INVESTIGATION OF BIOHEAT TRANSFER EQUATION OF PENNES VIA A NEW METHOD BASED ON WRM & HOMOTOPY PERTURBATION

Shahnazari M.[1], Aghanajafi C.[1], Azimifar M.[1] & Jamali H.[2]
[1] Department of Mechanical Engineering K.N.Toosi university, Tehran, Iran
[2] Department of Mechanical Engineering Industrial and Science university, Tehran, Iran

ABSTRACT
Heat transfer has an important role in biological systems of living beings. Precise evaluation of temperature distribution in live tissues can be effective in cure of lots of diseases as like as cancer. Therefore, it is required to have an adequate heat model to be able to study heat transfer in live tissues. Several models have been offered by now which among them, Pennes biothermal model has the most usage in evaluation of heat transfer in tissues of living beings. Some researcher have analyzed the Pennes equation of biothermal transfer by numeric methods. The goal of this study is to evaluate temperature distribution in live tissues while the skin is under effect of frequency thermal flux. To perform this study, the Pennes biothermal Equation has been analyzed by a wrm-homotopy perturbation combination method. Comparison of results obtained in this study with other methods, shows the capability of the method used to analyze these kind of problems. Finally effect of time, distance from skin surface, frequency and amplitude of heat flux and perfusion rate of blood on the temperature distribution will be evaluated.

Key words: *Biothermal equation, Frequency thermal flux, Homotopy Perturbation, Wrm.*

Nomenclature

c	dimensionless perfusion rate	X	dimensionless coordinate
c_b	specific heat of blood (J/kg.k)	x	coordinate (m)
c_t	specific heat of tissue (J/kg.k)	Z	dimensionless time
D	differential operator	α	thermal diffusivity (m^2/s)
G_2	Green's function, second kind	α_λ	normalized solution
k	thermal conductivity (J/s.m.k)	θ	dimensionless temperature
p	homotopy parameter	θ_{HP}	homotopy perturbation response
q_0	amplitude of heat flux (w/ m^2)	θ	homotopy perturbation collocation response
T	tissue temperature ($^\circ C$)	ρ_t	tissue density (kg/ m^3)
t	time (sec)	λ	regularization parameter
T_a	initial temperature ($^\circ C$)	ω	frequency of heat flux ($1/sec$)
w_b	blood perfusion rate (kg/sec. m^3)		

1. INTRODUCTION

The biological tissues have complex structure. They include vast cells which are separated by empty space. The mechanism of transfer of energy and mass is even more complex in these systems. Although, the science of heat transfer mechanism in biological tissues is important in medical methods which are using local heat in biological cells for treatment of diseases [1].

heat transfer in live tissues is a complex process which includes leading, movement, radiation, internal body mechanism, phase changing and setting main body temperature. Meanwhile, the main difference between live tissue

and other biological things is in influence of blood pressure rate on body temperature of different tissues and parts of the body [2], [3].
The history of vast science like medical engineering can be studied from different views and provide several understandings. Galileo was the first man who invent a thermometer made by
liquid inside a glass tube to compare the temperature of different people but unfortunately it was not enough useful. Besides, he stated that when the body is in current of the air, it will lose weight. Antoine Lavvazyh and Laplace have invested the study of Boyle and develop the principals of thermo metering for determining the lost heat from guinea pigs [1].
In 1948, Pennes states a model in which blood perfusion rate was a distribution of tangible heat transfer on a volume part based on volumetric blood perfusion rate and temperature difference between arterial blood and passing blood in veins. The model was based on supposition of heat balance between blood and tissues around it which the balance was making in entrance of capillaries [4]. In Pennes equation, it is supposed that blood is performing as local distributor, a spring or energetic well scalar [5], [6].
In Gautherie model, there is not a term for blood injection and the thermal conductivity coefficient of the tissue is supposed as a function based on local tissue`s heat [7].
While Chen and Holmes offered the model of biothermal transfer and Chato made his studies on blood veins heat transfer, Weinbaum and Jiji started series of study which leaded to invention of new biothermal model which was the model of two phases of blood and composite tissue [8], [9]. Xuan and Roetzel offered the theory of average volume in a case that there is not any balance between blood and surrounded tissues. In their model, the term of movement heat transfer has replaced the perfusion term [10]. In Barun and Ivanove`s study, a simple model for thermal specification of two parts of biological tissues, initial tissues without blood and tissues on which the veins are accidentally distributed, have proposed for study of thermal area made by external radiation [11]. Among the offered models, Pennes model has been criticized in years by some people mention in [7]. Despite of all criticizes and different views, the most of mathematical analysis in field of bioheat transfer have been performed by this equation [4], [12].
The homotopy perturbation has been presented by He in 1998 [13]. He presented a new perturbation model using the benefits of model of artificial parameter of Lieu [14] and Liao [15] model of Homotopy analysis. Shahnazari et al. applied a combination method based on Homotopy perturbation method for analysis the inverse heat transfer problems [16]. They introduced a special Homotopy for the conduction heat transfer equation appropriate to the boundary inverse problems and applied the Homotopy perturbation functions as estimated functions for using in weighted residual method.
in this study the goal is bringing up a suitable algorithm for using the functions obtained from Homotopy perturbation analysis as base functions in collocation method to analyse the Pennes equation and define temperature distribution in tissue. For this purpose some assumptions such as constant specific heat of blood and tissue, supposing tissue as a semi infinite body and blood vessels as heat source have been considered.

2. THEORY

The Pennes's bioheat transfer equation describes the thermal behavior based on the classical Fourier's law and it is based on the assumption that all heat transfer between the tissue and the blood occurs in the capillaries. In other words, this continuum approach neglects the local effects where "thermally significant" blood vessels do not appear in the temperature field.[17], [18].

In this article, tissue have been considered under perfusion as a semi infinite body in which the skin will be posed under frequency thermal flux [19]. Therefore, to calculating thermal diffusivity inside the tissue while the skin surface is subjected to frequency heat flux, the one-dimensional Pennes model with lateral and initial condition will be considered as below. The initial temperature of tissue has been assumed equal to the blood temperature, which is constant. As mentioned, the tissue has been supposed as a semi infinite body, so it can be assumed the temperature in tissue depth is finite.

$$\rho_t c_t \frac{\partial T}{\partial t} + w_b c_b (T - T_a) = k \frac{\partial^2 T}{\partial x^2} \tag{1}$$

$$T(x,0) = T_a \tag{2}$$

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0}=q_0\,e^{i\omega t} \tag{3}$$

for simplicity dimensional analysis has been applied. so Eq. (4) Eq. (5) and Eq. (6), which are dimensionless equations, will be investigated as the main problem.

$$\frac{\partial\theta}{\partial Z}+c\,\theta=\frac{\partial^2\theta}{\partial X^2} \tag{4}$$

$$\theta(X,0)=0 \tag{5}$$

$$\frac{\partial\theta}{\partial X}\bigg|_{X=0}=-\,e^{iZ} \tag{6}$$

the dimensionless quantities which have been used in above equation are presented by following equations:

$$Z=\omega t \quad \alpha=\frac{k}{\rho_t\,c_t} \quad X=\sqrt{\frac{\omega}{\alpha}}x \quad c=\frac{w_b\,c_b}{\rho_t\,c_t\,\omega} \quad \theta=\frac{k(T-T_a)}{q_0}\sqrt{\frac{\omega}{\alpha}} \tag{7}$$

In Homotopy perturbation method, Homotopy parameter is considered as a small parameter in the equations and according to the perturbation analysis, the response will be assumed as a weighted power series of Homotopy parameter to achieve an accurate result [16].

In order to analyse the problem by Homotopy perturbation method, the below Homotopy form has been suggested:

$$\theta_Z+P\,c\,\theta=\theta_{XX} \qquad 0\le P\le 1 \tag{8}$$

It will be shown from Eq. (8) that when the homotopy parameter is closing to 1, the Eq. (5) will be closed to Eq. (8). The response due to Homotopy perturbation analysis of Eqs. (8) is presented as a weighted power series of Homotopy parameters:

$$u=\theta_0+P\theta_1+P^2\theta_2+P^3\theta_3+... \tag{9}$$

Eq. (9) will be placed as a solution in Eq. (8) and the following equation will be obtained:

$$\begin{aligned}&(\theta_0+P\theta_1+P^2\theta_2+...)_Z+P\,c(\theta_0+P\theta_1+P^2\theta_2+...)\\&=(\theta_0+P\theta_1+P^2\theta_2+...)_{XX}\end{aligned} \tag{10}$$

It is possible to allocate all initial and boundaries conditions to one sentence in analyzing the perturbation homotopy and considered 0 for all other sentences or it is possible to divide the related function between sentences [16]. In this article, to increase the speed of convergence, the initial and boundary conditions of first sentence will be placed as result of homotopy perturbation. In consideration to given explanations, now we settle Eq. (10) based on equal powers of homotopy parameters which its result is as below [20]:

$$\begin{aligned}&P^0:\ \theta_{0Z}-\theta_{0XX}=0\\&X=0:\theta_{0X}=-e^{iZ}\\&Z=0:\theta_0=0\end{aligned} \tag{11}$$

$$\begin{aligned}&P^1:\ \theta_{1Z}-\theta_{1XX}=-c\theta_0\ \le\\&X=0:\theta_{1X}=0\\&Z=0:\theta_1=0\end{aligned} \tag{12}$$

$$
\begin{aligned}
&P^2: \ \theta_{2Z} - \theta_{2XX} = -c\theta_1 \\
&X = 0 : \theta_{2X} = 0 \\
&Z = 0 : \theta_2 = 0 \\
&\vdots
\end{aligned} \qquad (13)
$$

In the same way and by methods similar to above equations, the next sentences will be provided.
By using Green's functions, the result of Eqs. (11), (12) and (13) will be calculated as below [21]:

$$\theta_0(X,Z) = \frac{1}{\sqrt{\pi}} \int_0^Z e^{i\tau} (Z-\tau)^{-1/2} e^{-X^{2/4(Z-\tau)}} d\tau \qquad (14)$$

$$
\begin{aligned}
&\theta_1(X,Z) = \\
&\frac{1}{\sqrt{\pi}} \int_0^Z \int_0^\infty \theta_0(\xi,\tau) G_2(X,\xi,Z-\tau) d\tau \, d\xi
\end{aligned} \qquad (15)
$$

$$
\begin{aligned}
&\theta_2(X,Z) = \\
&\frac{1}{\sqrt{\pi}} \int_0^Z \int_0^\infty \theta_1(\xi,\tau) G_2(X,\xi,Z-\tau) d\tau \, d\xi
\end{aligned} \qquad (16)
$$

As mentioned, by closing homotopy parameter to 1, Eq. (8) will be closed to (5). Therefore, we can provide the result of homotopy perturbation number (5) as below :

$$\theta_{HP} = \lim_{p \to 1} u = \theta_0 + \theta_1 + \theta_2 + \ldots \qquad (17)$$

it is obvious in these cases, solution of the problem can't be a good estimation. even it's as a general solution for known conditions. For obviating this difficulty, some first sentences obtained from homotopy perturbation method are used as base function. then for collating the results, the collocation method will be applied [20]. as it has been explained, a new algorithm, wrm- homotopy perturbation combination, for evaluating the temperature distribution will be used. so the result of Eq. (5) will be considered as a series which its sentences include the results of analyzing the homotopy perturbation with unknown coefficient.

$$\theta = \theta_0 + \alpha_1 \theta_1 + \alpha_2 \theta_2 + \alpha_3 \theta_3 + \ldots \qquad (18)$$

The first 4 sentences will be considered as result of the problem in this article. To calculate the unknown items of the problem, we use the covering points in theory of weighted residual [20].
The operator differential dominating Eq. (5) will be defined as below :

$$D = \left(\frac{\partial}{\partial Z} - \frac{\partial^2}{\partial X^2} + c\right) \qquad (19)$$

The Eq. (18) will be placed in (19) and the residual will be defined as below:

$$D\theta(X,Z) - D\theta(X,Z) = R \qquad (20)$$

according to collocation theory, the residual amount will be leaded to 0 and in consideration to Eq. (5), we will have :

$$\left(\frac{\partial}{\partial Z} - \frac{\partial^2}{\partial X^2} + c\right)\theta(X,Z) = 0 \qquad (21)$$

To calculate the items, we will choose some points as collocation pints and put them in Eq. (21). It will be one equation against each point. Therefore, the result of Eq. (5) will be determined by solving the set of functions and unknowns. By solving this matrix equation, the coefficient which were used in Eq. (18) will be achieved.

$$A\alpha = b \tag{22}$$

3. REGULARIZATION

Matrix A in Eq. (22) is singular. so it's not possible to be solved in ordinary way such as least square method, and regularization methods is required for settling this difficulty. in this situation the main matter is the high sensitivity of solutions against disarray. while the coefficients matrix is singular, the solution norm which are achieved by ordinary methods , are much greater than exact solution norm. In this case the condition number is high, and it can be expressed that matrix A is close to linear dependence [22].

there are different methods for regularizing. these methods can be sorted in straight an iterative methods. among straight methods the regularization methods of Tikhonov, Least Square, Truncated Singular Value and among iterative methods, the regularization methods of Fourier, Coupled Gradient are more outstanding.

Among the mentioned methods, the Tikhonov method is most famous and capable which has been used in this article. In Tikhonov method the normalized solution, $\boldsymbol{\alpha}_\lambda$, is obtained by solving the following least square equation.

$$\boldsymbol{\alpha}_\lambda = \arg\min\{\|\mathbf{A}\boldsymbol{\alpha} - \mathbf{b}\|_2^2 + \lambda\|\boldsymbol{\alpha}\|_2^2\} \tag{23}$$

the Regularization Parameter, λ, controls the weight which is allocated for minimizing the extra constraint proportion to residual norm. so a method for finding a optimum parameter should be applied. in this article a logarithm curve between regularized solution and residual norm against different regularization parameter has been used. the value of optimum parameter is based on the corner point of the this curve. in order to find this point, on method is drawing the curve and discovering the mentioned point by iteration [23].

4. RESULTS AND DISCUSSION

In this article the thermal properties of blood and tissue will be considered as below [18]:

Table 1 property of blood and tissue

Property	Magnitude (unit)
c_b	3770 (J/kg.k)
c_t	3770 (J/kg.k)
k	0.5 (J/s.m.k)
q_0	5000 (w/ m^2)
T_a	37 ($^\circ C$)
w_b	0.5 (kg/sec. m^3)
ρ_t	1050(kg/ m^3)

In fig. 1, the deviation of temperature of the skin based on time and under effect of thermal flux with amplitude of 5000 w/ m^2 and frequency of 0.05 1/s, is shown. It is obvious in the graph that in first moments, there is a difference between the result of this article and study of Tzu-Ching Shih et al. [18] which decrease by the time. Meanwhile there are phase difference about some minutes in results.

Fig. 2 shows the non dimensional temperature deviation based on result of this article and result of Tzu-Ching Shih et al. [18] and also the non dimensional thermal flux based on time. Similar to Fig. 1, difference between two results in first moments and difference of time phases in later time is observed. The rate of inside tissue blood perfusion can be estimated based on phase shift between temperature and thermal flux considering to Tzu-Ching Shih et al. [18]. This graph is drawn based on thermal flux with amplitude and frequency of 5000 w/ m^2 and frequency of .05 1/s and perfusion rate of 0.5 kg/sec. m^3 in x=0.

In fig. 3, the variation of dimensionless temperature based on time has been shown. In this graph, the distribution of amplitude of dimensionless temperature varies while skin is under thermal flux with amplitude of 5000 w/ m^2 and frequency of 0.05 1/s against different amounts of non dimensional perfusion rate, is studied. It is shown that by increase of dimensionless perfusion rate, the amplitude of dimensionless temperature distribution will decrease. The high rate of perfusion will lead to heat transfer in less time. Therefore, temperature will decrease and the rate of non dimensional temperature diagram will decrease by increase of dimensionless perfusion rate parameter value.

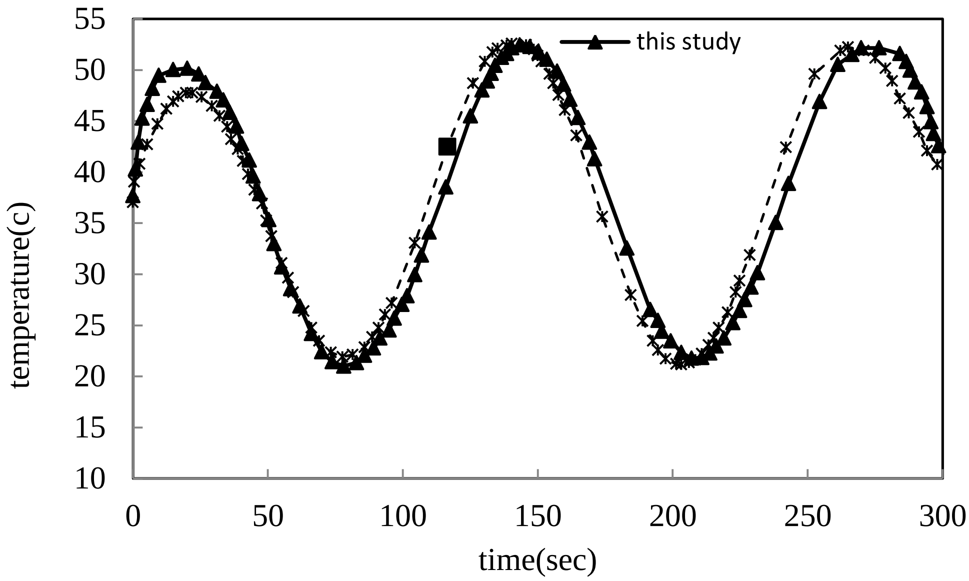

Fig. 1. The graph of temperature deviation of the skin

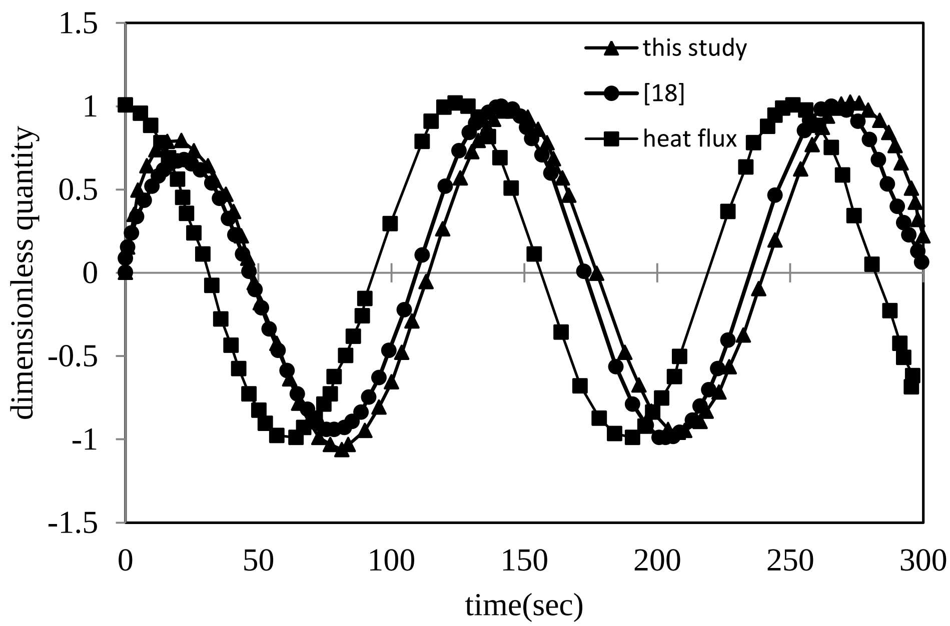

Fig. 2. The graph of non dimensional flux and temperature differences

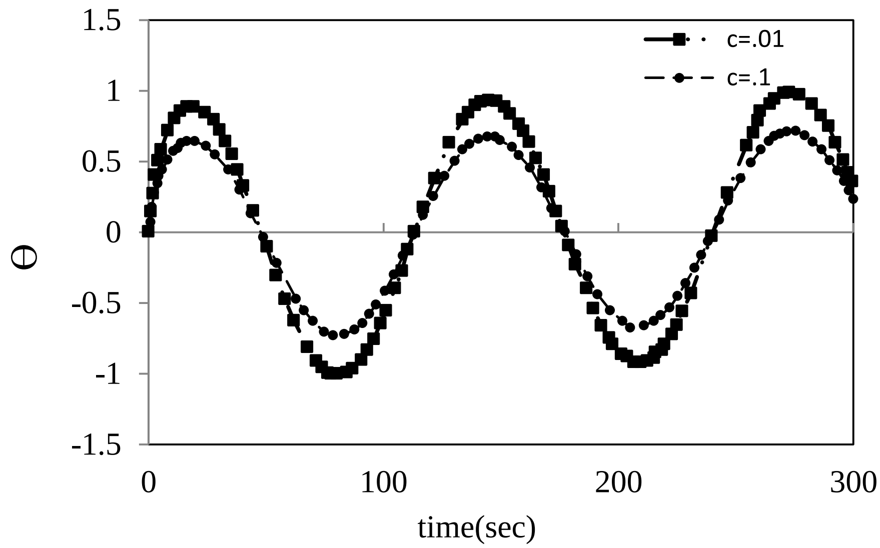

Fig. 3. The graph of non dimensional temperature deviation against different amount of non dimensional perfusion rate

In Fig. 4, non dimensional temperature distribution of skin based on time has been shown, in a case that skin is under thermal flux with amplitude of 5000 w/m^2 and frequency of 0.05 1/s and in different depth of the tissue, three depth of x=0, x=0.4 and x=0.8 cm. It is obvious that as much as the depth is increasing, the effectiveness of flux in more far layers of the skin is less [24]. Based on study of Tzu-Ching Shih et al. [18] in the first layer of tissue, X=3.1458, the range of temperature degree is about 4 degree less than surface of the skin. It means that it is about 35-39 $^{\circ}c$. In X=6.2917, the range of swinging temperature amplitude is about 1 degree. so in deeper layer of the skin, this range is more decreasing.

In Fig. 5, we can see the non dimensional temperature distribution, θ in Z=2π against perfusion rate of 0.5 kg/sec. m^3 and thermal flux of q=5000 w/m^2 in distance of 0 < x <1.5 in different frequencies. Considering to this graph, it is determining that while Z is fixed, by decrease of the thermal flux frequency, the stable swinging temperature amplitude in more depth of the tissue is presenting. This means that while the frequency of thermal flux is small, its influence is more. In fact, the high frequency leads to higher maximum amplitude of swinging temperature in first steps. Meanwhile in lower frequencies, the maximum of amplitude will decrease and depth of influx will increase [25].

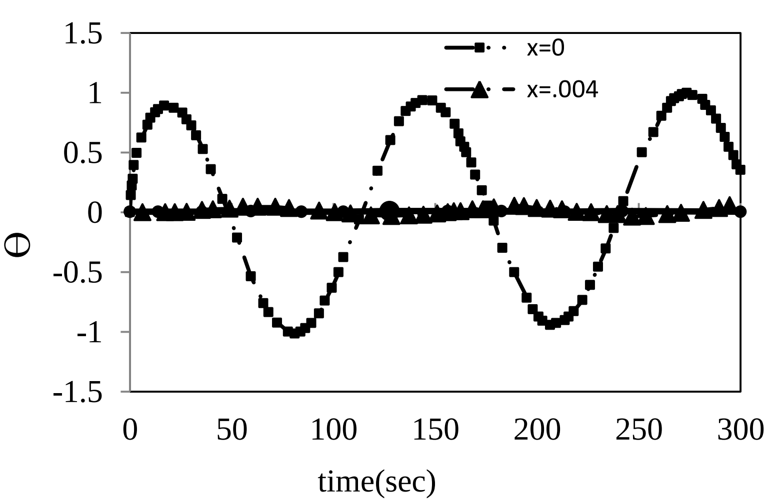

Fig. 4. The graph of non dimensional deviation in different depth of the tissue

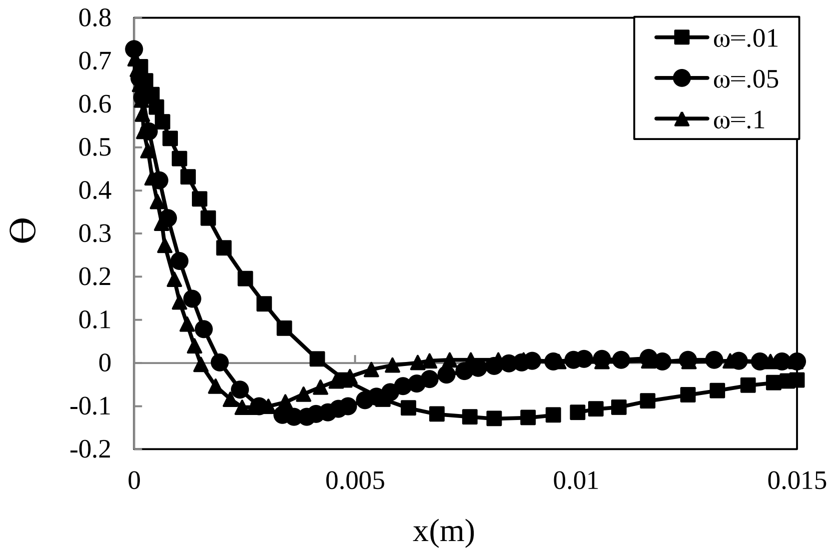

Fig. 5. The graph of differentiation of non dimensional temperature against different frequencies

5. CONCLUSIONS

In this article, the effect of frequency thermal flux on skin was studied. For this purpose, wrm -homotopy perturbation combination analysis to evaluate the heat transfer in tissues was used. Considering to obtained results and comparing them with other methods, it was concerned that the algorithm used in this article can be reliable. In fact difference between results of this method comparing to other study, in first moments is more observable and while time is increasing it will be decreasing.

As it was observed, while the frequency of heat flux was being raised, the amplitude of oscillatory temperature was decreasing and the frequency of response was increasing. Also investigating variation of oscillatory temperature response against distance of skin surface, showed that we can suppose the layer of tissue which are far from skin surface, about 1cm, are insulator. So we can consider the tissue as a finite body with the mentioned boundary condition at the far layers.

Based on obtained result, we can see that in first step of oscillatory temperature, the result is not fixed and in these cases, it is not possible to use the phase shift between thermal flux and dimensionless temperature for evaluating the perfusion rate.

REFERENCES

[1]. B. Khaefinejad, C. Aghanajafi, Laminar Flow with Simultaneously Effects of Mixed Convection and Thermal Radiation in a Channel, Journal of Fusion Energy, (2008), 83-90.

[2]. Shitzer, A. and R.C. Eberhart, eds, Heat transfer in medicine and biology,Analysis and application,Vols. 1 & 2, Plenum Press, New York, 1985.

[3]. Aghanajafi, C. · Hesampour, K., Heat Transfer Analysis of a Condensate Flow by VOF Method, Journal of Fusion Energy, (2006), .

[4]. Pennes, H.H., Analysis of tissue and arterial blood flow temperatures in the resting forearm", J of Appl. Physiology.,(1948), 93-122.

[5]. B. Rubinsky, NUMERICAL BIO-HEAT TRANSFER, University of California at Berkeley, CA 947, (2005).

[6]. Hessamoddin Abbassi, Cyrus Aghanajafi, Evaluation of Heat Transfer Augmentation in a Nanofluid-Cooled Microchannel Heat Sink, Journal of Fusion Energy, Volume 25, (), 3-4

[7]. Chato, J.C., History of heat transfer in bioengineering, in Bioengineering Heat Transfer, Y.I. Cho, Editor., Academic Press, Inc: Boston., (1992).

[8]. CHEN, M. M. & HOLMES, K. R. ,Microvascular contributions in tissue heat transfer, Ann. N. Y. Acad. Sci., (1980), 137–150.

[9]. Weinbaum, S. and L.M. Jiji, The matching of therma fields surrounding counter-current microvessels and the closure approximation in the Weinbaum-Jiji equation, ASME Trans J. of Biomech. Eng., (1989), 111: 127.

[10]. A. Nakayama, F. Kuwahara, A general bioheat transfer model based on the theory of porous media, International Journal of Heat and Mass Transfer 51, (2008), 3190–3199.

[11]. A.-R.A. Khaled, K. Vafai, The role of porous media in modeling flow and heat transfer in biological tissues, Int. J. Heat Mass Transfer, (2003),4989–5003.

[12]. P. R. Sharma, Sazid Ali, V. K. Katiyar ,Numerical Study of Heat Propagation in Living Tissue Subjected to Instantaneous Heating, Indian Journal of Biomechanics, (2009), .

[13]. He, J. H., Homotopy perturbation method: a new nonlinear analytical technique, Applied Mathematics and computation, Vol. 135, (2003),73-79.

[14]. Liu, G. L.,New research directions in singular perturbation theory: artificial parameter approach and inverse-perturbation technique, in: Conference of 7th Modern Mathematics and Mechanics, Shanghai, (1997).

[15]. Liao, S. J.,An explicit, totally analytic approximate solution for Blasius viscous flow problem, Int. J. Non-Linear Mech., Vol. 34, (1999), 759-778.

[16]. Shahnazari. M.,Ziabasharhagh.M, Amjadi golpayegani, A new collocation method for backward and boundary inverse heat conduction problems, Mechanical and Aerospace Engineering Journal, (2010), 25-34.

[17]. P.R.Sharma, Sazid Ali, V. K.Katiyar, Numerical Study of Heat Propagation in Living Tissue Subjected to Instantaneouse Heating, Indian Journal of Biomechanics, (2009).

[18]. Tzu-Ching Shih , Ping Yuan , Win-Li Lin, Hong-Sen Kou, Analytical analysis of the Pennes bioheat transfer equation with sinusoidal heat flux condition on skin surface, Medical Engineering & Physics 29, (2007), 946–953.

[19]. Hardy, J.D. and DuBois, E.F, Basal metabolism, radiation, convection and peripheral blood flow at temperatures, C. J. Nutr., 1938, 477-582.

[20]. Mohammad Reza ShahNazari, A novel Homotopy Perturbation Method: Kourosh´s Method for a Thermal Boundary Layer in a Saturated Porous Medium, International Journal of Engineering, (2009), 230-239.

[21]. Kevorkian, J.,Partial Differential Equations: Analytical Solution Techniques, Wadsworth & Brooks, California, (1989).

[22]. Hansen, P. C., Regularization Tools: A Matlab package for analysis and solution of discrete Ill-Posed problems, Numerical Algorithms, Vol. 6, pp. 1-35, 1994.

[23]. Hansen, P. C., O'leary, D. P., The use of the L-Curve in the regularization of discrete Ill-Posed problems, SIAM J. Sci. Comput, Vol. 14, pp. 487-503, 1993

[24]. Babak Safavisohi, Ehsan Sharbati, Cyrus Aghanajafi and Seyed Reza Khatami Firoozabadi, Effects of Boundary Conditions on Thermal Response of a Cellulose Acetate Layer Using Hottel's Zonal Method Layer Using Hottel's Zonal Method, Journal of Fusion Energy, vol25(3),145-153, 2006.

[25]. Ehsan Sharbati, Babak Safavisohi, Cyrus Aghanajafi, Transient Heat Transfer Analysis of a Layer by Considering the Effect of Radiation, vol 23,207-215, Sep 1, 2004.