DOUBLE SOLITON SOLUTIONS FOR THE (2+1)-DIMENSIONAL GENERALIZED BROER-KAUP SYSTEM

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ABSTRACT
In this article, the extended coupled sub-equations expansion method has been used to construct a series of double solution-like solutions, double triangular function solitons and complexiton solitons for some nonlinear partial differential equations via the (2+1)- dimensional Generalized Broer-Kaup System. With the help of symbolic computation as Maple, we obtain many new types of double soliton solutions, i.e. various combination of trigonometric periodic function and hyperbolic function, various combination of trigonometric periodic function and rational function, various combination of hyperbolic function and rational function.

Keywords: *The extended coupled sub-equations expansion method; Double soliton-like solutions; Double triangular function solutions; Complexiton soliton solutions; The (2+1)- dimensional Generalized Broer-Kaup System.*

1. INTRODUCTION

The nonlinear partial differential equations are frequently used for modeling natural and social phenomena and systems. Nonlinear wave phenomena appear in various scientific and engineering fields, such as solid-state physics, fluid mechanics, chemical kinetics, plasma physics, population models, nonlinear optics … etc. Many powerful methods have been presented by those authors such as the inverse scattering transform [3], the Backlund transform [7,20], the generalized Riccati equation [4,19], the Jacobi elliptic expansion [5,15], the extended tanh-function method [6,1,8], the F-expansion method [2,22], the exp–function expansion method [12,11], the sub-ODE method [17,13], the homogeneous balance method [16], the extended sine-cosine methods [21], the complex hyperbolic function method [24], the (G'/G)-expansion method [18,14,23] and so on. Recently, the extended coupled sub-equations expansion method as the extension of multiple Riccati equations expansion method is efficiently applied by many researchers to a great variety of NLPDEs [9]. In this article, we use the extended coupled sub-equations expansion method to calculate the different types of the exact solutions of some nonlinear partial differential equations in mathematical physics. Many new exact solutions are obtained to the (2+1)- dimensional Generalized Broer-Kaup System.

2. SUMMARY OF THE EXTENDED COUPLED SUB-EQUATIONS EXPANSION METHOD

In this section, we would like to outline the main steps of this method [25] as follows:

Step 1. We consider the following nonlinear partial differential equation

$$U(u, u_x, u_y, u_t, u_{xy}, u_{xt}, u_{yt}, u_{xx}, u_{yy}, u_{tt}, ...) = 0 \tag{1}$$

Step 2. We introduce a more generalized ansatz in terms of a finite form expansion in the following forms:

$$u(x, y, t) = a_0 + \sum_{k=1}^{n} \sum_{i+j=k} a_i^j \ \phi^i(\xi) \left[\frac{G'(\eta)}{G(\eta)} \right]^j, \tag{2}$$

where $a_0, a_i^j \ (i, j = 1, 2, 3, ..., n)$ are arbitrary constants to be determined later, while the new variables $\phi(\xi)$ and $G(\eta)$ satisfy the following two different differential equations:

$$\phi'(\xi) = q + r\,\phi(\xi) + p\,\phi(\xi)^2, \tag{3}$$

and

$$G''(\eta) + \lambda G'(\eta) + \mu G(\eta) = 0, \tag{4}$$

where q, r, p, λ, μ are arbitrary constants such that $p \neq 0$. The parameters ξ and η are given by $\xi = k_1 x + L_1 y + \lambda_1 t$ and $\eta = k_2 x + L_2 y + \lambda_2 t$, where $k_1, k_2, L_1, L_2, \lambda_1, \lambda_2$ are arbitrary constants.

Step 3. We will determine the positive integer *n* of the formal polynomial solution Eq.(2) by balancing the highest nonlinear terms and the highest-order partial derivative term in the given partial differential equation, and then give the formal solution.

Step 4. Substituting Eq.(2) along with Eq.(3) and Eq.(4) into Eq.(1) and then set all the coefficients of $[\phi(\xi)]^i \left[\frac{G'(\eta)}{G(\eta)}\right]^j \ (i, j = 0,1,2,3,...)$ to be zero. We get an over-determined system of algebraic equations with respect to $k_1, k_2, L_1, L_2, \lambda_1, \lambda_2, a_0$ and $a_i^j \ (i, j = 1,2,3,...,n)$.

Step 5. Solving the over-determined system of differential equations by using Maple to determine $k_1, k_2, L_1, L_2, \lambda_1, \lambda_2, a_0$ and $a_i^j \ (i, j = 1,2,3,...,n)$.

Step 6. It is well known that the general solutions of Riccati equation (3) and the linear second order differential equation (4) are list as follows:

(i) when $\Delta > 0$ and $\lambda^2 > 4\mu$, then

$$\phi(\xi) = -\frac{r + \sqrt{\Delta}\ \tanh(\frac{1}{2}\sqrt{\Delta}\ \xi)}{2p}, \quad \text{or} \quad \phi(\xi) = -\frac{r + \sqrt{\Delta}\ \coth(\frac{1}{2}\sqrt{\Delta}\ \xi)}{2p},$$

and

$$\frac{G'(\eta)}{G(\eta)} = -\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2} \left[\frac{C_1 \sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\ \eta) + C_2 \cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\ \eta)}{C_1 \cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\ \eta) + C_2 \sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\ \eta)}\right], \tag{5}$$

(ii) when $\Delta < 0$ and $\lambda^2 < 4\mu$, then

$$\phi(\xi) = -\frac{r - \sqrt{-\Delta}\ \tan(\frac{1}{2}\sqrt{-\Delta}\ \xi)}{2p}, \quad \text{or} \quad \phi(\xi) = -\frac{r - \sqrt{-\Delta}\ \cot(\frac{1}{2}\sqrt{-\Delta}\ \xi)}{2p},$$

and

$$\frac{G'(\eta)}{G(\eta)} = -\frac{\lambda}{2} + \frac{\sqrt{4\mu - \lambda^2}}{2} \left[\frac{-C_1 \sin(\frac{1}{2}\sqrt{4\mu - \lambda^2}\ \eta) + C_2 \cos(\frac{1}{2}\sqrt{4\mu - \lambda^2}\ \eta)}{C_1 \cos(\frac{1}{2}\sqrt{4\mu - \lambda^2}\ \eta) + C_2 \sin(\frac{1}{2}\sqrt{4\mu - \lambda^2}\ \eta)}\right], \tag{6}$$

(iii) when $\Delta < 0$ and $\lambda^2 > 4\mu$, then

$$\phi(\xi) = -\frac{r - \sqrt{-\Delta}\ \tan(\frac{1}{2}\sqrt{-\Delta}\ \xi)}{2p}, \quad \text{or} \quad \phi(\xi) = -\frac{r - \sqrt{-\Delta}\ \cot(\frac{1}{2}\sqrt{-\Delta}\ \xi)}{2p},$$

and

$$\frac{G'(\eta)}{G(\eta)} = -\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2} \left[\frac{C_1 \sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\ \eta) + C_2 \cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\ \eta)}{C_1 \cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\ \eta) + C_2 \sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\ \eta)}\right], \tag{7}$$

(iv) when $\Delta > 0$ and $\lambda^2 < 4\mu$, then

$$\phi(\xi) = -\frac{r + \sqrt{\Delta}\ \tanh(\frac{1}{2}\sqrt{\Delta}\ \xi)}{2p}, \quad \text{or} \quad \phi(\xi) = -\frac{r + \sqrt{\Delta}\ \coth(\frac{1}{2}\sqrt{\Delta}\ \xi)}{2p},$$

and

$$\frac{G'(\eta)}{G(\eta)} = -\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\left[\frac{-C_1 \sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\ \eta) + C_2 \cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\ \eta)}{C_1 \cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\ \eta) + C_2 \sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\ \eta)}\right], \tag{8}$$

where $\Delta = r^2 - 4pq$ and C_1, C_2 are constants.

(v) when $r = q = 0, \lambda^2 = 4\mu$ and $p, C_2 \neq 0$, then

$$\phi(\xi) = \frac{-1}{p\xi + E_1} \quad \text{and} \quad \frac{G'(\eta)}{G(\eta)} = -\frac{\lambda}{2} + \frac{C_2}{C_1 + C_2\eta}, \tag{9}$$

where E_1 is arbitrary constants. Similarly, we can find other general solutions, when $\Delta > 0$ and $\lambda^2 = 4\mu$ etc, which are omitted here for convenience.

3. APPLICATIONS

In this section, we use the extended coupled sub-equations expansion method to construct the exact solutions for the (2+1)-dimensional Generalized Broer-Kaup System which is very important in the mathematical physics.

3.1. Example. The (2+1)- dimensional Generalized Broer-Kaup System

In this section, we study the nonlinear (2+1)- dimensional Generalized Broer-Kaup equations [10]**:**

$$\begin{aligned} &u_t + 4\left(u_{xx} + u^3 - 3uu_x + 3uw + 3H\right)_x = 0,\\ &v_t + 4\left(v_{xx} + 3vu^2 + 3uv_x + 3vw\right)_x = 0,\\ &w_y - v_x = 0,\\ &H_y - (uv)_x = 0. \end{aligned} \tag{10}$$

By balancing the nonlinear terms and the highest order linear partial derivative terms of Eqs.(10), we get $n_1 = 1, n_2 = 2, n_3 = 2$ and $n_4 = 3$. Thus the exact solutions of the (2+1)-dimensional Generalized Broer-Kaup equations (10) take the following form:

$$u(t,x,y) = a_0 + a_1\phi(\xi) + a_2\left[\frac{G'(\eta)}{G(\eta)}\right]$$

$$v(t,x,y) = b_0 + b_1\phi(\xi) + b_2\left[\frac{G'(\eta)}{G(\eta)}\right] + b_3\phi^2(\xi) + b_4\left[\frac{G'(\eta)}{G(\eta)}\right]^2 + b_5\phi(\xi)\left[\frac{G'(\eta)}{G(\ )}\right]$$

$$w(t,x,y) = d_0 + d_1\phi(\xi) + d_2\left[\frac{G'(\eta)}{G(\eta)}\right] + d_3\phi^2(\xi) + d_4\left[\frac{G'(\eta)}{G(\eta)}\right]^2 + d_5\phi(\xi)\left[\frac{G'(\eta)}{G(\ )}\right]$$

$$H(t,x,y)=\alpha_0+\alpha_1\phi(\xi)+\alpha_2\left[\frac{G'(\eta)}{G(\eta)}\right]+\alpha_3\phi^2(\xi)+\alpha_4\left[\frac{G'(\eta)}{G(\eta)}\right]^2+\alpha_5\phi(\xi)\left[\frac{G'(\eta)}{G(\eta)}\right]$$
$$+\alpha_6\phi^3(\xi)+\alpha_7\phi^2(\xi)\left[\frac{G'(\eta)}{G(\eta)}\right]+\alpha_8\phi(\xi)\left[\frac{G'(\eta)}{G(\eta)}\right]^2+\alpha_9\left[\frac{G'(\eta)}{G(\eta)}\right]^3 \quad (11)$$

where $\xi=k_1x+L_1y+\lambda_1t$, $\eta=k_2x+L_2y+\lambda_2t$ and $k_1,k_2,L_1,L_2,\lambda_1,\lambda_2,q$, b_i,d_i $(i=0,1,2,...,5)$ and $\alpha_j(j=0,1,2,...,9)$ are arbitrary constants to be determined later. With the aid of Maple, we substitute Eqs.(11) along with Eq.(3) and Eq.(4) into Eqs.(10) and set the coefficients of $[\phi(\xi)]^n[\frac{G'(\eta)}{G(\eta)}]^m$ $(n,m=0,1,2,3,...)$ to be zero, yield a set of over-determined system of algebraic equations with respect to $k_1,k_2,L_1,L_2,\lambda_1,\lambda_2,a_i$, $\mathrm{d_i}$, $\mathrm{b_i}(i=0,1,2,...,5)$ and $\alpha_j(j=0,1,2,...,9)$. On using the
Maple software package, we solve the over-determined algebraic equations. Consequently, we get the following results:

Case (1)

$$\alpha_1=\frac{1}{L_1}\left(\frac{pk_1^3(pqL_1^2+\mu L_2^2)}{L_1}-\frac{prk_1(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_2}\right),$$

$$\alpha_2=\frac{1}{L_1}\left(-\frac{(pqL_1^2+\mu L_2^2)k_1^2k_2}{L_1}-\frac{\lambda k_2(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_1}\right),$$

$$\alpha_3=p^2rk_1^3-\frac{p^2k_1(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_1L_2},$$

$$\alpha_4=\frac{1}{L_2}\left(-\frac{k_2^2(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_1k_1}-\lambda L_2k_2^3\right),\qquad \alpha_5=\frac{-prL_1k_1^2k_2+p\lambda L_2k_1^2k_2}{L_1}$$

$$\alpha_6=p^3k_1^3,\qquad \alpha_7=-p^2k_1^2k_2,\qquad \alpha_8=pk_1k_2^2,\qquad \alpha_9=-k_2^3$$

$$a_0=\frac{-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2}{2L_1L_2k_1},\qquad a_1=-pk_1,\qquad a_2=k_2$$

$$\mathrm{b}_0=-\frac{(pqL_1^2+\mu L_2^2)k_1}{L_1},\qquad \mathrm{b}_1=-prL_1k_1,\qquad \mathrm{b}_3=-p^2L_1k_1,$$

$$b_2=-\lambda L_2k_2,\qquad \mathrm{b}_4=-L_2k_2,\qquad \mathrm{b}_5=0$$

$$d_0 = \frac{-3r^2L_1^2L_2k_1^3 - \lambda^2L_2^3k_1^3 - 8\mu L_2^3k_1^3 - L_1^3\lambda_2}{12L_1^2L_2k_1}, \qquad d_1 = -prk_1^2, \qquad d_2 = -\lambda k_2^2,$$

$$d_3 = -p^2k_1^2, \qquad d_4 = -k_2^2, \qquad d_5 = 0$$

$$\lambda_1 = \frac{1}{L_1^2L_2}\left(-8pqL_1^2L_2k_1^3 + 2r^2L_1^2L_2k_1^3 - 2\lambda^2L_2^3k_1^3 + 8\mu L_2^3k_1^3 + L_1^3\lambda_2\right), \qquad k_2 = \frac{L_2k_1}{L_1}. \tag{12}$$

where k_1, L_1, L_2 and λ_2 are arbitrary constants.

Case (2)

$$a_1 = 0, \qquad a_2 = k_2, \qquad b_0 = -\mu L_2 k_2, \qquad b_1 = 0,$$

$$b_2 = -\lambda L_2k_2, \qquad b_3 = 0, \qquad b_4 = -L_2k_2, \qquad b_5 = 0, \qquad \alpha_8 = 0,$$

$$\alpha_9 = -k_2^3, \qquad d_0 = \frac{-12a_0^2L_2k_2 + 12\lambda a_0L_2k_2^2 - 4\lambda^2L_2k_2^3 - 8\mu L_2k_2^3 - L_2\lambda_2}{12L_2k_2}, \qquad d_1 = 0,$$

$$d_2 = -\lambda k_2^2, \qquad d_3 = 0, \qquad d_4 = -k_2^2, \qquad d_5 = 0, \qquad \alpha_1 = 0,$$

$$\alpha_2 = \frac{-\lambda a_0L_2k_2^2 - \mu L_2k_2^3}{L_2}, \qquad \alpha_3 = 0, \qquad \alpha_4 = \frac{-a_0L_2k_2^2 - \lambda L_2k_2^3}{L_2}, \qquad \alpha_5 = 0,$$

$$\alpha_6 = 0, \qquad \alpha_7 = 0, \qquad \alpha_8 = 0, \qquad k_1 = 0. \tag{13}$$

Where $a_0, \alpha_0, \lambda_1, L_1, L_2,$ and λ_2 are arbitrary constants

Case (3)

$$a_1 = -pk_1, \qquad a_2 = 0, \qquad b_0 = -pqL_1k_1, \qquad b_1 = -prL_1k_1, \qquad b_2 = 0,$$

$$b_3 = -p^2L_1k_1, \qquad b_4 = 0, \qquad d_0 = \frac{-12a_0^2L_1k_1 - 12ra_0L_1k_1^2 - 8pqL_1k_1^3 - 4r^2L_1k_1^3 - L_1\lambda_1}{12L_1k_1},$$

$$d_1 = -prk_1^2, \qquad d_2 = 0, \qquad d_3 = -p^2k_1^2, \qquad d_4 = 0,$$

$$\alpha_1 = \frac{-pra_0L_1k_1^2 + p^2qL_1k_1^3}{L_1}, \qquad \alpha_3 = \frac{-p^2a_0L_1k_1^2 + p^2rL_1k_1^3}{L_1}, \qquad \alpha_4 = 0,$$

$$\alpha_5 = 0, \quad \alpha_6 = p^3 k_1^3, \quad \alpha_7 = 0, \quad \alpha_8 = 0,$$

$$\alpha_9 = 0, \quad L_2 = k_2 = 0., \quad b_5 = 0, \quad d_5 = 0. \tag{14}$$

Where a_0, α_0 , $\alpha_2, k_1, L_1, \lambda_1$ *and* λ_2 are arbitrary constants.

Case (4)

$$a_0 = \lambda k_2, \quad a_1 = 0, \quad a_2 = 2k_2, \quad b_0 = -2\mu L_2 k_2,$$

$$b_1 = 0, \quad b_2 = -2\lambda L_2 k_2, \quad b_3 = 0, \quad b_4 = -2L_2 k_2,$$

$$b_5 = 0, \quad d_0 = \frac{-4\lambda^2 L_2 k_2^3 - 8\mu L_2 k_2^3 - L_2 \lambda_2}{12 L_2 k_2}, \quad d_1 = 0,$$

$$d_2 = -2\lambda k_2^2, \quad d_3 = 0, \quad d_4 = -2k_2^2 \quad d_5 = 0,$$

$$\alpha_2 = \frac{-2\lambda^2 L_2 k_2^3 - 4\mu L_2 k_2^3}{L_2}, \quad \alpha_4 = -6\lambda k_2^3, \quad \alpha_9 = -4k_2^3 \tag{15}$$

$$\alpha_5 = 0, \quad \alpha_8 = 0, \quad \alpha_6 = 0, \quad L_1 = k_1 = 0 \ .$$

Where $\alpha_0, \alpha_1, \alpha_3\ \lambda_1, \lambda_2, k_2$ and L_2 are arbitrary constants

Case (5)

$$a_0 = -rk_1, \quad a_1 = -2pk_1, \quad a_2 = 0, \quad b_0 = -2pqL_1 k_1,$$

$$b_1 = -2prL_1 k_1, \quad b_2 = 0, \quad b_3 = -2p^2 L_1 k_1, \quad b_4 = 0,$$

$$b_5 = 0, \quad d_0 = \frac{-8p^3 q L_1^2 k_1^3 - 4p^2 r^2 L_1^2 k_1^3 - p^2 L_1^2 \lambda_1}{12 p^2 L_1^2 k_1}, \quad d_1 = -2prk_1^2,$$

$$d_2 = 0, \quad d_3 = -2p^2 k_1^2, \quad d_4 = 0, \quad d_5 = 0,$$

$$\alpha_1 = \frac{4p^2 q L_1 k_1^3 + 2pr^2 L_1 k_1^3}{L_1}, \quad \alpha_2 = 0, \quad \alpha_3 = 6P^2 r k_1^3$$

$$\alpha_4 = 0, \quad \alpha_5 = 0, \quad \alpha_6 = 4p^3k_1^3, \quad \alpha_7 = 0,$$

$$\alpha_8 = 0, \quad \alpha_9 = 0, \quad L_2 = k_2 = 0 \tag{16}$$

Where $\alpha_0, \lambda_1, \lambda_2, k_1$, and L_1 are arbitrary constants

Case (6)

$$a_0 = -rk_1, \quad a_1 = -2pk_1, \quad a_2 = 0, \quad b_0 = -2pqL_1k_1,$$

$$b_1 = -2prL_1k_1, \quad b_2 = 0, \quad b_3 = -2p^2L_1 k_1, \quad b_4 = 0,$$

$$b_5 = 0, \quad d_0 = \frac{-8p^3qL_1^2k_1^3 - 4p^2r^2L_1^2k_1^3 - p^2L_1^2\lambda_1}{12p^2L_1^2k_1}, \quad d_1 = -2prk_1^2,$$

$$d_2 = 0, \quad d_3 = -2p^2 k_1^2, \quad d_4 = 0, \quad d_5 = 0, \quad \alpha_5 = 0$$

$$\alpha_1 = \frac{4p^2qL_1k_1^3 + 2pr^2L_1k_1^3}{L_1}, \quad \alpha_3 = 6P^2r k_1^3, \quad \alpha_7 = 0, \quad \alpha_8 = 0$$

$$\alpha_6 = 4p^3k_1^3, \quad \alpha_9 = 0, \quad L_2 = k_2 = 0 \tag{17}$$

Where α_0, α_2, α_4 $\lambda_1, \lambda_2, k_1$ and L_1 are arbitrary constants

Case (7)

$$a_0 = \lambda k_2, \quad a_1 = 0, \quad a_2 = 2k_2, \quad b_0 = -2\mu L_2 k_2, \quad b_1 = 0,$$

$$b_2 = -2\lambda L_2 k_2, \quad b_3 = 0, \quad b_4 = -2L_2 k_2, \quad b_5 = 0$$

$$d_0 = \frac{-4\lambda^2 L_2 k_2^3 - 8\mu L_2 k_2^3 - L_2 \lambda_2}{12 L_2 k_2}, \quad d_1 = 0, d_2 = -2\lambda k_2^2, \quad d_3 = 0,$$

$$\mathrm{d}_5 = 0 \quad d_4 = -2k_2^2 \quad ,\alpha_1 = 0, \quad \alpha_3 = 0$$

$$\alpha_2 = \frac{-2\lambda^2 L_2 k_2^3 - 4\mu L_2 k_2^3}{L_2}, \quad \alpha_4 = -6\lambda k_2^3, \quad \alpha_9 = -4k_2^3$$

$$\alpha_5 = 0, \quad \alpha_8 = 0, \quad \alpha_6 = 0, \quad \mathrm{L}_1 = \mathrm{k}_1 = 0 \ . \tag{18}$$

Where $a_0, \lambda_1, \lambda_2, \mathrm{k}_2$ and L_2 are arbitrary constants. Note that, there are other cases which are omitted here for convenience. According to Eqs.(11-18) and the general solutions Eq.(5) - Eq.(9) listed in step 6, we obtain the following families of soliton-like solutions, double triangular function solutions and complexiton soliton solutions corresponding, for the nonlinear (2+1)-dimensional Generalized Broer-Kaup equations (10) as follows:

Case (1)

Family 1. When $\Delta > 0$ and $\lambda^2 > 4\mu$, then the double soliton-like solutions of Eqs.(10) have the following forms:

$$u_1(t,x,y) = \frac{-rL_1L_2k_1^2 - \lambda L_2^2 k_1^2 + 2\lambda L_1^2 k_2^2}{2L_1L_2k_1} - pk_1\left(-\frac{r + \sqrt{\Delta}\tanh\left(\frac{1}{2}\sqrt{\Delta}\,\xi\right)}{2p}\right) +$$
$$k_2\left[-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2}\left(\frac{C_1\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\ \eta\right) + C_2\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta\right)}{C_1\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta\right) + C_2\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta\right)}\right)\right]$$

$$v_1(t,x,y) = -\frac{\left(pqL_1^2 + \mu L_2^2\right)k_1}{L_1} - prL_1k_1\left(-\frac{r + \sqrt{\Delta}\tanh\left(\frac{1}{2}\sqrt{\Delta}\xi\right)}{2p}\right) + \left(-\lambda L_2 k_2\right)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right)-\left(p^2L_1k_1\right)\left(-\frac{r+\sqrt{\Delta}\tanh(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)^2$$

$$-L_2k_2\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right]^2$$

$$w_1(t,x,y)=\frac{-3r^2L_1^2L_2k_1^3-\lambda^2L_2^3k_1^3-8\mu L_2^3k_1^3-L_1^3\lambda_2}{12L_1^2L_2k_1}-prk_1^2\left(-\frac{r+\sqrt{\Delta}\tanh(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)-\lambda k_2^2$$

$$\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right]-p^2k_1^2\left(-\frac{r+\sqrt{\Delta}\tanh(\frac{1}{2}\sqrt{\Delta}\xi}{2p}\right)^2$$

$$-k_2^2\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right]^2$$

$$H_1(t,x,y)=\alpha_0+\left(\frac{1}{L_1}\left(\frac{p\left(p\,qL_1^2+\mu\,L_2^2\right)k_1^3}{L_1}-\frac{prk_1\left(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda\,L_1^2k_2^2\right)}{2L_2}\right)\right)\left(-\frac{r+\sqrt{\Delta}\tanh(\frac{1}{2}\sqrt{\Delta}\,\xi}{2p}\right)+$$

$$\left(\frac{1}{L_1}\left(-\frac{(pqL_1^2+\mu\,L_2^2)\,k_2k_1^2}{L_1}-\frac{\lambda k_2}{2L_1}\left(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2\right)\right)\right)\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\eta)}\right)\right]$$

$$+\left(p^2rk_1^3-\frac{p^2k_1(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_1L_2}\right)\left(-\frac{r+\sqrt{\Delta}\tanh(\frac{1}{2}\sqrt{\Delta}\,\xi)}{2p}\right)^2+\frac{1}{L_2}\left(-\lambda L_2k_2^3-\frac{k_2^2(-rL_1L_2k_1^2-\lambda\,L_2^2k_1^2+2\lambda\,L_1^2k_2^2}{2L_1k_1}\right)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\,\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right)^2+\left(\frac{-prL_1k_1^2k_2+p\lambda L_2k_1^2k_2}{L_1}\right)\left(-\frac{r+\sqrt{\Delta}\tanh(\frac{1}{2}\sqrt{\Delta}\,\xi}{2p}\right)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}\right)\right)+p^3k_1^3\left(-\frac{r+\sqrt{\Delta}\tanh(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)^3$$

$$-p^2k_1^2k_2\left(-\frac{r+\sqrt{\Delta}\tanh(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)^2\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}\right)\right)$$

$$+pk_1k_2^2\left(-\frac{r+\sqrt{\Delta}\tanh(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}\right)\right)^2+$$

$$-k_2^3\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}\right)\right)^3 \tag{19}$$

$$u_2(t,x,y)=\frac{-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2}{2L_1L_2k_1}-pk_1\left(-\frac{r+\sqrt{\Delta}\coth\left(\frac{1}{2}\sqrt{\Delta}\,\xi\right)}{2p}\right)+$$

$$k_2\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\;\eta\right)+C_2\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta}\right)}{C_1\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta}\right)+C_2\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta}\right)}\right)\right]$$

$$v_2(t,x,y)=-\frac{(pqL_1^2+\mu L_2^2)k_1}{L_1}-prL_1k_1\left(-\frac{r+\sqrt{\Delta}\coth\left(\frac{1}{2}\sqrt{\Delta}\xi\right)}{2p}\right)+(-\lambda L_2k_2)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}\right)\right)-p^2L_1k_1\left(-\frac{r+\sqrt{\Delta}\coth(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)^2$$

$$-L_2k_2\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}\right)\right]^2$$

$$w_2(t,x,y)=\frac{-3r^2L_1^2L_2k_1^3-\lambda^2L_2^3k_1^3-8\mu L_2^3k_1^3-L_1^3\lambda_2}{12L_1^2L_2k_1}-prk_1^2\left(-\frac{r+\sqrt{\Delta}\coth(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)-\lambda k_2^2$$
$$\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right]-p^2k_1^2\left(-\frac{r+\sqrt{\Delta}\coth(\frac{1}{2}\sqrt{\Delta}\xi}{2p}\right)^2$$
$$-k_2^2\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right]^2$$

$$H_2(t,x,y)=\alpha_0+\left(\frac{1}{L_1}\left(\frac{p\left(pqL_1^2+\mu L_2^2\right)k_1^3}{L_1}-\frac{prk_1\left(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2\right)}{2L_2}\right)\right)\left(-\frac{r+\sqrt{\Delta}\coth(\frac{1}{2}\sqrt{\Delta}\xi}{2p}\right)+$$

$$\left(\frac{1}{L_1}\left(-\frac{(pqL_1^2+\mu L_2^2)k_2k_1^2}{L_1}-\frac{\lambda k_2}{2L_1}\left(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2\right)\right)\right)\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right]$$
$$+\left(p^2rk_1^3-\frac{p^2k_1(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_1L_2}\right)\left(-\frac{r+\sqrt{\Delta}\coth(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)^2+\frac{1}{L_2}\left(-\lambda L_2k_2^3-\frac{k_2^2(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2}{2L_1k_1}\right)$$
$$\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right]^2+\left(\frac{-prL_1k_1^2k_2+p\lambda L_2k_1^2k_2}{L_1}\right)\left(-\frac{r+\sqrt{\Delta}\coth(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right)+p^3k_1^3\left(-\frac{r+\sqrt{\Delta}\coth(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)^3$$

$$-p^2k_1^2k_2\left(-\frac{r+\sqrt{\Delta}\coth(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)^2\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right)+$$

$$pk_1k_2^2\left(-\frac{r+\sqrt{\Delta}\coth(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right)^2+$$

$$-k_2^3\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right)^3 \qquad (20)$$

Family 2. When $\Delta<0$ and $\lambda^2<4\mu$, then the double soliton-like solutions of Eqs.(10) have the following forms:

$$u_3(t,x,y)=\frac{-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2}{2L_1L_2k_1}-pk_1\left(-\frac{r-\sqrt{-\Delta}\tan\left(\frac{1}{2}\sqrt{-\Delta}\,\xi\right)}{2p}\right)+$$

$$k_2\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right)+C_2\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right)}{C_1\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right)+C_2\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right)}\right)\right]$$

$$v_3(t,x,y)=-\frac{\left(pqL_1^2+\mu L_2^2\right)k_1}{L_1}-prL_1k_1\left(-\frac{r-\sqrt{-\Delta}\tan\left(\frac{1}{2}\sqrt{-\Delta}\xi\right)}{2p}\right)+\left(-\lambda L_2k_2\right)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta)}\right)\right)-p^2L_1k_1\left(-\frac{r-\sqrt{-\Delta}\tan(\frac{1}{2}\sqrt{-\Delta}\;\xi)}{2p}\right)^2$$

$$-L_2k_2\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right]^2$$

$$w_3(t,x,y)=\frac{-3r^2L_1^2L_2k_1^3-\lambda^2L_2^3k_1^3-8\mu L_2^3k_1^3-L_1^3\lambda_2}{12L_1^2L_2k_1}-prk_1^2\left(-\frac{r-\sqrt{-\Delta}\tan(\frac{1}{2}\sqrt{-\Delta}\xi)}{2p}\right)-\lambda k_2^2$$
$$\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right]-p^2k_1^2\left(-\frac{r-\sqrt{-\Delta}\tan(\frac{1}{2}\sqrt{-\Delta}\xi}{2p}\right)^2$$
$$-k_2^2\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu}-\lambda^2\eta)}\right)\right]^2$$

$$H_3(t,x,y)=\alpha_0+\left(\frac{1}{L_1}\left(\frac{p(pqL_1^2+\mu L_2^2)k_1^3}{L_1}-\frac{prk_1(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_2}\right)\right)\left(-\frac{r-\sqrt{-\Delta}\tan(\frac{1}{2}\sqrt{-\Delta}\,\xi}{2p}\right)+$$
$$\left(\frac{1}{L_1}\left(-\frac{(pqL_1^2+\mu L_2^2)k_2k_1^2}{L_1}-\frac{\lambda k_2}{2L_1}\left(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2\right)\right)\right)\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right]$$
$$+\left(p^2rk_1^3-\frac{p^2k_1(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_1L_2}\right)\left(-\frac{r-\sqrt{-\Delta}\tan(\frac{1}{2}\sqrt{-\Delta}\,\xi)}{2p}\right)^2+\frac{1}{L_2}\left(-\lambda L_2k_2^3-\frac{k_2^2(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2}{2L_1k_1}\right)$$
$$\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right]^2+\left(\frac{-prL_1k_1^2k_2+p\lambda L_2k_1^2k_2}{L_1}\right)\left(-\frac{r-\sqrt{-\Delta}\tan(\frac{1}{2}\sqrt{-\Delta}\xi)}{2p}\right)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right)+p^3k_1^3\left(-\frac{r-\sqrt{-\Delta}\tan(\frac{1}{2}\sqrt{-\Delta}\xi)}{2p}\right)^3$$

$$-p^2k_1^2k_2\left(-\frac{r-\sqrt{-\Delta}\tan(\frac{1}{2}\sqrt{-\Delta}\xi)}{2p}\right)^2\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right)+$$

$$pk_1k_2^2\left(-\frac{r-\sqrt{-\Delta}\tan(\frac{1}{2}\sqrt{-\Delta}\xi)}{2p}\right)\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right)^2+$$

$$-k_2^3\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right)^3$$

(21)

$$u_4(t,x,y)=\frac{-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2}{2L_1L_2k_1}-pk_1\left(-\frac{r-\sqrt{-\Delta}\cot\left(\frac{1}{2}\sqrt{-\Delta}\,\xi\right)}{2p}\right)+$$

$$k_2\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\;\eta\right)+C_2\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\;\eta\right)}{C_1\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\;\eta\right)+C_2\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\;\eta\right)}\right)\right]$$

$$v_4(t,x,y)=-\frac{\left(p\,q\,L_1^2+\mu\,L_2^2\right)k_1}{L_1}-p\,r\,L_1k_1\left(-\frac{r-\sqrt{-\Delta}\cot\left(\frac{1}{2}\sqrt{-\Delta}\;\xi\right)}{2p}\right)+\left(-\lambda\,L_2\;k_2\right)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\;\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\;\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\;\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\;\eta)}\right)\right)-p^2L_1k_1\left(-\frac{r-\sqrt{-\Delta}\cot(\frac{1}{2}\sqrt{-\Delta}\;\;\xi)}{2p}\right)^2$$

$$-L_2k_2\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\;\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\;\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\;\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\;\eta)}\right)\right]^2$$

$$w_4(t,x,y)=\frac{-3r^2L_1^2L_2k_1^3-\lambda^2L_2^3\,k_1^3-8\mu\,L_2^3\,k_1^3-L_1^3\,\lambda_2}{12\,L_1^2\,L_2k_1}-p\,r\,k_1^2\left(-\frac{r-\sqrt{-\Delta}\cot(\frac{1}{2}\sqrt{-\Delta}\,\xi)}{2p}\right)-\lambda\,k_2^2$$

$$\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right]-p^2k_1^2\left(-\frac{r-\sqrt{-\Delta}\cot(\frac{1}{2}\sqrt{-\Delta}\,\xi}{2p}\right)^2$$

$$-k_2^2\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu}-\lambda^2\eta)}\right)\right]^2$$

$$H_4(t,x,y)=\alpha_0+\left(\frac{1}{L_1}\left(\frac{p\left(p\,qL_1^2+\mu\,L_2^2\right)k_1^3}{L_1}-\frac{prk_1\left(-rL_1L_2k_1^2-\lambda\,L_2^2k_1^2+2\lambda\,L_1^2k_2^2\right)}{2L_2}\right)\right)\left(-\frac{r-\sqrt{-\Delta}\cot(\frac{1}{2}\sqrt{-\Delta}\,\xi}{2p}\right)+$$

$$\left(\frac{1}{L_1}\left(-\frac{(pqL_1^2+\mu\,L_2^2)\,k_2k_1^2}{L_1}-\frac{\lambda k_2}{2L_1}\left(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2\right)\right)\right)\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right]$$

$$+\left(p^2rk_1^3-\frac{p^2k_1(-rL_1L_2k_1^2-\lambda\,L_2^2k_1^2+2\lambda\,L_1^2k_2^2)}{2L_1L_2}\right)\left(-\frac{r-\sqrt{-\Delta}\cot(\frac{1}{2}\sqrt{-\Delta}\,\xi)}{2p}\right)^2+\frac{1}{L_2}\left(-\lambda L_2k_2^3-\frac{k_2^2(-rL_1L_2k_1^2-\lambda\,L_2^2k_1^2+2\lambda\,L_1^2k_2^2}{2L_1k_1}\right)$$

$$\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right]^2+\left(\frac{-prL_1k_1^2k_2+p\lambda L_2k_1^2k_2}{L_1}\right)\left(-\frac{r-\sqrt{-\Delta}\cot(\frac{1}{2}\sqrt{-\Delta}\xi)}{2p}\right)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right)+p^3k_1^3\left(-\frac{r-\sqrt{-\Delta}\cot(\frac{1}{2}\sqrt{-\Delta}\,\xi)}{2p}\right)^3$$

$$-p^2k_1^2k_2\left(-\frac{r-\sqrt{-\Delta}\cot(\frac{1}{2}\sqrt{-\Delta}\xi)}{2p}\right)^2\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right)+$$

$$pk_1k_2^2\left(-\frac{r-\sqrt{-\Delta}\cot(\frac{1}{2}\sqrt{-\Delta}\xi)}{2p}\right)\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right)^2+$$

$$-k_2^3\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right)^3$$

(22)

Family 3. When $\Delta<0$ and $\lambda^2>4\mu$, then the Complexiton solutions of Eqs.(10) have the following forms:

$$u_5(t,x,y)=\frac{-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2}{2L_1L_2k_1}-pk_1\left(-\frac{r-\sqrt{-\Delta}\tan\left(\frac{1}{2}\sqrt{-\Delta}\,\xi\right)}{2p}\right)+$$

$$k_2\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\ \eta\right)+C_2\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta\right)}{C_1\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta\right)+C_2\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta\right)}\right)\right]$$

$$v_5(t,x,y)=-\frac{\left(pqL_1^2+\mu L_2^2\right)k_1}{L_1}-prL_1k_1\left(-\frac{r-\sqrt{-\Delta}\tan\left(\frac{1}{2}\sqrt{-\Delta}\xi\right)}{2p}\right)+\left(-\lambda L_2k_2\right)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right)-p^2L_1k_1\left(-\frac{r-\sqrt{-\Delta}\tan(\frac{1}{2}\sqrt{-\Delta}\xi)}{2p}\right)^2$$

$$-L_2k_2\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right]^2$$

$$w_5(t,x,y)=\frac{-3r^2L_1^2L_2k_1^3-\lambda^2L_2^3k_1^3-8\mu L_2^3k_1^3-L_1^3\lambda_2}{12L_1^2L_2k_1}-prk_1^2\left(-\frac{r-\sqrt{-\Delta}\tan(\frac{1}{2}\sqrt{-\Delta}\xi)}{2p}\right)-\lambda k_2^2$$
$$\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right]-p^2k_1^2\left(-\frac{r-\sqrt{-\Delta}\tan(\frac{1}{2}\sqrt{-\Delta}\xi}{2p}\right)^2$$
$$-k_2^2\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right]^2$$

$$H_5(t,x,y)=\alpha_0+\left(\frac{1}{L_1}\left(\frac{p\left(pqL_1^2+\mu L_2^2\right)k_1^3}{L_1}-\frac{prk_1\left(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2\right)}{2L_2}\right)\right)\left(-\frac{r-\sqrt{-\Delta}\tan(\frac{1}{2}\sqrt{-\Delta}\xi}{2p}\right)+$$
$$\left(\frac{1}{L_1}\left(-\frac{(pqL_1^2+\mu L_2^2)k_2k_1^2}{L_1}-\frac{\lambda k_2}{2L_1}\left(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2\right)\right)\right)\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right]$$
$$+\left(p^2rk_1^3-\frac{p^2k_1(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_1L_2}\right)\left(-\frac{r-\sqrt{-\Delta}\tan(\frac{1}{2}\sqrt{-\Delta}\xi)}{2p}\right)^2+\frac{1}{L_2}\left(-\lambda L_2k_2^3-\frac{k_2^2(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2}{2L_1k_1}\right)$$
$$\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right]^2+\left(\frac{-prL_1k_1^2k_2+p\lambda L_2k_1^2k_2}{L_1}\right)\left(-\frac{r-\sqrt{-\Delta}\tan(\frac{1}{2}\sqrt{-\Delta}\xi)}{2p}\right)$$
$$\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right)+p^3k_1^3\left(-\frac{r-\sqrt{-\Delta}\tan(\frac{1}{2}\sqrt{-\Delta}\xi)}{2p}\right)^3$$
$$-p^2k_1^2k_2\left(-\frac{r-\sqrt{-\Delta}\tan(\frac{1}{2}\sqrt{-\Delta}\xi)}{2p}\right)^2\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right)+$$

$$pk_1k_2^2\left(-\frac{r-\sqrt{-\Delta}\tan(\frac{1}{2}\sqrt{-\Delta}\xi)}{2p}\right)\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}\right)\right)^2+$$

$$-k_2^3\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}\right)\right)^3 \tag{23}$$

$$u_6(t,x,y)=\frac{-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2}{2L_1L_2k_1}-pk_1\left(-\frac{r-\sqrt{-\Delta}\cot\left(\frac{1}{2}\sqrt{-\Delta}\,\xi\right)}{2p}\right)+$$

$$k_2\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\;\eta\right)+C_2\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta}\right)}{C_1\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta}\right)+C_2\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta}\right)}\right)\right]$$

$$v_6(t,x,y)=-\frac{\left(pqL_1^2+\mu L_2^2\right)k_1}{L_1}-prL_1k_1\left(-\frac{r-\sqrt{-\Delta}\cot\left(\frac{1}{2}\sqrt{-\Delta}\xi\right)}{2p}\right)+\left(-\lambda L_2k_2\right)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}\right)\right)-p^2L_1k_1\left(-\frac{r-\sqrt{\Delta}\cot(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)^2$$

$$-L_2k_2\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}\right)\right]^2$$

$$w_6(t,x,y)=\frac{-3r^2L_1^2L_2k_1^3-\lambda^2L_2^3k_1^3-8\mu L_2^3k_1^3-L_1^3\lambda_2}{12L_1^2L_2k_1}-prk_1^2\left(-\frac{r-\sqrt{-\Delta}\cot(\frac{1}{2}\sqrt{-\Delta}\xi)}{2p}\right)-\lambda k_2^2$$

$$\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right]-p^2k_1^2\left(-\frac{r-\sqrt{-\Delta}\cot(\frac{1}{2}\sqrt{-\Delta}\xi}{2p}\right)^2$$

$$-k_2^2\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right]^2$$

$$H_6(t,x,y)=\alpha_0+\left(\frac{1}{L_1}\left(\frac{p(pqL_1^2+\mu L_2^2)k_1^3}{L_1}-\frac{prk_1\left(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2\right)}{2L_1L_2}\right)\right)\left(-\frac{r-\sqrt{-\Delta}\cot(\frac{1}{2}\sqrt{-\Delta}\xi}{2p}\right)$$

$$+\left(\frac{1}{L_1}\left(-\frac{(pqL_1^2+\mu L_2^2)k_2k_1^2}{L_1}-\frac{-\lambda k_2}{2L_1}\left(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2\right)\right)\right)\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right]$$

$$+\left(p^2rk_1^3-\frac{p^2k_1(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_1L_2}\right)\left(-\frac{r-\sqrt{-\Delta}\cot(\frac{1}{2}\sqrt{-\Delta}\xi)}{2p}\right)^2+\frac{1}{L_2}\left(-\lambda L_2k_2^3-\frac{k_2^2(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_1k_1}\right)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta}\right)\right)^2+\left(\frac{-prL_1k_1^2k_2+p\lambda L_2k_1^2k_2}{L_1}\right)\left(-\frac{r-\sqrt{-\Delta}\cot(\frac{1}{2}\sqrt{-\Delta}\,\xi)}{2p}\right)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\mu)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right)+p^3k_1^3\left(-\frac{r-\sqrt{-\Delta}\cot(\frac{1}{2}\sqrt{-\Delta}\,\xi)}{2p}\right)^3-$$

$$p^2k_1^2k_2\left(-\frac{r-\sqrt{-\Delta}\cot(\frac{1}{2}\sqrt{-\Delta}\,\xi)}{2p}\right)^2\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right)+$$

$$pk_1k_2^2\left(-\frac{r-\sqrt{-\Delta}\cot(\frac{1}{2}\sqrt{-\Delta}\,\xi)}{2p}\right)\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right)^2+$$

$$-k_2^3\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right)^3$$

(24)

Family 4. When $\Delta > 0$ and $\lambda^2 < 4\mu$, then the Complexiton solutions of Eqs.(10) have the following forms:

$$u_7(t,x,y) = \frac{-rL_1L_2k_1^2 - \lambda L_2^2k_1^2 + 2\lambda L_1^2k_2^2}{2L_1L_2k_1} - pk_1\left(-\frac{r+\sqrt{\Delta}\tanh\left(\frac{1}{2}\sqrt{\Delta}\,\xi\right)}{2p}\right) +$$

$$k_2\left[-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right) + C_2\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right)}{C_1\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right) + C_2\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right)}\right)\right]$$

$$v_7(t,x,y) = -\frac{\left(pqL_1^2 + \mu L_2^2\right)k_1}{L_1} - prL_1k_1\left(-\frac{r+\sqrt{\Delta}\tanh\left(\frac{1}{2}\sqrt{\Delta}\xi\right)}{2p}\right) + \left(-\lambda L_2k_2\right)$$

$$\left(-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right) + C_2\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right)}{C_1\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right) + C_2\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right)}\right)\right) - p^2L_1k_1\left(-\frac{r+\sqrt{\Delta}\tanh\left(\frac{1}{2}\sqrt{\Delta}\,\xi\right)}{2p}\right)^2$$

$$-L_2k_2\left[-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right) + C_2\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right)}{C_1\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right) + C_2\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right)}\right)\right]^2$$

$$w_7(t,x,y) = \frac{-3r^2L_1^2L_2k_1^3 - \lambda^2L_2^3k_1^3 - 8\mu L_2^3k_1^3 - L_1^3\lambda_2}{12L_1^2L_2k_1} - prk_1^2\left(-\frac{r+\sqrt{\Delta}\tanh\left(\frac{1}{2}\sqrt{\Delta}\xi\right)}{2p}\right) - \lambda k_2^2$$

$$\left[-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta\right) + C_2\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta\right)}{C_1\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta\right) + C_2\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta\right)}\right)\right] - p^2k_1^2\left(-\frac{r+\sqrt{\Delta}\tanh\left(\frac{1}{2}\sqrt{\Delta}\xi\right)}{2p}\right)^2$$

$$-k_2^2\left[-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right) + C_2\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right)}{C_1\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta\right) + C_2\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right)}\right)\right]^2$$

$$H_7(t,x,y)=\alpha_0+\left(\frac{1}{L_1}\left(\frac{p\left(pqL_1^2+\mu L_2^2\right)k_1^3}{L_1}-\frac{prk_1\left(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2\right)}{2L_1L_2}\right)\right)\left(-\frac{r+\sqrt{\Delta}\tanh(\frac{1}{2}\sqrt{\Delta}\xi}{2p}\right)$$

$$+\left(\frac{1}{L_1}\left(-\frac{(pqL_1^2+\mu L_2^2)k_2k_1^2}{L_1}-\frac{-\lambda k_2}{2L_1}\left(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2\right)\right)\right)\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right]$$

$$+\left(p^2rk_1^3-\frac{p^2k_1(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_1L_2}\right)\left(-\frac{r+\sqrt{\Delta}\tanh(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)^2+\frac{1}{L_2}\left(-\lambda L_2k_2^3-\frac{k_2^2(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_1k_1}\right)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\cos(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}{C_1\cos(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\sin(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta}}\right)\right)^2+\left(\frac{-prL_1k_1^2k_2+p\lambda L_2k_1^2k_2}{L_1}\right)\left(-\frac{r+\sqrt{\Delta}\tanh(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu\mu})+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})}\right)\right)+p^3k_1^3\left(-\frac{r+\sqrt{\Delta}\tanh(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)^3-$$

$$p^2k_1^2k_2\left(-\frac{r+\sqrt{\Delta}\tanh(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)^2\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right)+$$

$$pk_1k_2^2\left(-\frac{r+\sqrt{\Delta}\tanh(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right)^2+$$

$$-k_2^3\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right)^3$$

(25)

$$u_8(t,x,y)=\frac{-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2}{2L_1L_2k_1}-pk_1\left(-\frac{r+\sqrt{\Delta}\coth\left(\frac{1}{2}\sqrt{\Delta}\,\xi\right)}{2p}\right)+$$

$$k_2\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right)+C_2\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right)}{C_1\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right)+C_2\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right)}\right)\right]$$

$$v_8(t,x,y)=-\frac{\left(pqL_1^2+\mu L_2^2\right)k_1}{L_1}-prL_1k_1\left(-\frac{r+\sqrt{\Delta}\coth\left(\frac{1}{2}\sqrt{\Delta}\xi\right)}{2p}\right)+\left(-\lambda L_2k_2\right)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta)}\right)\right)-p^2L_1k_1\left(-\frac{r+\sqrt{\Delta}\coth(\frac{1}{2}\sqrt{\Delta}\ \xi)}{2p}\right)^2$$

$$-L_2k_2\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right]^2$$

$$w_8(t,x,y)=\frac{-3r^2L_1^2L_2k_1^3-\lambda^2L_2^3k_1^3-8\mu L_2^3k_1^3-L_1^3\lambda_2}{12L_1^2L_2k_1}-prk_1^2\left(-\frac{r+\sqrt{\Delta}\coth(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)-\lambda k_2^2$$

$$\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right]-p^2k_1^2\left(-\frac{r+\sqrt{\Delta}\coth(\frac{1}{2}\sqrt{\Delta}\xi}{2p}\right)^2$$

$$-k_2^2\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu}-\lambda^2\eta)}\right)\right]^2$$

$$H_8(t,x,y)=\alpha_0+\left(\frac{1}{L_1}\left(\frac{p\left(pqL_1^2+\mu L_2^2\right)k_1^3}{L_1}-\frac{prk_1\left(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2\right)}{2L_1L_2}\right)\right)\left(-\frac{r+\sqrt{\Delta}\coth(\frac{1}{2}\sqrt{\Delta}\ \xi}{2p}\right)+$$

$$\left(\frac{1}{L_1}\left(-\frac{(pqL_1^2+\mu L_2^2)k_2k_1^2}{L_1}-\frac{\lambda k_2}{2L_1}\left(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2\right)\right)\right)\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos\ (\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right]$$

$$+\left(p^2rk_1^3-\frac{p^2k_1(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_1L_2}\right)\left(-\frac{r+\sqrt{\Delta}\coth(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)^2+\frac{1}{L_2}\left(-\lambda L_2k_2^3-\frac{k_2^2(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_1k_1}\right)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta}\right)\right)^2+\left(\frac{-prL_1k_1^2k_2+p\lambda L_2k_1^2k_2}{L_1}\right)\left(-\frac{r+\sqrt{\Delta}\coth(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right)+p^3k_1^3\left(-\frac{r+\sqrt{\Delta}\coth(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)^3-$$

$$p^2k_1^2k_2\left(-\frac{r+\sqrt{\Delta}\coth(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)^2\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right)+$$

$$pk_1k_2^2\left(-\frac{r+\sqrt{\Delta}\coth(\frac{1}{2}\sqrt{\Delta}\xi)}{2p}\right)\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right)^2+$$

$$-k_2^3\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right)^3$$

(26)

Where $\Delta=r^2-4pq$ and C_1,C_2 are constants.

Family 5. When $r=q=0$, then the double soliton-like solutions of Eqs.(24) have the following forms:

(i) If $\lambda^2>4\mu$, we get

$$u_9(t,x,y)=\frac{-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2}{2L_1L_2k_1}-pk_1\left(\frac{-1}{p\xi+H_1}\right)+$$

$$k_2\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\ \eta\right)+C_2\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta\right)}{C_1\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta\right)+C_2\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta\right)}\right)\right]$$

$$v_9(t,x,y) = -\frac{\left(pqL_1^2+\mu L_2^2\right)k_1}{L_1} - prL_1k_1\left(\frac{-1}{p\xi+E_1}\right) + \left(-\lambda L_2k_2\right)$$
$$\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right) - p^2L_1k_1\left(\frac{-1}{p\xi+H_1}\right)^2$$
$$-L_2k_2\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right]^2$$

$$w_9(t,x,y) = \frac{-3r^2L_1^2L_2k_1^3-\lambda^2L_2^3k_1^3-8\mu L_2^3k_1^3-L_1^3\lambda_2}{12L_1^2L_2k_1} - prk_1^2\left(\frac{-1}{p\xi+E_1}\right) - \lambda k_2^2$$
$$\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right] - p^2k_1^2\left(\frac{-1}{p\xi+E_1}\right)^2$$
$$-k_2^2\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right]^2$$

$$H_9(t,x,y) = \alpha_0 + \left(\frac{1}{L_1}\left(\frac{p\left(p\,qL_1^2+\mu L_2^2\right)k_1^3}{L_1} - \frac{p\,rk_1\left(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2\right)}{2L_1L_2}\right)\right)\left(\frac{-1}{p\xi+E_1}\right)$$
$$+\left(\frac{1}{L_1}\left(-\frac{(pqL_1^2+\mu L_2^2)k_2k_1^2}{L_1} - \frac{-\lambda k_2}{2L_1}\left(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2\right)\right)\right)\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right]$$
$$+\left(p^2rk_1^3 - \frac{p^2k_1(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_1L_2}\right)\left(\frac{-1}{p\xi+E_1}\right)^2 + \frac{1}{L_2}\left(-\lambda L_2k_2^3 - \frac{k_2^2(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_1k_1}\right)$$
$$\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta}\right)\right)^2 + \left(\frac{-prL_1k_1^2k_2+p\lambda L_2k_1^2k_2}{L_1}\right)\left(\frac{-1}{p\xi+E_1}\right)$$
$$\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\mu)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right) + p^3k_1^3\left(\frac{-1}{p\xi+E_1}\right)^3 -$$

$$p^2k_1^2k_2\left(\frac{-1}{p\xi+E_1}\right)^2\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right)+$$

$$pk_1k_2^2\left(\frac{-1}{p\xi+E_1}\right)\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right)^2+$$

$$-k_2^3\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}{C_1\cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)+C_2\sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu}\eta)}\right)\right)^3 \quad (27)$$

(ii) If $\lambda^2<4\mu$, we get

$$u_{10}(t,x,y)=\frac{-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2}{2L_1L_2k_1}-pk_1\left(\frac{-1}{p\xi+E_1}\right)+$$

$$k_2\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right)+C_2\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right)}{C_1\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right)+C_2\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta\right)}\right)\right]$$

$$v_{10}(t,x,y)=-\frac{(pqL_1^2+\mu L_2^2)k_1}{L_1}-p\,rL_1k_1\left(\frac{-1}{p\xi+E_1}\right)+(-\lambda L_2k_2)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right)-(p^2L_1k_1)\left(\frac{-1}{p\xi+E_1}\right)^2$$

$$-L_2k_2\left[-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\,\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right]^2$$

$$w_{10}(t,x,y)=\frac{-3r^2L_1^2L_2k_1^3-\lambda^2L_2^3k_1^3-8\mu L_2^3k_1^3-L_1^3\lambda_2}{12L_1^2L_2k_1}-prk_1^2\left(\frac{-1}{p\xi+E_1}\right)-\lambda k_2^2$$

$$\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right]-p^2k_1^2\left(\frac{-1}{p\xi+E_1}\right)^2$$

$$-k_2^2\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu}-\lambda^2\eta)}\right)\right]^2$$

$$H_{10}(t,x,y)=\alpha_0+\left(\frac{1}{L_1}\left(\frac{p(pqL_1^2+\mu L_2^2)k_1^3}{L_1}-\frac{prk_1(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_1L_2}\right)\right)\left(\frac{-1}{p\xi+E_1}\right)$$

$$+\left(\frac{1}{L_1}\left(-\frac{(pqL_1^2+\mu L_2^2)k_2k_1^2}{L_1}-\frac{-\lambda k_2}{2L_1}\left(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2\right)\right)\right)\left[-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right]$$

$$+\left(p^2rk_1^3-\frac{p^2k_1(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_1L_2}\right)\left(\frac{-1}{p\xi+E_1}\right)^2+\frac{1}{L_2}\left(-\lambda L_2k_2^3-\frac{k_2^2(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_1k_1}\right)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta}\right)\right)^2+\left(\frac{-prL_1k_1^2k_2+p\lambda L_2k_1^2k_2}{L_1}\right)\left(\frac{-1}{P\xi+E_1}\right)$$

$$\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right)+p^3k_1^3\left(\frac{-1}{P\xi+E_2}\right)^3-$$

$$p^2k_1^2k_2\left(\frac{-1}{p\xi+E_1}\right)^2\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\ \eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right)+$$

$$pk_1k_2^2\left(\frac{-1}{p\xi+E_1}\right)\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\cos(\frac{1}{2}\sqrt{4\mu-\lambda}\ \eta)}{C_1\cos(\frac{1}{2}\sqrt{\lambda^2-4\mu\eta})+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right)^2+$$

$$-k_2^3\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}{C_1\cos(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)+C_2\sin(\frac{1}{2}\sqrt{4\mu-\lambda^2}\eta)}\right)\right)^3$$

(28)

Family 6. when $r=q=0,\lambda^2=4\mu$ and $p,C_2\neq 0$, then the double soliton-like solutions of Eqs.(10) have the following forms:

$$u_{11}(t,x,y)=\frac{-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2}{2L_1L_2k_1}-pk_1\left(\frac{-1}{p\xi+E_1}\right)+k_2\left[-\frac{\lambda}{2}+\frac{C_2}{C_1+C_2\eta}\right]$$

$$v_{11}(t,x,y)=-\frac{\left(pqL_1^2+\mu L_2^2\right)k_1}{L_1}-prL_1k_1\left(\frac{-1}{p\xi+E_1}\right)+\left(-\lambda L_2k_2\right)\left(-\frac{\lambda}{2}+\frac{C_2}{C_1+C_2\eta}\right)-\left(p^2L_1k_1\right)\left(\frac{-1}{p\xi+E_1}\right)^2$$
$$-L_2k_2\left[-\frac{\lambda}{2}+\frac{C_2}{C_1+C_2\eta}\right]^2$$

$$w_{11}(t,x,y)=\frac{-3r^2L_1^2L_2k_1^3-\lambda^2L_2^3k_1^3-8\mu L_2^3k_1^3-L_1^3\lambda_2}{12L_1^2L_2k_1}-prk_1^2\left(\frac{-1}{p\xi+E_1}\right)-\lambda k_2^2\left[-\frac{\lambda}{2}+\frac{C_2}{C_1+C_2\eta}\right]$$
$$-p^2k_1^2\left(\frac{-1}{p\xi+E_1}\right)^2-k_2^2\left[-\frac{\lambda}{2}+\frac{C_2}{C_1+C_2\eta}\right]^2$$

$$H_{11}(t,x,y)=\alpha_0+\left(\frac{1}{L_1}\left(\frac{p\left(pqL_1^2+\mu L_2^2\right)k_1^3}{L_1}-\frac{prk_1\left(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2\right)}{2L_1L_2}\right)\right)\left(\frac{-1}{p\xi+E_1}\right)$$
$$+\left(\frac{1}{L_1}\left(-\frac{(pqL_1^2+\mu L_2^2)k_2k_1^2}{L_1}-\frac{-\lambda k_2}{2L_1}\left(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2\right)\right)\right)\left[-\frac{\lambda}{2}+\frac{C_2}{C_1+C_2\eta}\right]$$
$$+\left(p^2rk_1^3-\frac{p^2k_1(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_1L_2}\right)\left(\frac{-1}{p\xi+E_1}\right)^2+\frac{1}{L_2}\left(-\lambda L_2k_2^3-\frac{k_2^2(-rL_1L_2k_1^2-\lambda L_2^2k_1^2+2\lambda L_1^2k_2^2)}{2L_1k_1}\right)$$
$$\left(-\frac{\lambda}{2}+\frac{C_2}{C_1+C_2\eta}\right)^2+\left(\frac{-prL_1k_1^2k_2+p\lambda L_2k_1^2k_2}{L_1}\right)\left(\frac{-1}{P\xi+E_1}\right)\left(-\frac{\lambda}{2}+\frac{C_2}{C_1+C_2\eta}\right)+p^3k_1^3\left(\frac{-1}{P\xi+E_2}\right)^3-$$

$$p^2k_1^2k_2\left(\frac{-1}{p\xi+E_1}\right)^2\left(-\frac{\lambda}{2}+\frac{C_2}{C_1+C_2\eta}\right)+pk_1k_2^2\left(\frac{-1}{p\xi+E_1}\right)\left(-\frac{\lambda}{2}+\frac{C_2}{C_1+C_2\eta}\right)^2-k_2^3\left(-\frac{\lambda}{2}+\frac{C_2}{C_1+C_2\eta}\right)^3 \quad (29)$$

We should point out that the solutions obtained in this paper are not only the (19)-(29). We only list some new ones corresponding to case1, to show that our method is efficiency in constructing the double solitary- like wave solutions, double trigonometric function solutions and complexiton soliton solutions of the nonlinear (2+1)-dimensional Generalized Broer-Kaup equations (10)

4. CONCLUSION

In this paper, we use the extended coupled sub-equations expansion method to calculate some new exact solutions for the nonlinear partial differential equations via the nonlinear (2+1)-dimensional Generalized Broer-Kaup equations . With the aid of Maple, we have obtained new exact solutions of these equations

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