CO-INTEGRATION AND ECONOMETRIC MODEL

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ABSTRACT
Most of the time the time series of macroeconomic variables are related to Non-Stationary. Non-Stationary variable causes several problems in estimating relationships between economic variables and using regression technique. One of these problems is the consensus on macroeconomic theory that should exist a stable long-run relationship between the levels of certain economic variables. In this regard, in case of failure to use proper econometric methods to investigate and estimate the relationship between the variables, it is possible that the irrelevant variables which have only similar time trend are identified to have meaningful relationship, or it is possible that the variables are relevant and the economic theory supports this relationship. However, existence of the time trend results in incorrect assessment of the intensity of the relationship or its direction. In terms of econometrics, both these two relationships are called spurious regression. Certain Co-integration discussions have been to remove this problem, whose basis is the discussion of stationary or non- stationary of time series. In this article, a general and brief view is focused on the set of discussions relevant to the convergence and it has been tried that the aforementioned discussion is more expressed for the scholars in this field in a clear and applicable manner. In this regard, first of all the concept of uniform co-integration in the short term and long term relationships is focused and after that the co-integration vectors and their general features are specified. Eventually, by introducing the Error Correction Model (ECM) and Johansson Test which assesses the long term an stable relationship among the variables by which the discussion on co-integration are completed.

Key words: *Co-integration, Non-Stationary, Time series, Error-Correction Model.*

1. INTRODUCTION

Since most of the time series of macroeconomic variables are related to Non-Stationary and cause problems in estimation and testing of quantitative relationships between economic variables and their analysis, it is essential to achieving Non-Stationary series to be transformed into stationary series. Recent developments in time series analysis shows that the time series is related to variables being in a particular Non-Stationary pattern, the technique can be used to estimate a model that is known to co-integration. This technique can directly be used to examine or modify the basic theoryBefore examining this issue, it seems to be necessary to express some notes, first, understanding the difference between the time series and other statistical data is of great importance, often statistical methods for working with a series of data resulting from independent and observable experiments, are formulated and in Statistical analysis, universe characteristics is acquired by examining typical statistical properties. The data are usually not important in this scenario, but in the time series, that each one is related to a specific period of time, respectively, the data arrangement is important. It attempts to drive characteristics of stochastic process from characteristics of series or observable string limited random variable and by this way; one model is built to introduce the process of generating data or same stochastic process. Secondly, in order to formulate a regression equation in econometric modeling, explanatory variables should be considered and the relationship between these variables in different equations in a system of simultaneous equations is considered. The explanatory variables are defined by economic theories and models are targeted to describe and predict the behavior of the statistical data related to the underlying dependent variable. In analysis of time series in contrast to modeling in convenient econometric, models and methods are provided that make future predicting of one variable based on past data for same variable, possible. Additionally, in analysis of time series, one method is also provided to solve lacking stability in stochastic structure in modeling data generating process. These features were important for analyzing time series data and their dynamics. Thirdly, if a time series is a set of samples that each one has one observation and random process of the samples is unknown, how the statistical characteristics of the samples, such as mean, variance and covariance can be estimated? Granger and Newbold(1974) believe that Theoretically, doing these estimates is possible based on changing in mean, variance and covariance series during the time. In this sense, the concept of stationary as one of the limitations for the analysis of time series, are presented. In time series analysis to describe or predict the future state of a variable, as it was said, it depends on the previous values, thus, there should be observations of a variable over time correlation and structure and shape of correlation should remain constant over time. When there is a stationary random process, it means that the process for generating these data has fixed structure over time. However, the scientific advances made in the field of econometric has identified that many of economic variables time series, are Non-Stationary and this causes the same technique as the co-integration has extension in experimental works and Non-stationary series econometric interpreting.

In this report, we investigate the concepts and features of this technique and the application of relevant tests are raised and examined.

2. CO-INTEGRATION, SHORT-TERM AND LONG-TERM RELATIONSHIPS

As previously mentioned, Non-Stationary variable causes several problems in estimating relationships between economic variables and using regression technique. One of these problems is the consensus on macroeconomic theory that should exist a stable long-run relationship between the levels of certain economic variables. Usually, this theory states that the number of unique pairs or sets of variables, not least in the long run to get away from each other a lot, while in the short-term because of some factors, these variables are diverging away from each other and, Non-Stationary variables means that they are in the process can be both random and deterministic. So primarily, these variables must be eliminated from the processMost common method for doing so is differentiating method. However, since the process represents a long term moving of variable, differentiating actually eliminates the long term feature of a time series and as a result, working with time series, in this case, does not provide information on the long-run relationship between them. In such circumstances, it is intended to investigate the long-run relationship between two or more Non-Stationary variables, what should we do? To solve this problem, the concept of uniform Co-integration, is discussed. In fact, this method provides the possibility of consideration long-term relationships between Non-Stationary variables. In other words, when the variables have same co-integration, examining their long-term relationship with time series data will be easy. Even if these conditions are Non-Stationary variables, the estimates results obtained from econometric conventional methods, can be interpreted.Uniform Co-integration is a certain class of vector unit root process. The specification of the error correction models (Error-Correction Model) from Davidson, Srba, Hendry, Yeo (1978). have been proposed implicitly been mentioned. However, explicit recognition of the concept and its formal generalization by Granger in (1983). and Engel - Granger work in 1987, has been introduced. Co-integration is defined as: if we have a vector (N × 1) time series $Y_n t$ that each of the elements (variables) of the vector are Non-Stationary, but by first one differentiating be stationary.ie, each of these variables Co-integration are the first degree I(1), any linear combination of these variables is a first degree convergent variable, I(1), and for some linear combination like $\alpha'_\Lambda y_{\Lambda t}$, the series may be stationary or zero I(0). In other words:

$$y_t = (y_{1t}, y_{2t}, \dots \dots, y_{nt}) \quad , \quad [y_{it}] \sim I(1)$$
$$\alpha_\Lambda \neq 0_\Lambda \Rightarrow \alpha'_\Lambda y_{\Lambda t} \sim I(0)$$

As a result the linear combination between several variables is stationary and also these factors may not be divergent. This means that the time series of a linear combination of these variables fluctuate around a constant mean, in other words these variables over time have converging path. So in general we can say that when a linear combination of Non-Stationary variables has similar degree of Co-integration and it is stationary, these variables are called Co-integration. For example, a Non-Stationary time series with M time difference that is transformed into a stationary series and it is Co-integration with M degree. It is shown with I(M).

A system of two variables equations below is simple example of the Co-integration process:

$$y_{1t} = \lambda y_{2t} + u_{1t} \quad (1)$$
$$y_{2t} = y_{2t-1} + u_{2t} \quad (2)$$

In the above two equations, u_{1t} and phrases u_{2t} are disturbances term that are assumed to have mean zero and constant variance and are uncorrelated and this indicates that the variable y_{2t} is a random walk, i.e.:

$$\Delta y_{2t} = y_{2t} - y_{2t-1} = u_{2t} \quad (3)$$

Calculated by differentiating from the equation (1) , we have:

$$\Delta y_{1t} = \lambda \Delta y_{2t} + \Delta u_{1t} = \lambda u_{2t} + u_{1t} - u_{1t-1} \quad (4)$$

Right side of equation (4) represents a first order moving average process. Thus, both variables y_{1t} and y_{2t} have co-integration of first degree, I(1), and linear combination of them $(y_{1t} - \lambda y_{2t})$ is stationary too. Here vector *a*= *a=(1,-*λ') is co-integration vector.

For better understanding, we look at the equations (1) and (2). It should be noted that $E_{2t} = u_{2t}$ is forecast error which is based on the lag values y_1 and y_2 ,while $E_{1t} = \lambda u_{2t} + u_{1t}$ is forecast error for y_{1t}.
Right side of equation (4) can be written as:

$$(\lambda u_{2t} + u_{1t}) - u_{1t-1} = \varepsilon_{1t} - (\varepsilon_{1t-1} - \lambda\varepsilon_{2t-1}) = (1-L)\varepsilon_{1t} + \lambda L\varepsilon_{2t} \quad (5)$$

Where L is the lag operator.
By substituting the above equation in equation (4) and inserting this equation with equation (3) in the form of an vector system, for extracting a moving average vector process $(\Delta y_{1t}, \Delta y_{2t})'$ will be as follows:

$$\begin{bmatrix}\Delta y_{1t}\\ \Delta y_{2t}\end{bmatrix} = \Psi(L)\begin{bmatrix}\varepsilon_{1t}\\ \varepsilon_{2t}\end{bmatrix} \quad (6)$$

$$\Psi(L) \equiv \begin{bmatrix}1-L & \lambda L\\ 0 & 1\end{bmatrix} \quad (7)$$

Vector autoregressive for differentiated data can be written as follows:

$$\Phi(L)\Delta y_{nt} = \varepsilon_t$$

$$\Phi(L) = [\Psi(L)]^{-1}$$

However, in the process above, polynomials matrix for moving average operator, Ψ *(Z)* has a unit root and therefore $|\Psi(1)| = \begin{bmatrix}(1-1) & \lambda\\ 0 & 1\end{bmatrix} = 0$ here, matrix of moving average operator has not converse and there is not any autoregressive vector in particular order for explaining Δy_t.
Because of the certain autoregressive vector in differentiating form may offer poor approximation for Co-integration system of equations (1) and (2), the system that variable y_2 contains some information that could be useful for prediction y_1.

If we adjust the autoregressive vector with lag values, we can easily identify stationary being the Δy_t.

Note that given to relation, $u_{1,t-1} = \lambda_{1,t-1} - \lambda y_{2,t-1}$ equations (3) and (4) can be expressed as follows.

$$\begin{bmatrix}\Delta y_{1t}\\ \Delta y_{2t}\end{bmatrix} = \begin{bmatrix}-1 & \lambda\\ 0 & 0\end{bmatrix}\begin{bmatrix}y_{1,t-1}\\ y_{2,t-1}\end{bmatrix} + \begin{bmatrix}\lambda u_{2t} + u_{1t}\\ u_{2t}\end{bmatrix} \quad (8)$$

In general, the above equation provides a vector system which contains the lagged values and its changes. Therefore Δy_t describes as a form of a vector autoregressive.
In general, the discussion of Co-integration, one point is expressed that there are many factors that cause permanent changes in individual in one-by-one vector elements as y_t , but the long-run equilibrium relationship between these elements and based on linear combination can be found. there are many practical examples of such a system including consumption expenditure model by Davidson, Hendry, Srba and Yeo(1978). results that those authors are acquires is that consumption and income behavior have a unit root, But in the long run, taking almost a constant ratio from income and difference between the logarithm of consumption variable income variable is a stationary process.

3. GENERAL FEATURE OF CO-INTEGRATION VECTOR

As before if each element of the variables vector (y_t) has similar degree of Co-integration and there is as **a** nonzero vector with dimension (n × 1) that some linear combinations of variable vectors are stationary, is called a Co-integration vector. In fact, this vector is obtained in the form of converting the system; provide the parameters of a long-term relationship and balance offered by the theory. Co-integration vector is not unique vector and if $a'y_t$ is stationary, for other non-zero scalars such as b,linear combination b$a'y_t$ 'is too stationary and if **a** is Co-integration vector, **ba** is a Co-integration vector too.
If the vector y_t contains more than two elements (variables), so we can consider two non-zero vectors a_1, a_2 that are linearly independent, (i.e there is not any scalar like b whereby $a_2 = ba_1$) such that both $a'_1 y_t$ and $a'_2 y_t$ are

stationary. Actually, there can be h vectors (n × 1) linearly independent $(a_{\wedge^1}, a_{\wedge^2}, \dots\dots\dots\dots\dots, a_{\wedge^h})$ so that $A'y_t$ is a vector (h × 1) and is stationary where A' is a matrix (h × n) with full column rank as follows:

$$A' = \begin{bmatrix} a'_{\wedge^1} \\ a'_{\wedge^2} \\ . \\ . \\ . \\ a'_{\wedge^h} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{1n} \\ a_{21} & a_{22} & a_{2n} \\ . & . & . \\ . & . & . \\ . & . & . \\ a_{h1} & a_{h2} & a_{hn} \end{bmatrix} \qquad (9)$$

Once again, vectors $(a_{\wedge^1}, a_{\wedge^2}, \dots\dots\dots\dots\dots, a_{\wedge^h})$ are not unique and if $A'y_t$ is stationary, for any non-zero vector (1 × h) as **b '**, scalar $b'A'y_t$ would be stationary too. Then, the vector of **Ab= π** (n × 1), can also be defined as an co-integration vector.

Suppose that there is a matrix (h × n) as A', that its rows are linearly independent, so that the vector $A'y_t$ is a stationary vector (h × 1), also assume **c'** is a arbitrary vector (1 × h) which is linearly independent of the rows of matrix A', then $c'y_t$ is a Non-Stationary scalar. Therefore, we can say that there is h Co-integration relationships between the elements of the vector y_t and vectors $(a_{\wedge^1}, a_{\wedge^2}, \dots\dots\dots\dots\dots, a_{\wedge^h})$ form a vector space and remaining n-h are Non-Stationary.

It should be noted that although Co-integration vectors are not unique, the number of them should be less than the number of elements of desirable variables vectors i.e y_t, in other words, n> h. if the number of these vectors is equal to the number of elements of vector y_t this implied that y_t would be stationary vector, ie $[y_{it}] \sim I(0)$ is and this would be a contradiction here.

Vector autoregressive process and Co-integration concepts:

A time series vector that has Co-integration, does not have properties like autoregressive process. Suppose that the time series vector y_t is Non-Stationary, but by differentiating it turns into Stationary, ie:

$$[y_{\Lambda t}] \sim I(1)\ ,\ [\Delta y_{\Lambda t}] \sim I(0)$$

also Suppose that:

$$\Phi(L)(1-L)y_{\Lambda t} = \varepsilon_{\Lambda t} \quad , \quad \Phi(L)\Delta y_{\Lambda t} = \varepsilon_{\Lambda t}$$

If vector **a** is defined as **a** co-integration vector, we have:

$$a' y_{\Lambda t} \sim I(0) \qquad (10)$$

$$\Phi(L)(1-L)a'y_{\Lambda t} = a'\varepsilon_{\Lambda t} = u_{\Lambda t} \qquad (11)$$

By definition, if $w = \Phi(L)a'y_{\Lambda t}$ In this case, according to equation (10), w_t is stationary, i.e $w_t \sim I(0)$, but on the other hand:

$$(1-L)\, w_t = u_t \Rightarrow w_t \sim I(1)$$

So here an index is created. Therefore, changes of Co-integration series cannot have autoregressive process. However, the compatibility between Co-integration system and Autoregressive process about variables cannot exist, but this compatibility (self) is true for variables. Now we want to examine if one model has Co-integration property with the same behavior as autoregressive process, what the results are.

Suppose that the autoregressive time series vector $y_{\Lambda t}$ is in order p and Non-Stationary vector:

$$y_{\Lambda t} = \propto_{\Lambda} + \Phi_1 y_{\Lambda t-1} + \Phi_2 y_{\Lambda t-2} + \cdots \dots\dots.. + \Phi_p y_{\Lambda t-p} + \varepsilon_{\Lambda t} \qquad (12)$$

Or:

$$\Phi(L)\, y_{\Lambda t} = \propto_{\Lambda} + \varepsilon_{\Lambda t} \qquad (13)$$

In the above equation:

$$\Phi\,(L) = I_n - \Phi_1 L - \Phi_2 L^2 - \cdots \dots\dots\dots. - \Phi_p L^p \qquad (14)$$

Based on Wold theorem(1954), assume that the vector Δy_t can be expressed as follows:

$$\Delta y_t = \delta + \sum_{i=0}^{\infty} \Psi i \varepsilon_{t-i}$$

$$(1 - L)y_t = \delta + \Psi(L)\varepsilon_t \qquad (15)$$

Multiplying equation (15) in φ (L), we have:

$$(1 - L)\Phi(L)y_t = \Phi(L)\delta + \Phi(L)\Psi(L)\varepsilon_t$$

Due to the constant value of δ:

$$\Phi(L)\delta = \Phi(1)\delta$$

Therefore:

$$(1 - L)\Phi(L)y_t = \Phi(1)\delta + \Phi(L)\Psi(L)\varepsilon_t \qquad (16)$$

By substituting equation (13) in equation (16), the following equation is obtained:

$$(1 - L)(\propto + \varepsilon_t) = \Phi(1)\delta + \Phi(L)\Psi(L)\varepsilon_t \qquad (17)$$

Since (1-L) α = 0, the above equation would be as follows:

$$(1 - L)\varepsilon_t = \Phi(1)\delta + \Phi(L)\Psi(L)\varepsilon_t \qquad (18)$$

if φ (1) δ=0, above relationship would indicate the identification, i.e:

$$(1 - L)I_n = \Phi(L)\Psi(L) \qquad (19)$$

If L=1, we have:

$$\Phi(1)\Psi(1) = 0 \qquad (20)$$

If we show each rows of vector Φ (1) with π ', in this case, according to equation (20) and Φ (1) σ =0, we have the following relations:

$$\pi' \Psi(1) = 0'$$

$$\pi' \sigma = 0$$

This means that π is a Co-integration vector. If the vectors $a_1, a_2, \dots \dots .., a_n$ form the Co-integration vectors space, so π can be expressed as a linear combination of $a_1, a_2, \dots \dots .., a_n$ i.e there is a vector (1 × h) like **b**, so that:

$$\pi = [a_1, a_2, \dots \dots .., a_n]b$$

$$\pi' = b'A'$$

Here A' is a matrix with dimensions (h×n) that its ith row is vector a_i'. Applying the reasoning for any of rows matrix Φ (1), there is a matrix with dimension (n × h) like B based on:

$$\Phi(1) = BA' \qquad (21)$$

Since equation (20) has pointed out that the matrix Φ (1) is a singular matrix (n × n), then the determinant of matrix Φ (Z) contains a unit root for which the determinant is zero:

$$z = 1 \Rightarrow \left\| I_n - \Phi_1 z^1 - \Phi_2 z^2 - \cdots \dots \dots \dots - \Phi_p z^p \right\| = 0$$

4. ERROR CORRECTION MODEL

As we said, when two or more variables have a similar degree of Co-integration, their long-run relationship estimates with time series data is readily available and doesn't have problem of spurious regression. However the long-run relationships between these variables will be stationary, but in this case the variables and their relationships in short term are ignored, while these estimates in short term may have significant bias or removing dynamic factors from model is problematic in small samples.

However, even if there are no such problems, it is still well desirable that short term variations are evaluated, because long-term relationships between variables and equilibrium conditions barely is visible. Moreover, short-term changes of variables can gain useful information. A very simple form of short-term two-variable model can be considered as follows.

$$y_t = \propto_0 + \lambda_0 X_t + \lambda_1 X_{t-1} + \propto_1 y_{t-1} + u_t \quad , \quad u_t \sim N(0, \sigma^2) \qquad (22)$$

In the above equation, coefficient λ_0 is short-term response of y_t relative to changes of X_t that long term relationship between x_t , y_t is as follows:

$$y_t = B_0 + B_1 x_t$$

Here B_1 is response to changes in the long-term relative to x_t. However, if the system reaches to equilibrium, in the case of the short-term model, we have $y_t = y_{t-1}$ and $x_t = x_{t-1}$ and equation 22 is transformed as follows:

$$(1 - \propto_1) y_t = \propto_0 + (\lambda_0 + \lambda_1) x_t + u_t$$

$$y_t = \frac{\propto_0}{1 - \propto_1} + \frac{(\lambda_0 + \lambda_1)}{1 - \propto_1} x_t + \frac{u_t}{1 - \propto_1}$$

If we assume: $B = \frac{\lambda_0 + \lambda_1}{1 - \propto_1}$, $B_0 = \frac{\propto_0}{1 - \propto_1}$, $v_t = \frac{1}{1 - \propto_1} u_t$

So we have:

$$y_t = B_0 + Bx_t + v_t$$

In other words, assuming the case of short-term relationship between the variables tend to move toward an equilibrium state, ($\propto_1$<1), hence long-term model will be derived from short term model. Short-term dynamic model (22) can be extended by adding longer lagged and more variables, but doing this will lead to problems. This may be combined collinearity variable between the current amounts and lagged when more variables are considered, it is likely that some or all variables are Non-Stationary and therefore we face with the problem of spurious regression. So inevitably we enter variables as first differentiating of them in the model that cause elimination long-term relationships between variables. One solution to this problem is to investigate the relationships between variables to make a model that collectively consider the long and short term factors. Such a model is called Error Correction Model.

To clarify the issue, we assume that the long-run equilibrium relationship between x_t and y_t is $y_t = Bx_t$ which shows long run path. Bx_t and y_t are the actual value.

The difference between these two variables,($y_t = Bx_t$), gives useful information., If we want to have a long term relationship that reaches a steady state, in this way, there will be a long-term tendency to eliminate the distance between y_t and Bx_t. Therefore, it can be argued that the difference (y_{t-1}–Bx_{t-1}) is the amount of the error term for long term path that such an error could be useful as an explanatory variable for the orientation of changing variable *Yt* that enters into the model.

In general, the error correction mechanism is in the form of a modification process that collects dynamic moving of variables by equilibrium relationship between them.

Assume that the variables x_t and y_t have co-integration in first degree and become stationary by first degree difference. However, if there is long-run equilibrium relationship between them as $y_t = Bx_t$, error correction model for these two variables will be as follows:

$$\Delta y_t = \propto \Delta x_t + \lambda(y_{t-1} - Bx_{t-1}) + \varepsilon_t$$

According to the assumption made in the model, both Δy_t and Δx_t are stationary and disturbance term in model is assumed as well behave. However, if Co-integration vector is *(I , -B)* the variable $(y_{t-1} - Bx_{t-1})$ of model is stationary and the model is easily estimated. Δx_t explains the short-term response of y_t, while error correction term, $(y_{t-1} - Bx_{t-1})$ is model modification to the equilibrium. Coefficient λ explains contribution of non-equilibrium states in the previous period which is corrected in current period.

Given the above, we can say Co-integration between several variables is suggested that there is a mechanism to correct errors or deviations in the long term so that they can be prevented from increasing process.

A vector error correction model may be generalized and examined in the case that there is a vector of variables and lagged are more. This model is actually a special case of autoregressive model that its variables have model parameters have Co-integration property. A vector autoregressive model can be written as:

$$y_{\Lambda t} = \xi_1 \Delta y_{\Lambda t-1} + \xi_2 \Delta y_{\Lambda t-2} + \cdots \ldots \ldots + \xi_{p-1} \Delta y_{\Lambda t-p+1} + \propto_\Lambda + \rho y_{\Lambda t-1} + \varepsilon_{\Lambda t} \qquad (23)$$

In the above equation:

$$\rho = \Phi_1 + \Phi_2 + \cdots \ldots \ldots + \Phi_p$$

$$\xi_s = -[\Phi_{s+1} + \Phi_{s+2} + \cdots \ldots \ldots + \Phi_p] \quad , s = 1,2, \ldots \ldots, P-1$$

By subtracting of $y_{\Lambda t-1}$ from both sides of equation (23) we have:

$$\Delta y_{\Lambda t} = \xi_1 \Delta y_{\Lambda t-1} + \xi_2 \Delta y_{\Lambda t-2} + \cdots \dots \dots + \xi_{p-1} \Delta y_{\Lambda t-p+1} + \alpha_\Lambda + \xi_0 y_{\Lambda t-1} + \varepsilon_{\Lambda t} \quad (24)$$

Where:

$$\xi_0 = \rho - I_n = -(I_n - \Phi_1 - \Phi_2 - \cdots \dots \dots \dots - \Phi_p) = -\Phi(1) \quad (25)$$

As previously mentioned, if h co-integration relationship between variables (vector y_t) exist, (Equation 21) is $\Phi(1) = BA'$, so by using this equation and substituting equation 25 in equation 24, we have:

$$\Delta Y_{\Lambda t} = \xi_1 \Delta y_{\Lambda t-1} + \xi_2 \Delta y_{\Lambda t-2} + \cdots \dots \dots \dots . . + \xi_{p-1} \Delta y_{\Lambda t-p+1} + \alpha - BA' \Delta y_{\Lambda t-1} + \varepsilon_{\Lambda t} \quad (26)$$

If y_t vector is a vector of time series Co-integration, so the vector $Z_t \equiv A' y_t$ is a vector (h × 1) with stationary situation and therefore equation 26 can be written as:

$$\Delta Y_{\Lambda t} = \xi_1 \Delta y_{\Lambda t-1} + \xi_2 \Delta y_{\Lambda t-2} + \cdots \dots \dots \dots . . + \xi_{p-1} \Delta y_{\Lambda t-p+1} + \alpha - BZ_{\Lambda t-1} + \varepsilon_{\Lambda t} \quad (27)$$

Above equation represents an Error Correction Mechanism about Co-integration system. If we assume:

$$E(\Delta y_{\Lambda t}) = \alpha_\Lambda - BE(Z_{\Lambda t-1}) = 0_\Lambda$$

And also assuming Z_t vector is stationary, we have: $E(Z_{\Lambda t}) = \mu_\Lambda$ and this shows that:

$$\alpha_\Lambda - BE(Z_{\Lambda t-1}) = \alpha_\Lambda - B\mu_\Lambda = 0_\Lambda \Rightarrow \alpha_\Lambda = B\mu_\Lambda$$

Therefore, equation 27 will be as follows:

$$\Delta Y_{\Lambda t} = \xi_1 \Delta y_{\Lambda t-1} + \xi_2 \Delta y_{\Lambda t-2} + \cdots \dots \dots \dots . . + \xi_{p-1} \Delta y_{\Lambda t-p+1} - B(Z_{\Lambda t-1}\mu_\Lambda) + \varepsilon_{\Lambda t} \quad (28)$$

In the above equation, $(Z_{\Lambda t-1} - \mu_\Lambda)$ is error term and matrix elements of B are vector of modification coefficients.

5. CO-INTEGRATION TESTS

Although there are similarities between unit roots and the Co-integration tests, these tests are not identical. Actually the unit root tests are performed on single variable time series and in other words, it examines that one variable is stationary or not, while in Co-integration tests, between groups of variables which unconditionally have unit root, are discussed. Several tests of Co-integration is addressed that there are similarities and differences between them. These tests may include tests, Johansen, Engel - Granger, Stock and Watson noted that given the multivariate model are described in the following:

$$Y_{\Lambda t} = A_1 y_{\Lambda t-1} + A_2 y_{t-2} + \cdots \dots \dots \dots . . + A_p y_{\Lambda t-p} + \varepsilon_{\Lambda t} \quad (29)$$

The Co-integration tests are addressed in both cases. The first case is a situation in which Co-integration vectors and their elements are defined and (Co-integration factor) are marked out and based on, the Co-integration variables vector time series is tested. For example, if the vector of $(y_{\Lambda t})$ has Co-integration and become stationary by first differentiating $(y_{\Lambda t} \sim I(1))$. and by this assumption that Co-integration vector is identified (vector a). Therefore:

$$a'_\Lambda y_{\Lambda t} \sim I(0)$$

It must be shown that the $y_{\Lambda t} \sim I(1)$ or not and if so, whether it would be $a'_\Lambda y_{\Lambda t} \sim I(0)$ or not.

The latter is a condition in which Co-integration vector is undefined. Where the Co-integration vector is identified, it is easily possible to test. But on many issues (especially in economic theories), in which case Co-integration vector is not which primarily to be estimated and then the Co-integration test variables is performed.

In this part, Co-integration test methods of the two Engel - Granger and the Johansen will investigate.

6. CO-INTEGRATION TESTS ENGEL - GRANGER

Engel – Granger test is the simplest and most common method for co-integration. The Co-integration test method has two stages. the first stage is to test whether any variables are stationary or not. It is done by unit root tests on each variable. Secondly, as soon as it is identified that variables are Non-Stationary and degree of Co-integration, it is examined whether trends in these variables are associated with each other or not. In other words, the long-run relationship between these variables and the vector Co-integration is analyzed. As earlier, we assume that the Co-integration vector is explicitly specified. In this case, the Co-integration test using this method would be very easy, but in this case it is often not clear Co-integration of the vector coefficients and it usually identifies by OLS estimation.

Engel – Granger Method generally is used for co-integration test between the two variables. To clarify the use of this method, we assume that the relationship between y_t and x_t. Uniform Co-integration is established as follows:

$$y_t = \beta x_t + u_t \qquad (29\text{-}1)$$

To test the Co-integration of the above equation is the same as the one proposed by default, the first step of the test is whether the time series variables are stationary or not and if y_t and x_t are Non-Stationary, the degree of Co-integration of them will be examined. Secondly being stationary difference between time series variables that is same disturbance term (u_t) in above equation, $u_t = y_t - \beta x_t$, is being tested:In other words, if the time series u_t is stationary, variables y_t and x_t would not be converge in the long-term movement, although in the short term there is distance between themselves from each other by some reasons. So It is sufficient that the unit root test in case of disturbance term, (u_t) is done If in time series unit root test, the null hypothesis (of Non-Stationary u_t) is rejected, and we conclude that the two variables (y_t and x_t) have Co-integration.If the Co-integration vector of considered relation (1,-β) is known, it is easily possible to calculate u_t. But this Co-integration vector is not clear; we need to estimate disturbance term u_t by estimating equation 29-1 and then we do unit root tests for time series of these estimates.Engel – Granger have introduced Durbin –Watson statistic for uniform Co-integration regression (CRDW) test. They have also introduced Adjusted Dickey - Fuller (ADF) test for unit root test as a criterion of Co-integration. About CRDW test, if value of this statistic is small, the assumption of uniform Co-integration is rejected, but if the value is close to 2, this assumption can be accepted.

7. JOHANSEN TEST

One of the Co-integration test methods in case that the co-integration vector is unknown is the Johansen test. In this way, we seek to determine the number of co-integration vectors; this test method is estimated by maximum likelihood as described by the vector autoregressive model which is based on an error correction model dealing directly. It is assumed that y_t vector is a uniform co-integration with degree one so that Δy_t is a I(0).

As previously, if the number of co-integration vectors is equal to the number of variables, (h = n) these variables will be stationary and there is no Co-integration. But in the case that number of Co-integration vectors is less than the number of variables (h <n), The variables are Non-Stationary and can have the uniform Co-integration properties. In Johansen's method, we equal number of vectors with less than the number of variables and the relationship will be estimated based on the maximum likelihood estimation and once the number of vectors equal to the number of variables we consider the Co-integration and estimate. These estimates are then calculated and tested. This test is called the likelihood ratio test. The logarithm of the ratio that is multiplied in -2 has the distribution of x^2.

This test method is based on a vector autoregressive model and is directly dealing with error correction model:

$$Y_{\Lambda t} = A_1 y_{\Lambda t-1} + A_2 y_{\Lambda t-2} + \cdots \dots \dots \dots . + A_k y_{\Lambda t-k} + u_{\Lambda t} \qquad (30)$$

$$u_t \sim IN(0_\Lambda, \Sigma)$$

The above model can be converted into a vector error correction model (VECM).

$$\Delta Y_{\Lambda t} = \Pi_1 \Delta y_{\Lambda t-1} + \Pi_2 \Delta y_{\Lambda t-2} + \cdots \dots \dots \dots . + \Pi_{k-1} \Delta y_{\Lambda t-k+1} + \Pi y_{\Lambda t-1} + u_{\Lambda t} \qquad (31)$$

Where:

$$\Pi_i = (I - A_1 - \cdots \dots \dots . - A_{k-1}) \ , i = 1,2, \dots \dots \dots ., k-1$$

$$\Pi = (I - A_1 - \cdots \dots \dots . - A_k)$$

Equation (31) is first difference of Autoregressive model in which $\Pi y_{\Lambda t-1}$ is added to. If there is a long-term relationship between the variables of vector y_t, such term indicate the error correction amount relative to the long term path in previous period.

Johansen method also called trace matrix, is concentrated on the Π matrix. The objective is to determine whether long-run relationship between the variables and the matrix on the information are presented or not. The main focus is on the rank of the matrix. If trace matrix is (h × n), the vector Co-integration assumption entails that the rank of this matrix is smaller than its dimension (h <n). Johansen procedure is applied in practice, a model that includes the intercept and deterministic factors are:

$$\Delta Y_{\Lambda t} = \Pi_1 \Delta y_{\Lambda t-1} + \cdots \dots \dots + \Pi_{k-1} \Delta y_{\Lambda t-k+1} + \Pi y_{\Lambda t-1} + \Phi D_t + u_{\Lambda t} \qquad (32)$$

Or in a more simplified form:

$$\Delta Y_{\Lambda t} = \Sigma \Pi_i \Delta y_{\Lambda t-i} + \Pi y_{\Lambda t-1} + \Phi D_t + u_{\Lambda t} \qquad (33)$$

In above model, D is deterministic factors matrix. What is important here is to understand the relationship between Co-integration concept and matrix rank. For this purpose, we rearrange equation(33) in terms of $\Pi y_{\Lambda t-1}$:

$$\Delta Y_{\Lambda t-1} = \Delta y_{\Lambda t} - \Sigma \Pi_i \Delta y_{\Lambda t-i} - \Phi D_t + u_{\Lambda t} \qquad (34)$$

If we assume that the vector y_t contains Non-Stationary parameters or $y_{\Lambda t} \sim I(1)$, in this case, by assuming that $u_{\Lambda t}$ is well behave, the right side of equation 32 contains only stationary variables then the left-hand side of this equation will be I(0) if $\Pi y_{\Lambda t-1}$ provide a set of combinations of variables with uniform Co-integration in other words, each of the rows of the matrix should indicate co-integration vectors for y_t variables.

In this connection, three cases can occur on the rank of the matrix that follows, each with its own conclusion is:

First case: the rank of the trace matrix is equal to zero. All the elements of this matrix is equal to zero. In this context, there is not long-term relationship between the variables and the error correction model is zero or in other words, $\Delta y_{\Lambda t}$ cannot change under $y_{\Lambda t}$ difference in long term.

Second case is a full rank trace matrix. i.e Co-integration vectors is equal to the number of variables (h = n), or in other words, the trace matrix is nonsingular matrix. In extreme conditions, there can be a Co-integration vector and all linear combinations in vector y_t are stationary and this means that all variables are stationary. Since we previously assumed that variables are Non-stationary it has made contradiction.

Third case: the rank of the trace matrix is non-zero and is not perfect, thus Co-integration vectors is not complete yet exist, but their number is less than the number of variables (0 <h <n). In this context, long-term relationship between Non-Stationary variables is acceptable.

Given to the above cases, the third case would be desired. If we realize this, the trace matrix will be convertible as follows: $\Pi = \alpha\beta'$

Where α and β are matrices (n × h) are the matrix elements of α shows the modification coefficients in the error correction term and elements of matrix β are long term coefficient.

However, in order to determine which of the above three cases can occur, it should get rank of trace matrix and for this purpose it is necessary to calculate the eigenvalues (characteristic roots) for this matrix. If the eigenvalues are shown with λ, we have:

$$\Pi y_{\Lambda t} = \lambda y_{\Lambda t}$$

$$(\Pi - \lambda I) y_{\Lambda t} = 0_{\Lambda}$$

Given the assumption of a non-zero vector y_t, matrix (Π- λI) should be singular, i,e its determinant is equal to zero. if this matrix is nonsingular, then matrix $(\Pi - \lambda I) y_{\Lambda t}$ is a vector (n × 1) that indicate the n independent linear combination of variables $y_{\Lambda t}$ which will be zero, and this means that all elements of vector $y_{\Lambda t}$ must be zero so that are not compatible with the basic premise.

However, considering that the $(\Pi - \lambda I)$ *is* singular matrix and its determinant is zero, the n roots can be found for λ. For every λ ≠ 0 , must be linearly independent one row or column in the matrix Π And as we know the number of rows or columns of the matrix shows the rank of it. Thus, the number of non-zero roots will be equal to rank of trace matrix. If the number of non-zero roots of ($\lambda_i \neq 0$) is equal to h, the rank of the matrix will be h. and n-h is the other roots that are correspond with non-independent rows and columns of matrix Π, are equal to zero. Now, given the above, incomplete rank tests (the third case), can be studied. By solving the matrix equation |Π-λI| = o, n values obtained for λ. Then for the test, singular matrix has h maximum independent row or column. In other words, h-vectors of co-integration in the model, is available, it should test the null hypothesis that (n-h) other root of λ is zero, it means:

$$H_0: \hat{\lambda}_i = \circ \quad , \qquad i = (h+1), (h+2), \ldots\ldots, n$$

To analyze this case, we assume, n estimated roots for λ is arranged as follows:

$$\hat{\lambda}_1 > \hat{\lambda}_2 > \cdots > \hat{\lambda}_n$$

If the vector of y_t has not any compound uniform co-integration, All of the estimated λ must be zero. In such a case *Ln (1-* $\hat{\lambda}_i$) equal to zero. Also if we have the h-vector of uniform co-integration, It will be:

$0 < \lambda_i^{\wedge} < 1 \quad , \quad i = 1,2, \dots \dots \dots , h$

For These values are estimated $\lambda_i^{\wedge}$, $\ln(1 - \lambda_i^{\wedge})$ is negative. for other $\lambda_i^{\wedge}$ that equal to zero, It will be:

$Ln(1\text{-}\lambda_{h+1}^{\wedge}) = Ln(1\text{-}\lambda_{h+2}^{\wedge}) = \cdots \dots \dots = Ln(1 - \lambda_n^{\wedge}) = 0$

In fact, if we want to get the rank of singular matrix, we examine that how many logarithms of above, are equal to zero. For this order, The two statistic tests are presented as follows:

Trace test : $\lambda_{tr}(h) = -T \sum_{i=h+1}^{n} Ln(1 - \lambda_i^{\wedge})$

Maximum eigenvalue test : $\lambda_{max}(h, h+1) = -T Ln(1 - \lambda_{h+1}^{\wedge})$

In the above, T denotes the number of time periods in relation to the data. If $\lambda_i = 0$ then $\lambda_{tr}^{(h)}$ equal to zero and the value λ_i is greater than zero, the $\lambda_{tr}^{(h)}$ will increase. Comparison of estimated $\lambda_{tr}^{(h)}$ with critical values given in the table for this purpose was developed by Johansen and juselius (1996), the test is complete and the estimated value for $\lambda_{tr}^{(h)}$ is smaller than its corresponding critical value in the table, the null hypothesis of equal rank with trace matrix h cannot be rejected, and We conclude that the rank of trace matrix is equal to h. If the estimated $\lambda_{tr}^{(h)}$ is greater than the critical value, the null hypothesis for h+1 will be done and we assumes zero. If the subsequent assumption of the null hypothesis of equal rank matrix with h +1 is not rejected, it can be concluded that the rank of this matrix is equal to h +1. But if you reject the null hypothesis at this stage, we do the test again for the null hypothesis h +2. Accordingly, this action will be followed to determine the trace matrix rank. Similarly, much of the second test $\lambda_i^{\wedge}$ much closer to zero, λ_{max} will be lower and when the amount is larger than the corresponding critical value λ_{max} , Null hypothesis of equal rank with matrix h is rejected. The second test includes alternative hypothesis and rejecting the null hypothesis of equal rank h, h +1 is accepted assumption of equal rank.

Generally, the Johansen procedure has several advantages. first; the Co-integration of these relations can be distinguished. In this method there is a possibility to test hypotheses concerning the number of equilibrium relations. The second advantage of this method is that the statistics presented in this method have exactly known distribution which has function only one parameter. Due to these advantages, the vector models Co-integration diagnostic tests will be follow up.

8. CONCLUSION

As stipulated in this study, in order to escape from being trapped in the false regression, the uniform co-integration discussion shall be considered. This expression means that the current time series variables well pursue or accompany each other in a econometric model during the time, so that it is a certain part of the disorder resulted from this model within the stationary time. Stationary time series means fix average and variance during the time and that the covariance among the interruptions of a variable merely depends on the term of interruption.

Usually the stationary tests of a time series include Dickey Fuller and extended Dickey Fuller tests. Along with the discussions on co-integration, the discussion of the long term and short term relationships is addressed for which the vector error correction models are mentioned, while notwithstanding the collective and long term relationship among the model variables, it determines the short term error correction factor, while such factor indicates the manner of error correction process from short term to long term trend.

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