ERRATUM TO:
"COMMON FIXED POINT THEOREM FOR SUB COMPATIBLE AND SUB SEQUENTIALLY CONTINUOUS MAPS IN FUZZY METRIC SPACE USING IMPLICT RELATION"

Saurabh Manro*
*School of Mathematics and Computer Applications, Thapar University, Patiala (Punjab), India
(sauravmanro@hotmail.com)

On critical examination of the results given in our paper [2], we notice one crucial error. We need to carry out the following correction:Unfortunately, Theorems 4.1 and 4.3 are not true in its present form. To illustrate this viewpoint, we furnish the following example which disproves Theorem 4.1 and 4.3.

Example X:[1] Let $X=[0,\infty)$. For each $t>0$ and $x,y\in X$, define

$$M(x,y,t)=\begin{cases}\dfrac{t}{t+|x-y|}, & t>0,\\ 0 & t=0.\end{cases}$$

Then *(X, M, *)* be fuzzy metric space. Let *A* = *B* and *S* = *T* and define *A* and *S* as

$$A(x)=\begin{cases}0 & if \quad x=0\\ 1+x & if \quad x\in(0,1]\\ 2x-1 & if \quad x\in(1,\infty)\end{cases} \quad \text{and} \quad S(x)=\begin{cases}1-x & if \quad x\in[0,1)\\ 3x-2 & if \quad x\in[1,\infty).\end{cases}$$

Clearly, *(A, S)* is subcompatible by taking sequence $\left\{x_n=1+\dfrac{1}{n}\right\}$ for *n* = 1, 2, 3, …. . Also, *A* and *S* are discontinuous at *x = 1*. Also *pair (A, S)* is subsequentially continuous by taking $\left\{x_n=\dfrac{1}{n}\right\}$ for n = 1, 2, 3, …. But the maps *(A, S)* do not have a coincidence point. Hence, Theorem 4.1 (and Theorem 4.3) is not true in its present form.

However, Theorem 4.1 can be corrected in two ways as follows:

Theorem 1: Let *A, B, S* and *T* be four self maps of a fuzzy metric space *(X, M,*)* with continuous t-norm * defined by $t*t\geq t$ for all $t\in[0,1]$. If the pairs *(A, S)* and *(B, T)* are compatible and subsequentially continuous, then

(a) *A* and *S* have a coincidence point;
(b) *B* and *T* have a coincidence point.
(c) For some $\phi\in\Phi$ and for all $x,y\in X$, $t>0$

$$\phi\left\{\begin{matrix}M(Ax,By,t),M(Sx,Ty,t),M(Ax,Sx,t),\\ M(By,Ty,t),M(By,Sx,t),M(Ax,Ty,t)\end{matrix}\right\}\geq 0$$

Then, *A, B, S* and *T* have a unique common fixed point.

Theorem 2: Let *A, B, S* and *T* be four self maps of a fuzzy metric space *(X,M,*)* with continuous *t*-norm * defined by $t*t\geq t$ for all $t\in[0,1]$. If the pairs *(A, S)* and *(B, T)* are subcompatible and reciprocally continuous, then

(a) *A* and *S* have a coincidence point;
(b) *B* and *T* have a coincidence point.
(c) For some $\phi\in\Phi$ and for all $x,y\in X$, $t>0$

$$\phi\begin{Bmatrix} M(Ax,By,t), M(Sx,Ty,t), M(Ax,Sx,t), \\ M(By,Ty,t), M(By,Sx,t), M(Ax,Ty,t) \end{Bmatrix} \geq 0$$

Then, *A*, *B*, *S* and *T* have a unique common fixed point.

Similarly, Theorem 4.3 can also be corrected in two ways as follows:

Theorem 3: Let *A*, *B*, *S* and *T* be four self maps of a fuzzy metric space *(X, M,*)* with continuous t-norm * defined by $t * t \geq t$ for all $t \in [0,1]$. If the pairs *(A, S)* and *(B, T)* are compatible and subsequentially continuous, then

(d) *A* and *S* have a coincidence point;
(e) *B* and *T* have a coincidence point.
(f) For some $\phi \in \Phi$ and for all $x, y \in X$, $t > 0$

$$\phi\begin{Bmatrix} M(Ax,By,t), M(Sx,Ty,t), \dfrac{M(Sx,Ax,t)+M(Ty,By,t)}{2}, \\ M(By,Sx,t), M(Ax,Ty,t) \end{Bmatrix} \geq 0$$

Then, *A*, *B*, *S* and *T* have a unique common fixed point.

Theorem 4: Let *A*, *B*, *S* and *T* be four self maps of a fuzzy metric space *(X,M,*)* with continuous *t*-norm * defined by $t * t \geq t$ for all $t \in [0,1]$. If the pairs *(A, S)* and *(B, T)* are subcompatible and reciprocally continuous, then

(d) *A* and *S* have a coincidence point;
(e) *B* and *T* have a coincidence point.
(f) For some $\phi \in \Phi$ and for all $x, y \in X$, $t > 0$

$$\phi\begin{Bmatrix} M(Ax,By,t), M(Sx,Ty,t), \dfrac{M(Sx,Ax,t)+M(Ty,By,t)}{2}, \\ M(By,Sx,t), M(Ax,Ty,t) \end{Bmatrix} \geq 0$$

Then, *A*, *B*, *S* and *T* have a unique common fixed point.

There is no need to give the proof of the above stated theorems as the proof furnished in [1] survives except the noted fallacy.

Now, we furnish two illustrative examples to highlight the utility of Theorem 1 (Theorem 3) and Theorem 2 (Theorem 4):

Example 1:[1] Let $X = [0,\infty)$. For each $t > 0$ and $x, y \in X$, define

$$M(x,y,t) = \begin{cases} \dfrac{t}{t+|x-y|}, & t > 0, \\ 0 & t = 0. \end{cases}$$

Then *(X, M, *)* be fuzzy metric space. Let *A* = *B* and *S* = *T*. Define *A* and *S* as follows:

$$A(x) = \begin{cases} \dfrac{x}{3} & if \quad x \in [0,1] \\ 2x-1 & if \quad x \in (1,\infty) \end{cases}, S(x) = \begin{cases} \dfrac{x}{2} & if \quad x \in [0,1] \\ 3x-2 & if \quad x \in (1,\infty). \end{cases}$$

Clearly *A* and *S* are discontinuous at *x = 1*.

Let $\{x_n\}$ be a sequence in X defined by $\left\{x_n = \frac{1}{n}\right\}$ for $n = 1, 2, 3, \ldots$

Then, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = 0, 0 \in X$ and

$ASx_n \to 0 = A(0), SAx_n \to 0 = S(0)$

when $n \to \infty$, and clearly, $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$.

Now, let $\{x_n\}$ be a sequence in X defined by $\left\{x_n = 1 + \frac{1}{n}\right\}$ for $n = 1, 2, 3, \ldots$

Then, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = 1, 1 \in X$ and $ASx_n \to 1 \neq A(1)$ when $n \to \infty$, so A and S are not reciprocally continuous and clearly, $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$.

Also, the pair *(A, S)* is compatible as well as sub sequentially continuous but not reciprocally continuous. Therefore, all the conditions of Theorem 1 (and Theorem 3) are satisfied and $x = 0$ is unique common fixed point of pair *(A, S)*.

Example 2:[1] Let $X = R$ (Set of Real Numbers). For each $t > 0$ and $x, y \in X$, define

$$M(x, y, t) = \begin{cases} \dfrac{t}{t + |x - y|}, & t > 0, \\ 0 & t = 0. \end{cases}$$

Then *(X, M, *)* be fuzzy metric space. Let $A = B$ and $S = T$. Define A and S as follows:

$$A(x) = \begin{cases} x + 1 & if \quad x \in (-\infty, 1) \\ 2x - 1 & if \quad x \in [1, \infty) \end{cases}, S(x) = \begin{cases} \dfrac{x}{2} & if \quad x \in (-\infty, 1) \\ 3x - 2 & if \quad x \in [1, \infty). \end{cases}$$

Clearly A and S are discontinuous at $x = 1$.

Let $\{x_n\}$ be a sequence in X defined by $\left\{x_n = 1 + \frac{1}{n}\right\}$ for $n = 1, 2, 3, \ldots$

Then, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = 1, 1 \in X$ and $ASx_n \to 1 = A(1), SAx_n \to 1 = S(1)$

when $n \to \infty$, and clearly, $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$. Also, clearly by taking sequence $\{x_n\}$ in X defined by $\left\{x_n = \frac{1}{n} - 2\right\}$ for $n = 1, 2, 3, \ldots$

Clearly, $\lim_{n\to\infty} M(ASx_n, SAx_n, t) \neq 1$

Then, A and S are reciprocally continuous as well as subcompatible but not compatible.

Therefore, all the conditions of Theorem 2 (and Theorem 4) are satisfied and $x = 1$ is unique common fixed point of pair *(A, S)*.

REFERENCES

[1]. S. Manro, S. Kumar and S.S. Bhatia, An Addendum to subcompatibility and fixed point theorem in Fuzzy metric Space, *Advances in fuzzy mathematics* (accepted for publication).

[2]. K. Wadhwa, F. Beg and H. Dubey, Common fixed point theorem for sub compatibile and subsequentially continuous maps in fuzzy metric space using implicit relation, *International journal of Research and Reviews in Applied Mathematics*,**9**(1)(2011), 87-92.