscous dissipation, Joule heating and heat generation are considered Hall and ion-slip terms were ignored in applying Ohm's law as it has no marked effect for small and moderate values of the magnetic field.

Hayat T *et al* [16] investigated the unsteady flow of an incompressible fluid in a circular duct embedded in a porous medium. The fluid was made to conduct in the presence of magnetic field and the influence of Hall current was investigated. Laplace transform technique was used to determine the exact solution for; constantly accelerating flow, sudden started flow, flow rate of trapezoidal variation with time and oscillatory flow. After examining the governing equation for an unsteady incompressible, electrically conducting fluid in circular duct, they discovered that the flow, Hall Effect and porosity depend on the material parameters.

Angail [17], studied the magneto hydrodynamic model of boundary – layer equations for conducting viscous fluids to find the effect of free convection currents with two relaxation times on the flow of a viscous conducting fluid. He adopted the solution of one-dimensional transient problem to a whole space with a plane distribution of heat sources. He observed that as the Alfven velocity the velocity of the fluid increased. This was mainly due to the fact that the effect of magnetic field tends to decelerate in fluid particles. He also noted that the velocity increased as the Grashof number (Gr) increased while it decreases when the Prandtl number (Pr) increases.

Eldabe N.T. *et al* [18] discussed numerical solutions for problems which involved both the heat and mass transfer in hydro magnetic flow of a micro polar fluid past a stretching surface with Ohmic heating and viscous dissipation using Chebychev finite difference method. Various magnetic field parameters were used and they concluded that decrease in magnetic field parameters resulted to increase in temperature and concentration and decrease in velocity profiles.

Jordon [19] analyzed the effects of thermal radiation and viscous dissipation on MHD free-convection flow over a semi-infinite vertical porous plate. The network simulation method was used to solve the boundary-layer equations based on the finite difference method. It was found that an increase in viscous dissipation leads to an increase of both velocity and temperature profiles, an increase in the magnetic parameter leads to an increase in the temperature profiles and a decrease in the velocity profiles. Finally an increase in the suction parameter leads to an increase in the local skin-friction and Nusselt number.

2. PROBLEM FORMULATION: APPROXIMATIONS AND ASSUMPTIONS

In simplifying the equations governing the fluid flow in this study, the following assumptions will be made;

1. Liquid metals and ionized gases have permeability μ_e, so that we write $\hat{B} = \mu_e \hat{H}$.
2. All velocities considered are much smaller compared to velocity of light q^2 / c^2 <<1.
3. The fluid flow is restricted to a laminar domain.

4. The fluid is incompressible (density is assumed to be constant).
5. There is no external applied electric field.
6. The plate is electrically non-conducting.
7. The fluid is considered to be electrically conducting with no surplus electric charge distribution.
8. Viscosity μ is assumed to be constant.
9. Thermal conductivity k is assumed to be constant.
10. There are no chemical reactions taking place in the fluid.
11. Body forces action on the fluid caused by gravity and magnetic fields are assumed vital in the analysis.
12. The force $\sigma\hat{E}$ due to electric field is negligible compared to the force $\hat{J}\times\hat{B}$ due to the magnetic field.

3. GENERAL GOVERNING EQUATIONS

The equations governing the flow of an incompressible, electrically conducting fluid in the presence of a strong, non-uniform magnetic field are highlighted in this chapter. The equations are equation of continuity, equation of momentum, energy equation, concentration equation, Maxwell's equation and Ohm's law.

3.1. Equation of Continuity

This equation is based on two propositions:
That the mass of the fluid is conserved. That is mass can neither be created nor destroyed.
That the flow is continuous that is empty spaces do not occur between particles which were in contact.
In this case we consider an incompressible fluid (i.e. density is a constant), therefore the equation of continuity in tensor form

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}=0 \qquad (3.1)$$

$$0+\rho\frac{\partial u}{\partial x}=0$$

$$\Rightarrow\frac{\partial u}{\partial x}=0 \qquad (3.2)$$

If this motion is three dimensions, equation 3.2 in *x, y, z* component becomes

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (3.3)$$

Equation of continuity in vector form

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}=0.$$

Since density is constant then $\frac{\partial\rho}{\partial t}=0$

The equation reduces to $\nabla\cdot q=0$ where $\nabla=\underset{\sim}{i}\frac{\partial}{\partial x}+\underset{\sim}{j}\frac{\partial}{\partial y}+\underset{\sim}{k}\frac{\partial}{\partial z}$ (3.3)

hence in three dimension

$$\left(\underset{\sim}{i}\frac{\partial}{\partial x}+\underset{\sim}{j}\frac{\partial}{\partial y}+\underset{\sim}{k}\frac{\partial}{\partial z}\right)\left(u\underset{\sim}{i}+v\underset{\sim}{j}+w\underset{\sim}{k}\right)=0$$

$$=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (3.4)$$

If the flow is along *x* – axis, and the plate is infinite it extends and lies along *x* and *z* axis thus the physical condition depends only on *y*, then the equation 3.4 reduces to

$$\frac{\partial v}{\partial y}=0 \qquad (3.4.a)$$

Since the velocity gradient is along y – axis. On integrating 3.4a we get
V = constant = V_0 (3.4.b)

3.2 Equation of momentum

The equation of motion is based on the Newton second law of motion.
The net rate of change of momentum must equal the net sum of forces acting on the fluid. These equations are also known as Navier-Stokes Equation.
In vector notation, the equation of motion considering the body force due to gravity and electromagnetic force only is written as

$$\frac{\partial q}{\partial t}+\left(q.\nabla\right)q=-\frac{1}{\rho}\nabla p+\nu\nabla^2 q+F\ . \tag{3.5}$$

where $\frac{\partial q}{\partial t}$ is the temporal acceleration.

$\left(q.\nabla\right)q$ is the convective acceleration and allows for acceleration even when the flow is steady.

$-\frac{1}{\rho}\nabla p$ is pressure gradient.

$\nu\nabla^2 q$ is force due to viscosity.

F is the body force
Considering the body forces due to gravity and the electromagnetic force only, then these two forces replace the body force.

Hence $$\frac{\partial q}{\partial t}+\left(q.\nabla\right)q=-\frac{1}{\rho}\nabla p+\nu\nabla^2 q+F_e+F_g \tag{3.6}$$

The electromagnetic force $F_e=\rho_e E+J\times B$ (3.7)

where J = current density
B = Magnetic field
E = electric field
ρ_e = space charge

Which is approximated to $F_e=J\times B$. (3.8)

Hence the Navier Stoke equation becomes

$$\frac{\partial q}{\partial t}+\left(q.\nabla\right)q=-\frac{1}{\rho}\nabla p+\nu\nabla^2 q-\rho g+\left(J\times B\right)\ . \tag{3.9}$$

The electrostatic force $\rho_e E$ in many flow problems is negligibly small in comparison with the electromagnetic force$\left(J\times B\right)$. The interaction between the magnetic field and the flow field is important in the dynamics of a conducting fluid. When the electromagnetic force F_e is in the direction perpendicular to both the magnetic field H and the current density J there will be a significant influence on the flow field. Thus the direction of the magnetic field H will have a significant influence on the flow field.

3.2. Equation of energy

This equation is derived from the first law of thermodynamics
The amount of heat added to a system dQ equals to the change in internal energy dE plus the work done dW

$$dQ=dE+dW \tag{3.10}$$

If heat produced by external forces is ignored then in tensor form is written as

$$\rho\frac{\partial h}{\partial t}+\frac{\partial\left(\rho U_j h\right)}{\partial x_j}=\frac{\partial p}{\partial t}+\frac{\partial\left(\rho U_j p\right)}{\partial x_j}-\frac{\partial q_j}{\partial x_j}+\phi \tag{3.11}$$

Where ϕ is the viscous dissipation in three dimension and it is given by

$$\phi = \mu\left\{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right\}^2+\left\{\left(\frac{\partial u}{dy}\right)+\left(\frac{\partial v}{\partial y}\right)\right\}^2+\left\{\left(\frac{\partial v}{\partial z}\right)+\left(\frac{\partial w}{\partial y}\right)\right\}^2+\left\{\left(\frac{\partial w}{\partial x}\right)+\left(\frac{\partial v}{\partial z}\right)\right\}^2.$$

When we apply thermodynamic definition of *h*.

$dQ = dE + dW$ becomes

$$h = E + \frac{p}{\rho}. \tag{3.12}$$

where *E* is the specific internal energy. In differential form equation (3.12) becomes

$$dh = dE + \frac{dp}{\rho} + pd\left(\frac{1}{\rho}\right). \tag{3.13}$$

3.4. Concentration equation

This is based on the principle of mass conservation for each species in a fluid mixture. In tensor form the diffusion equation is

$$\frac{\Delta C_j}{\Delta t} = \frac{\partial J_j}{\partial x_j}. \tag{3.14}$$

3.5. Ohm's law

The law characterizes the ability of a material to transport electric charge under the influence of an applied electric field. For an electrically conducting material at rest the current density is

$$\hat{J} = \sigma\hat{E} \tag{3.15}$$

In moving electrically conducting fluids the magnetic field induces a voltage in the conductor of the magnitude

$\hat{q}\times\hat{B}$

The generalized Ohm's law is

$$\hat{J} = \sigma\ \hat{E}+\hat{q}\times\hat{B}\ . \tag{3.16}$$

3.6. Maxwell's equation

These equations provide links between the electric and magnetic fields independent of the properties of the matter. In this we consider the following set of equations

$$\nabla\times\hat{H} = \hat{J}$$

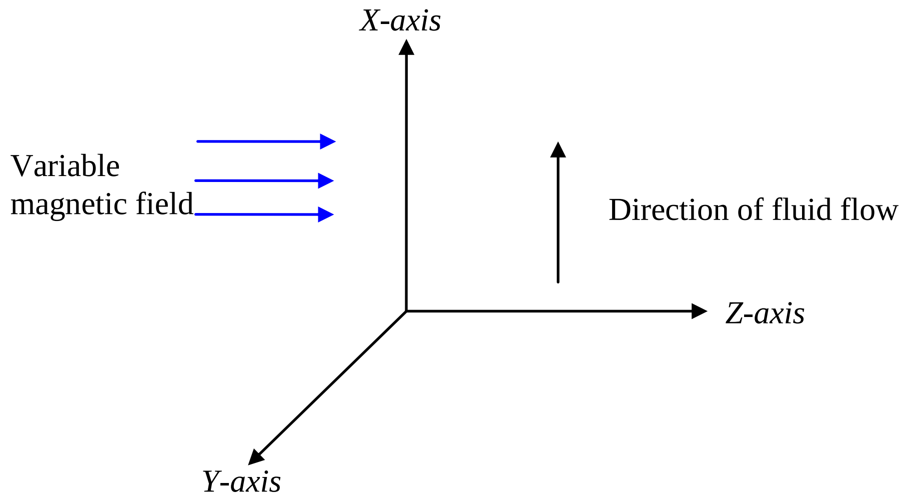
$$\nabla.\hat{B} = 0$$

$$\nabla\times\hat{E} = -\frac{\partial\hat{B}}{\partial t}. \tag{3.17}$$

4. SPECIFIC EQUATIONS GOVERNING FLUID FLOW

In this section the specific equations governing incompressible free convection fluid flow in the presence of a variable magnetic field are derived. The magnetohydrodynamic flow is considered past an infinite vertical porous plate.

The *x* – axis is taken along the plate in vertical upward direction, which is the direction of the flow



4.1. Magnetic transport equation

Combining Maxwell's equation and Ohm's law

$$\nabla \times \hat{H} = \hat{J} = \sigma\left(\hat{E} + \hat{V} \times \hat{B}\right) \tag{4.1}$$

where

$\hat{H}$	=	Magnetic field intensity
σ	=	Electrical conductivity
$\hat{B}$	=	Magnetic induction vector
$\hat{E}$	=	Electric field

$$\Rightarrow \left(E + \hat{V} \times \hat{B}\right) = \frac{1}{\sigma}\left(\nabla \times \hat{H}\right)$$

4.2. Momentum equation

Since the fluid is in motion it possess momentum, hence we consider the equation of momentum.

$$\rho\left(\frac{\partial \hat{q}}{\partial t}\right) + \rho\left(\hat{q} \cdot \nabla\right)\hat{q} = -\frac{\partial P}{\partial x} + \rho\upsilon\nabla^2\hat{q} - \rho g + \hat{J} \times \hat{B}. \tag{4.2}$$

The momentum equation is evaluated at the edge of the boundary layer where $\rho \to \rho_\infty$ and $V \to 0$. This is because at the boundary layer the velocity of the fluid is at its minimum.

The pressure gradient in the y – direction results from the change of elevation

Thus $\frac{\partial P}{\partial x} = \rho_\infty g$

where ρ_∞ is the density near the plate.

Combining the pressure term and the body force term, gives

$$-\rho g - \left(-\rho_\infty g\right) = \rho_\infty g - \rho g$$

$$= g\left(\rho_\infty - \rho\right)$$

The velocity profile at various Y – positions is

$$\left(\hat{q} \cdot \nabla\right)\hat{q} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$\upsilon\nabla^2\hat{q} = \upsilon\frac{\partial^2 v}{\partial x^2}$$

Equation 4.2 becomes

$$\rho\frac{\partial v}{\partial t}+\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=\rho\upsilon\frac{\partial^2 v}{\partial x^2}+g\left(\rho_\infty-\rho\right)+\hat{J}\times\hat{B}\,. \tag{4.3}$$

The volumetric coefficient of thermal expansion

$$\beta=-\frac{1}{\rho}\left(\frac{\Delta\rho}{\Delta T}\right)$$

where T is temperature

$$=-\frac{1}{\rho}\left(\frac{\rho_\infty-\rho}{T_\infty-T}\right)$$

$$=\frac{1}{\rho}\left(\frac{\rho_\infty-\rho}{T-T_\infty}\right)$$

$$\Rightarrow \rho_\infty-\rho=\beta\rho\left(T-T_\infty\right)$$

The current density $J=\sigma\left(v\times B\right)$, But $B=\mu_e H$

$$J=\sigma\left(v\times\mu_e H\right).$$

Hence the force term $J\times B=\sigma\left(v\times\mu_e H\right)\times\mu_e H$

$$=\sigma\mu_e^2H^2v\,.$$

Thus equation 4.3 reduces to

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=\rho\upsilon\frac{\partial^2 v}{\partial x^2}+g\beta\rho\left(T-T_\infty\right)+\sigma\mu_e^2H^2v$$

Dividing both sides by ρ to get

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\frac{\partial^2 v}{\partial x^2}+g\beta\left(T-T_\infty\right)+\frac{\sigma\mu_e^2H^2v}{\rho}\,. \tag{4.4}$$

In the Z – axis direction, the force term $J\times B$ becomes $\sigma\mu_e^2H^2w$

Where w is the velocity of the fluid in the Z direction

Hence the equation of momentum in the Z – axis will be

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=\upsilon\frac{\partial^2 w}{\partial x^2}-\frac{\sigma\mu_e^2H^2w}{\rho}\,. \tag{4.5}$$

4.3. Non – Dimensionalization

The non – dimensionalization process is important because the results obtained for a surface experiencing one set of conditions can be applied to a geometrically similar surface under different conditions. These conditions may be nature of the fluid, the fluid velocity or size of the surface.

In MHD flow problems the some non-dimensional parameters are important and are defined as follows.

4.3.1. Reynolds number, Re

It is the ratio of inertia force to the viscous force if for any flow this number is less than one the inertia force is negligible and if it is large, one can ignore viscous force.

$$\mathrm{Re}=\frac{\rho UL}{\mu}=\frac{UL}{\upsilon}.$$

4.3.2. Grashof number, Gr

It is the ratio of buoyancy forces to viscous forces. If it is large then there is strong convective current.

$$Gr = \frac{\upsilon g \beta \left(T_w^* - T_\infty\right)}{U^3}.$$

4.3.3. Prandtl number, Pr

It is the ratio of viscous force to thermal force. The Prandtl number is large when thermal conductivity is less than one and viscosity is large and it is small when viscosity is less than one and thermal conductivity is large.

$$\Pr = \frac{\mu C_p}{k}.$$

4.3.4. Magnetic parameter/Hartmann Number, *M*

It is the ratio of magnetic force to the viscous force.

$$M^2 = \frac{\sigma \mu_e^2 H_0^2 \upsilon}{U^2 \rho}.$$

4.3.5. Eckert number, Ec.

It is the ratio of the kinetic energy to thermal energy.

$$Ec = \frac{U^2}{C_p\left(T_w^* - T_\infty^*\right)}.$$

If we let all the variables with the superscript (*) star to represent dimensional variables then the non-dimensionalization can be based on the following scaling variables.

$$t = \frac{t^* U^2}{\upsilon}, \qquad x = \frac{x^* U}{\upsilon}, \qquad y = \frac{y^* U}{\upsilon} \qquad u = \frac{u^*}{U}$$

$$v = \frac{v^*}{U}, \qquad w = \frac{w^*}{U}, \qquad H = \frac{H^*}{H_0} \text{ and } \qquad \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}.$$

Equations 4.4 and 4.5 can be written using dimensional variables as

$$\frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = \upsilon \frac{\partial^2 v^*}{\partial x^{*2}} + g\beta\left(T^* - T_\infty^*\right) + \frac{\sigma \mu_e^2 H^{*2} v^*}{\rho} \tag{4.6}$$

$$\frac{\partial w^*}{\partial t^*} + u^* \frac{\partial w^*}{\partial x^*} + v^* \frac{\partial w^*}{\partial y^*} = \upsilon \frac{\partial^2 w^*}{\partial x^{*2}} - \frac{\sigma \mu_e^2 H^{*2} w^*}{\rho} \tag{4.7}$$

These are non-dimensionalized as follows

$$\frac{\partial^2 H^*}{\partial x^{*2}} = \frac{\partial}{\partial x^*}\left(\frac{\partial\left(H H_0\right)}{\partial x} \cdot \frac{\partial x}{\partial x^*}\right) = \frac{\partial}{\partial x^*}\left(H_0 \frac{\partial H}{\partial x}\right) \cdot \frac{U}{\upsilon}$$

$$= \frac{H_0 U}{\upsilon} \cdot \frac{\partial^2 H}{\partial x^2} \cdot \frac{\partial x}{\partial x^*}$$

$$= \frac{H_0 U}{\upsilon} \cdot \frac{U}{\upsilon} \cdot \frac{\partial^2 H}{\partial x^2} = \frac{H_0 U^2}{\upsilon^2} \frac{\partial^2 H}{\partial x^2}$$

$$\frac{\partial\left(v^*H^*\right)}{\partial y^*}=\frac{\partial\left(vU\cdot HH_0\right)}{\partial y}\cdot\frac{\partial y}{\partial y^*}=UH_0\frac{\partial\left(vH\right)}{\partial y}\cdot\frac{U}{\upsilon}$$

$$=\frac{U^2H_0}{\upsilon}\left(v\frac{\partial H}{\partial y}+H\frac{\partial v}{\partial y}\right)$$

$$\frac{\partial v^*}{\partial t^*}=\frac{\partial\left(vu\right)}{\partial t}\cdot\frac{\partial t}{\partial t^*}=U\frac{\partial v}{\partial t}\cdot\frac{U^2}{\upsilon}=\frac{U^3}{\upsilon}\frac{\partial v}{\partial t}$$

$$u^*\frac{\partial v^*}{\partial x^*}=Uu\frac{\partial\left(vU\right)}{\partial x}\cdot\frac{\partial x^*}{\partial x^*}=uU^2\frac{\partial v}{\partial x}\cdot\frac{U}{\upsilon}=\frac{uU^3}{\upsilon}\frac{\partial v}{\partial x}$$

$$v^*\frac{\partial v^*}{\partial y^*}=vU\frac{\partial\left(vU\right)}{\partial y}\cdot\frac{\partial y}{\partial y^*}=vU^2\frac{\partial v}{\partial y}\cdot\frac{U}{\upsilon}=\frac{vU^3}{\upsilon}\frac{\partial v}{\partial y}$$

$$\frac{\partial^2 v^*}{\partial x^{*2}}=\frac{\partial}{\partial x^*}\frac{\partial\left(vU\right)}{\partial x}\cdot\frac{\partial x}{\partial x^*}=\frac{\partial}{\partial x^*}\cdot U\frac{\partial v}{\partial x}\cdot\frac{U}{\upsilon}=\frac{U^2}{\upsilon}\frac{\partial^2 v}{\partial x^2}\cdot\frac{\partial x}{\partial x^*}$$

$$=\frac{U^2}{\upsilon}\cdot\frac{U}{\upsilon}\frac{\partial^2 v}{\partial x^2}=\frac{U^3}{\upsilon^2}\frac{\partial^2 v}{\partial x^2}$$

$$H^*=H_0H\,,\quad T^*-T^*_\infty=\left(T^*_w-T^*_\infty\right)\theta$$

Substituting the above, equation (4.7) becomes

$$\frac{U^3}{\upsilon}\frac{\partial v}{\partial t}+\frac{uU^3}{\upsilon}\frac{\partial v}{\partial x}+\frac{vU^3}{\upsilon}\frac{\partial v}{\partial y}=\upsilon\frac{U^3}{\upsilon^2}\frac{\partial^2 v}{\partial x^2}+g\beta\left(T^*_w-T^*_\infty\right)\theta+\frac{\sigma\mu_e^2H^2H_0^2Uv}{\rho}$$

Multiplying by $\dfrac{\upsilon}{U^3}$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{\mu}{\rho\upsilon}\left(\frac{\partial^2 v}{\partial x^2}\right)+\frac{g\beta\upsilon}{U^3}\left(T^*_w-T^*_\infty\right)\theta+\frac{\sigma\upsilon\mu_e^2H^2H_0^2v}{\rho U^2}$$

But $Gr=\dfrac{\upsilon g\beta\left(T^*_w-T_\infty\right)}{U^3}$ and $M^2=\dfrac{\sigma\upsilon\mu_e^2H_0^2}{\rho U^2}$

Thus

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{\partial^2 v}{\partial x^2}+Gr\theta+M^2H^2v\quad.\qquad(4.8)$$

$$\frac{\partial w^*}{\partial t^*}=\frac{\partial\left(wU\right)}{\partial t}\cdot\frac{\partial t}{\partial t^*}=U\frac{\partial w}{\partial t}\cdot\frac{U^2}{\upsilon}=\frac{U^3}{\upsilon}\frac{\partial w}{\partial t}$$

$$u^*\frac{\partial w^*}{\partial x^*}=uU\cdot\frac{\partial\left(wU\right)}{\partial x}\cdot\frac{\partial x}{\partial x^*}=uU^2\frac{\partial w}{\partial x}\cdot\frac{U}{\upsilon}=\frac{uU^3}{\upsilon}\frac{\partial w}{\partial x}$$

$$v^*\frac{\partial w^*}{\partial y^*}=vU\frac{\partial\left(wU\right)}{\partial y}\cdot\frac{\partial y}{\partial y^*}=vU^2\frac{\partial w}{\partial y}\cdot\frac{U}{\upsilon}=\frac{vU^3}{\upsilon}\frac{\partial w}{\partial y}$$

$$\upsilon\frac{\partial^2 w^*}{\partial x^{*2}}=\upsilon\frac{\partial}{\partial x^*}\cdot\frac{\partial\left(wU\right)}{\partial x}\cdot\frac{\partial x}{\partial x^*}=\upsilon\frac{\partial}{\partial x^*}U\frac{\partial w}{\partial x}\cdot\frac{U}{\upsilon}=U^2\frac{\partial}{\partial x}\cdot\frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial x^*}=\frac{U^3}{\upsilon}\frac{\partial^2 w}{\partial x^2}$$

$$H^*=HH_0\,,\qquad w^*=wU$$

Substituting these, equation 4.7 becomes

$$\frac{U^3}{\upsilon}\frac{\partial w}{\partial t}+\frac{uU^3}{\upsilon}\frac{\partial w}{\partial x}+\frac{vU^3}{\upsilon}\frac{\partial w}{\partial y}=\frac{U^3}{\upsilon}\frac{\partial^2 w}{\partial x^2}-+\frac{\sigma\mu_e^2 H^2 H_0^2 Uw}{\rho}$$

Multiplying both sides by $\dfrac{\upsilon}{U^3}$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=\frac{\partial^2 w}{\partial x^2}-\frac{\sigma\mu_e^2 H^2 H_0^2\cdot Uw\cdot\upsilon}{\rho U^3}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=\frac{\partial^2 w}{\partial x^2}-\frac{\sigma\mu_e^2 H_0^2\upsilon H^2 w}{\rho U^2}$$

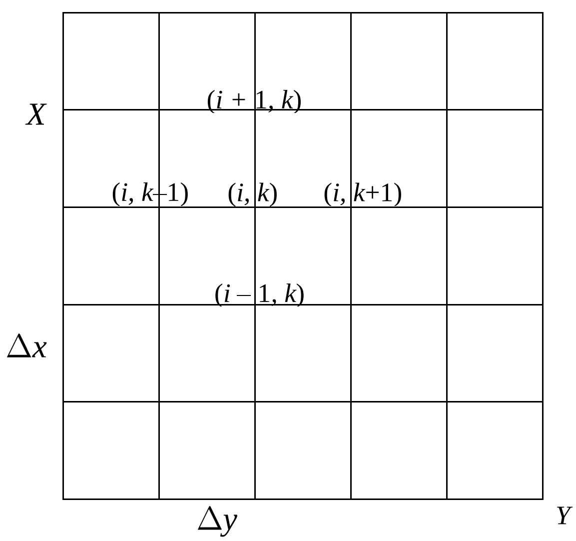
But $\dfrac{\sigma\mu_e^2 H_0^2\upsilon}{\rho U^2}=M^2$

Thus

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=\frac{\partial^2 w}{\partial x^2}-M^2H^2w \quad . \tag{4.9}$$

5. Finite difference approximations

To give a relationship between the partial derivative in the differential equation and the function values at the adjacent nodal points, we use a uniform mesh. In this case the x-y plane is divided into a network of uniform rectangular cells of width Δy and height Δx .



Where i refers to X
k refers to Y

If Δx represent increment in X then X=i Δx .
If Δy represent increment in Y then Y=k Δy .
Using Taylor's series expansion of dependent variables about a grid point *(i,k)* we have

$$\phi(i,k-1)=\phi(i,k)-\phi^{/}(i,k)\Delta y+\frac{1}{2}\phi^{//}(i,k)(\Delta y)^2-\frac{1}{6}\phi^{///}(i,k)(\Delta y)^3+... \tag{5.1}$$

$$\phi(i,k+1)=\phi(i,k)+\phi^{/}(i,k)\Delta y+\frac{1}{2}\phi^{//}(i,k)(\Delta y)^2+\frac{1}{6}\phi^{///}(i,k)(\Delta y)^3+... \tag{5.2}$$

Subtracting equation 5.1 from equation 5.2 we have

$$2\phi^{/}(i,k)=\phi(i,k+1)-\phi(i,k-1)$$

$$\phi^{/}(i,k)=\frac{\phi(i,k+1)-\phi(i,k-1)}{2\Delta y}+Hot \tag{5.3}$$

Adding equation 5.1 to equation 5.2 results to

$$\phi^{//}(i,k)(\Delta y)^2+2\phi(i,k)+Hot=\phi(i,k-1)+\phi(i,k+1)$$

$$\phi^{//}(i,k)=\frac{\phi(i,k+1)-2\phi(i,k)+\phi(i,k-1)}{(\Delta y)^2}+Hot \tag{5.4}$$

The central difference formulae for the first and second derivative with respect to x are

$$\phi^{/}=\frac{\phi(i+1,k)-\phi(i-1,k)}{2\Delta x}+Hot \tag{5.5}$$

$$\phi^{//}=\frac{\phi(i+1,k)-2\phi(i,k)+\phi(i-1,k)}{(\Delta x)^2}+Hot \quad . \tag{5.6}$$

Let the mesh point variable at time t_n be denoted by $\phi_{(i,k)}^{n}$. Then the forward difference for the first order derivative with respect to time t will be

$$\phi_{(i,k)}^{/n}=\frac{\phi_{(i,k)}^{n+1}-\phi_{(i,k)}^{n}}{\Delta t}+Hot\,. \tag{5.7}$$

Using forward finite difference for the first order time derivative and central finite difference for the first and second order partial derivatives, then the governing equations 4.8 and 4.9 can be written in the finite difference form as follows

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{\partial^2 v}{\partial x^2}+Gr\theta+M^2H^2v \text{ becomes}$$

$$\frac{v_{(i,k)}^{n+1}-v_{(i,k)}^{n}}{\Delta t}+u_0\left[\frac{v_{(i+1,k)}^{n}-v_{(i-1,k)}^{n}}{2\Delta x}\right]+v_{(i,k)}^{n}\left[\frac{v_{(i,k+1)}^{n}-v_{(i,k-1)}^{n}}{2\Delta y}\right]=\frac{v_{(i+1,k)}^{n}-2v_{(i,k)}^{n}+v_{(i-1,k)}^{n}}{(\Delta x)^2}$$

$$+Gr\theta_{(i,k)}^{n}+M^2H_{(i,k)}^{n}{}^2v_{(i,k)}^{n}$$

Making $v_{(i,k)}^{n+1}$ the subject we get

$$v_{(i,k)}^{n+1}=v_{(i,k)}^{n}+\left\{\begin{aligned}&-u_0\left[\frac{v_{(i+1,k)}^{n}-v_{(i-1,k)}^{n}}{2\Delta x}\right]-v_{(i,k)}^{n}\left[\frac{v_{(i,k+1)}^{n}-v_{(i,k-1)}^{n}}{2\Delta y}\right]\\&+\frac{v_{(i+1,k)}^{n}-2v_{(i,k)}^{n}+v_{(i-1,k)}^{n}}{(\Delta x)^2}+Gr\theta_{(i,k)}^{n}+M^2H_{(i,k)}^{n}{}^2v_{(i,k)}^{n}\end{aligned}\right\}\Delta t\,. \tag{5.8}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=\frac{\partial^2 w}{\partial x^2}-M^2H^2w \text{ becomes}$$

$$\frac{w_{(i,k)}^{n+1}-w_{(i,k)}^{n}}{\Delta t}+u_0\left[\frac{w_{(i+1,k)}^{n}-w_{(i-1,k)}^{n}}{2\Delta x}\right]+v_{(i,k)}^{n}\left[\frac{w_{(i,k+1)}^{n}-w_{(i,k-1)}^{n}}{2\Delta y}\right]=\frac{w_{(i+1,k)}^{n}-2w_{(i,k)}^{n}+w_{(i-1,k)}^{n}}{(\Delta x)^2}$$

$$-M^2H_{(i,k)}^{n}{}^2w_{(i,k)}^{n}$$

Making $w_{(i,k)}^{n+1}$ the subject we have

$$w_{(i,k)}^{n+1} = w_{(i,k)}^{n} + \left\{ \begin{array}{l} -u_0 \left[\dfrac{w_{(i+1,k)}^{n} - w_{(i-1,k)}^{n}}{2\Delta x} \right] - v_{(i,k)}^{n} \left[\dfrac{w_{(i,k+1)}^{n} - w_{(i,k-1)}^{n}}{2\Delta y} \right] \\ + \left[\dfrac{w_{(i+1,k)}^{n} - 2w_{(i,k)}^{n} + w_{(i-1,k)}^{n}}{(\Delta x)^2} - M^2 {H_{(i,k)}^{n}}^2 w_{(i,k)}^{n} \right] \end{array} \right\} \Delta t \ . \qquad (5.9)$$

In this case we assume that the induced magnetic field is negligible so that the fluid is permeated by a strong variable magnetic field H of magnitude H_o

The generalized Ohm's law including the effects of Hall current parameter is

$$J + \frac{m_*}{H_o}\ J \times H_o\ = \sigma \left[E + \mu_e q \times H_o + \frac{1}{e\eta_e} \nabla P_e \right] \qquad (5.10)$$

where $m_* = \omega_e \tau_e$ is Hall Current parameter.

For short circuit problem, the applied electric field E=0 and for partially ionized gases the electron pressure gradient may be neglected.

Hence equation (5.10) becomes

$$J + \frac{m_*}{H_o}\ J \times H_o\ = \sigma\ \mu_e q \times H_o\ . \qquad (5.11)$$

In component form

$$\begin{pmatrix} 0 \\ J_y \\ J_z \end{pmatrix} + \frac{m_*}{H_o} \begin{vmatrix} i & j & k \\ 0 & J_y & J_z \\ H_o & 0 & 0 \end{vmatrix} = \sigma \mu_e \begin{vmatrix} i & j & k \\ 0 & v & w \\ H_o & 0 & 0 \end{vmatrix}$$

This gives

$$J_y + m_* J_z = \sigma \mu_e w H_o \qquad (5.12)$$

and $$J_z - m_* J_y = -\sigma \mu_e v H_o \qquad (5.13)$$

Solving equations (5.12) and (5.13) simultaneously we get

$$J_y = \frac{\sigma \mu_e H_o\ \ m_* v + w}{m_*^2 + 1} \quad \text{and} \quad J_z = \frac{\sigma \mu_e H_o\ \ m_* w - v}{m_*^2 + 1}$$

But $J = \sigma \mu_e q H$,

$$\Rightarrow J_y = \sigma \mu_e v H_y = \frac{\sigma \mu_e H_o\ \ m_* v + w}{m_*^2 + 1}$$

Hence $H_y = \dfrac{H_o\ \ m_* v + w}{m_*^2 + 1\ v}$.

$J_z = \sigma \mu_e w H_z = \dfrac{\sigma \mu_e H_o\ \ m_* w - v}{m_*^2 + 1}$ this yield

$$H_z = \frac{H_o\ \ m_* w - v}{m_*^2 + 1\ w} .$$

Substituting H_y and H_z in equations 5.8 and 5.9 we get the final equations as

$$v_{(i,k)}^{n+1} = v_{(i,k)}^{n} + \left\{ \begin{array}{l} -u_0 \left[\dfrac{v_{(i+1,k)}^{n} - v_{(i-1,k)}^{n}}{2\Delta x} \right] - v_{(i,k)}^{n} \left[\dfrac{v_{(i,k+1)}^{n} - v_{(i,k-1)}^{n}}{2\Delta y} \right] \\ + \dfrac{v_{(i+1,k)}^{n} - 2v_{(i,k)}^{n} + v_{(i-1,k)}^{n}}{(\Delta x)^2} + Gr\theta_{(i,k)}^{n} + M^2 \left(\dfrac{H_o \;\; m_* v_{(i,k)}^{n} + w_{(i,k)}^{n}}{m_*^{\,2} + 1 \;\; v_{(i,k)}^{n}} \right)^2 v_{(i,k)}^{n} \end{array} \right\} \Delta t \; . \; (5.14)$$

$$w_{(i,k)}^{n+1} = w_{(i,k)}^{n} + \left\{ \begin{array}{l} -u_0 \left[\dfrac{w_{(i+1,k)}^{n} - w_{(i-1,k)}^{n}}{2\Delta x} \right] - v_{(i,k)}^{n} \left[\dfrac{w_{(i,k+1)}^{n} - w_{(i,k-1)}^{n}}{2\Delta y} \right] \\ + \left[\dfrac{w_{(i+1,k)}^{n} - 2w_{(i,k)}^{n} + w_{(i-1,k)}^{n}}{(\Delta x)^2} - M^2 \left(\dfrac{H_o \;\; m_* w_{(i,k)}^{n} - v_{(i,k)}^{n}}{m_*^{\,2} + 1 \;\; w_{(i,k)}^{n}} \right)^2 w_{(i,k)}^{n} \right] \end{array} \right\} \Delta t \qquad . \; (5.15)$$

Since X-axis is along the infinite vertical porous plate then x varies from 0 to infinity. If we set i=21 to correspond to $x = \infty$ and $\Delta x = \Delta y = 0.1$ then we have

$x = i\Delta x = 21 \times 0.1 = 2.1$

The initial condition (where t=0)

At x=0 $\quad v_{(0,k)}^{0} = 1, \quad w_{(0,k)}^{0} = 0, \quad u_0 = 5$

At x>0 $\quad v_{(i,k)}^{0} = 0, \quad w_{(i,k)}^{0} = 0, \quad u_0 = 5$

For i>0 and all k, the boundary conditions (where t>0) takes the form

At x=0 $\quad v_{(0,k)} = 1, \quad w_{(0,k)} = 1, \quad u_0 = 5$

At $x = \infty (21) \quad v_{(21,k)} = 0, \quad w_{(21,k)} = 0, \quad u_0 = 5$, For all n.

The computations are done when Δt is small. Set Δt =0.00125.

The Prandtl number is taken as 0.71 which correspond to air.

Magnetic parameter M^2 =10 which signifies a strong variable magnetic field.

Grashof number, Gr > 0(0.4) corresponding to convective cooling of the plate.

Reynolds number, Re <1 (0.6) so that the inertia force is negligible.

The Hall current parameter m_* is varying from 0, 0.25, 0.5, 0.75, 1.0.

6. RESULTS AND DISCUSSION

We analyzed the results accruing from the iterations to get the effects of Hall current on the primary velocity and secondary velocity. The results are presented in tables and graphs.

TABLE 5.1: TABLE FOR PRIMARY VELOCITY

X	V1	V2	V3	V4	V5
0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
1	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
2	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
3	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
4	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
5	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
6	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
7	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
8	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
9	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
10	0.00000001	0.00000001	0.00000001	0.00000001	0.00000001
11	0.00000007	0.00000007	0.00000007	0.00000007	0.00000007
12	0.05609995	0.03896634	0.01467953	0.00384491	0.00087389
13	0.04401007	0.03058397	0.01153308	0.00302267	0.00068744
14	0.05445099	0.03782142	0.01423591	0.00372992	0.00084951
15	0.05833446	0.04053308	0.01525194	0.00400049	0.00091394
16	0.06643570	0.04566854	0.01703638	0.00446799	0.00102655
17	0.07484943	0.05089758	0.01893019	0.00493957	0.00114450
18	0.09454902	0.05998714	0.02119153	0.00549167	0.00128723
19	0.12031026	0.07184199	0.02378313	0.00610872	0.00145823
20	0.22872354	0.10908002	0.02764635	0.00682719	0.00166190
21	0.41883271	0.17207046	0.03305566	0.00766075	0.00190543

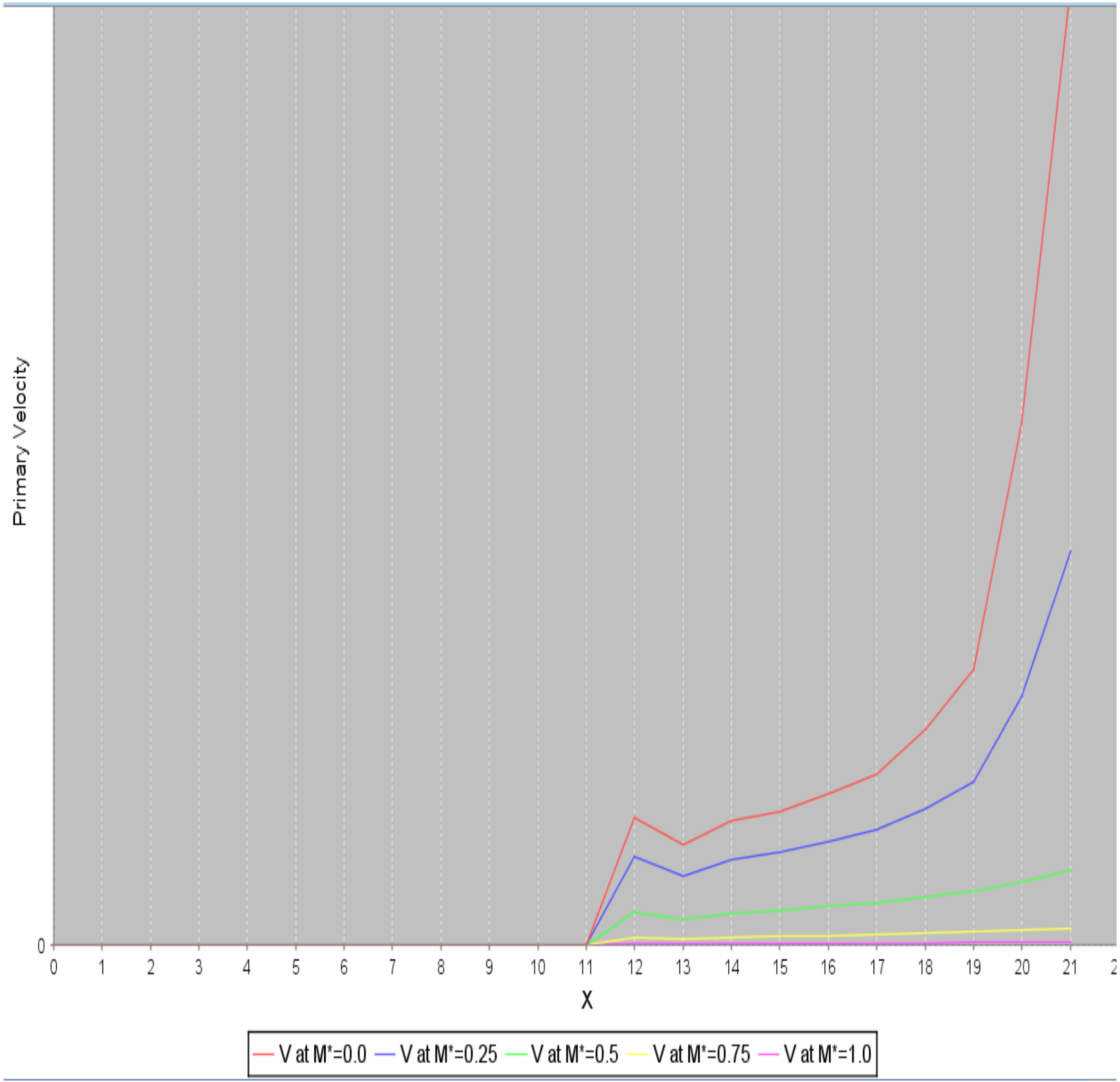


Figure 5.1: Graph of primary velocity

Increase in Hall current parameter results decrease in the primary velocity and the velocity increases as move away from the plate. This is because increase in Hall current results to increase in cyclotron frequency which reduces the velocity of the fluid.

TABLE 5.2:TABLE FOR SECONDARY VELOCITY

X	W1	W2	W3	W4	W5
0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
1	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
2	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
3	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
4	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
5	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
6	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
7	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
8	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
9	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
10	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
11	-0.00070307	-0.00062279	-0.00044996	-0.00028798	-0.00017577
12	-0.00073237	-0.00064845	-0.00046811	-0.00029942	-0.00018273
13	0.03369989	0.01610532	0.00196172	-0.00012285	-0.00017359
14	0.03326077	0.01574366	0.00167272	0.00013172	-0.00016804
15	0.01432770	0.00660706	-0.00009189	-0.00028583	-0.00016341
16	-0.00584263	-0.00365039	0.01175670	-0.00016877	-0.00009129
17	-0.00884405	-0.00361002	0.01114382	0.00020451	-0.00000452
18	-0.00350270	0.00200358	0.01112390	-0.00013111	0.00064059
19	0.02029088	-0.00878591	0.01107907	0.00067989	0.00069953
20	0.01701811	-0.00333301	0.01043704	0.00052956	0.00075490
21	-0.01141618	0.02063509	0.00960671	0.00012998	0.00080618

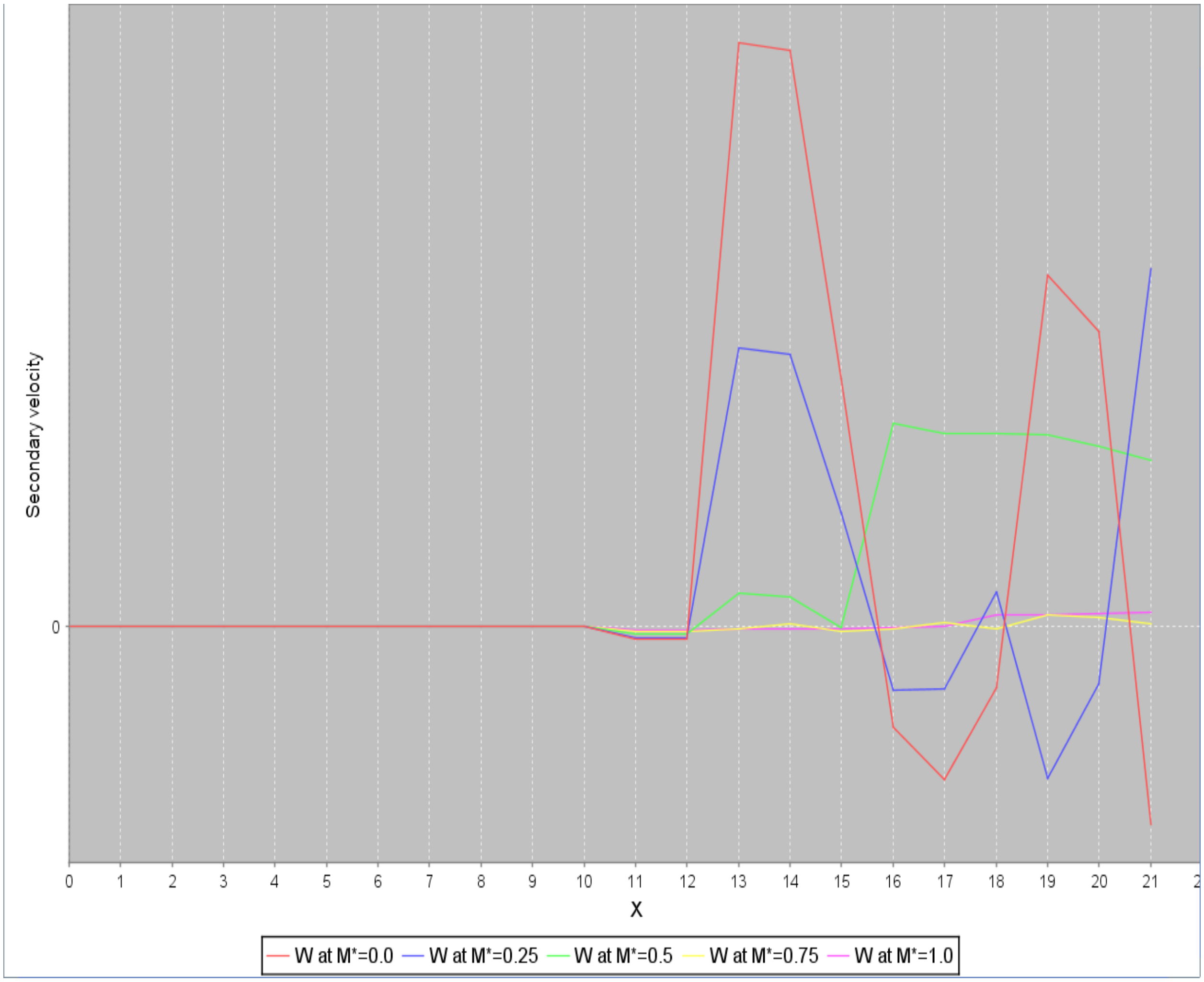


Figure 5.2: Graph of secondary velocity

For small values of the Hall current, the secondary velocity fluctuates as we move away from the plate. But as the Hall current is increased the secondary velocity tend to be constant (zero). The fluid particles next to the plate have a velocity of zero, however since the flow is laminar, velocity increases and decreases away from the plate due to the presence of the variable magnetic field.

6.1. CONCLUSION

An analysis of the effects of Hall current on the primary and secondary velocities of fluid flowing across a variable magnetic field has been carried out. The specific governing equations have been solved by numerically and the results have been analysed. In this study, it has noted that increase in the Hall current affects the velocity of the fluid. However due to lack of equipments, no experimental results have been obtained hence there is no comparison between these numerical values and experimental values.

6.1.2. RECOMMENDATIONS

MHD fluid flows involves wide area of study among which have not been considered in this project. It is recommended that future researchers extend this work considering the following;

1. The experimental results of our research to help in the comparison to these numerical results.
2. Fluid flow between two moving plates across a variable magnetic field.
3. Flow of fluids with variable thermal conductivity.
4. Flow of fluids with variable viscosity.
5. Hall current effects on flows in turbulent boundary layer.
6. Flow of fluids when the variable magnetic field is at an angle to the plate.

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