RELIABILITY AND AVAILABILITY ANALYSIS OF A STANDBY REPAIRABLE SYSTEM WITH DEGRADATION FACILITY

M.A. El-Damcese & M.S. Shama
Department of Mathematics, Faculty of Science, Tanta University, Tanta, Egypt

ABSTRACT
This paper investigates reliability and availability of a repairable system with degradation facility. Failure times and repair times of failed units are assumed to be exponentially distributed. There are two types of repair. The first is due to failed state, the second is due to degraded state. The expressions of availability and reliability characteristics such as the system reliability and the mean time to failure are derived. We used several cases to analyze graphically the effect of various system parameters on the availability system, reliability system and mean time to failure. We also investigated the sensitivity analysis for the system reliability with changes in a specific value of the system parameters.

Keywords: *Availability, reliability, mean time to system failure, sensitivity analysis, degradation facility.*

1. INTRODUCTION

Standby is a technique that has been widely applied to improve system reliability and availability in system design. In most cases, components in standby system are assumed to have two states up and down. In this paper, we assume that, components in standby system have three states up, degraded and down. The transition from up state to degraded state represents a partial failure and the transition from up state to down state or from degraded state to down state represents a complete failure. Several researchers [1,2,3] define different types of failures .there are three basic types of failure ,i.e. wear out failure, random failure, and infant mortalities . We also cannot neglect the effect of various failures such as major failure, minor failure, catastrophic failure, and critical human failure, and so on.
In the past decades, many articles concerning the reliability and availability of standby systems have been published. Among them, Galikowsky et al.[4] analyzed the series systems with cold standby components .Wang and Sivazlian [5] examined the reliability characteristics of a multiple-server $(M+W)$ unit system with exponential failure and exponential repair time distributions.ke et al. [6] studied the reliability measures of a repairable system with warm standby switching failures and reboot delay. Ke et al. [8] analyzed the machine repair problem with unreliable multi-repairmen. Wang et al. [9] considered the single server machine repair problem with working vacation. Wang et al. [7] introduced the warm-standby machine repair problem with Balking, reneging and standby switching failures. Hsu et al. [10] examined an availability system with reboot delay, standby switching failures and an unreliable repair facility, which consists of two active components and one warm standby. Jain et al. [11] studied the degraded model with warm standbys and two repairmen. Wang et al. [12] compared four different system configurations with warm standby components and standby switching failures.
There are three objectives of this paper. The first objective is to develop the explicit expressions for availability function, reliability functionand mean time to failure using Laplace transform techniques. The second objective is to perform a parametric investigation which presents numerical results to analyze the effects of the various system parameters on the system reliability and on the mean time to failure. The third objective is to perform a sensitivity analysis in the system reliability and themean time to failure along with changes in specific values of the system parameters.

1.1 Problem description

We consider a system which consists of M identical units operating simultaneously in parallel, W cold standby units, R_1 repairmen in the first service line who repair failed units and R_2 repairmen in the second service line who repair degraded units .The assumptions of the model are described as follows. We suppose that the failures of the operating units from up state to down, failures from up state to degraded state and failures from degraded state to down state occur independently of the states of other units and follow exponential distributions with $\lambda_1, \lambda_2, \lambda_3 (where \lambda_1 < \lambda_2 < \lambda_3)$,respectively. An operating unit replaced by a cold standby if and only if it comes to down state and one cold standby unit is available, and then it is immediately sent to the first service line where it is repaired with time-to-repair which is exponentially distributed with parameter μ_1 .When an operating unit degraded it is repaired with time-to-repair which is exponentially distributed with parameter μ_2 during it working.

Moreover, we assume that the secession of failure times and repair times are independently distributed random variable. Let us assume that failed units arriving at the repairmen form a single waiting line and are repaired in the order of their breakdowns; i.e. according to the first-come, first-served discipline. Suppose that the repairmen in the first service line can repair only one failed unit at a time and the repairmen in the second service line can repair only one degraded unit at a time in addition to the repair is independent of the failure of the units. Once a unit is repaired, it is as good as new. System availability is investigated according to the assumptions that system is safe when all M operating units are working.

Notations

M : the number of operating units in the initial state.

W : the number of cold standby units in the initial state.

R_1 : the number of repairman in the first service line who repair failed units.

R_2 : the number of repairman in the second service line who repair degraded units.

λ_1: the failure rate of the unit from up state to failed state.

λ_2 : the failure rate of the unit from up state to degraded state the (partial failure rate).

λ_3: the failure rate of the unit from degraded state to failed state .

μ_1 : the repair rate of failed unit.

μ_2 : the repair rate of degraded unit.

$\mu_{1,n}$: mean repair rate when there are n failed units in the system

$\mu_{2,n}$: mean repair rate when there are n degraded units in the system.

$P_{i,j}(t)$: probability that there are i degraded units and j failed units in the system at time t where $i = 0,1,2,...,M$, $j = 0,1,...,W.$

$P_{i,W+1}(t)$:failed states of the system that represent probability that there are i degraded units and all standby units failed in addition to one unit of operating unit failed in the system at time t where $i = 0,1,2,...,M-1$

s : Laplace transform variable.

$P^*_{i,j}(t)$: Laplace transform of $P_{i,j}(t)$.

Y : time to failure of the system.

$A_Y(t)$: availability function of the system.

$R_Y(t)$: reliability function of the system.

$MTTF$: mean time to failure.

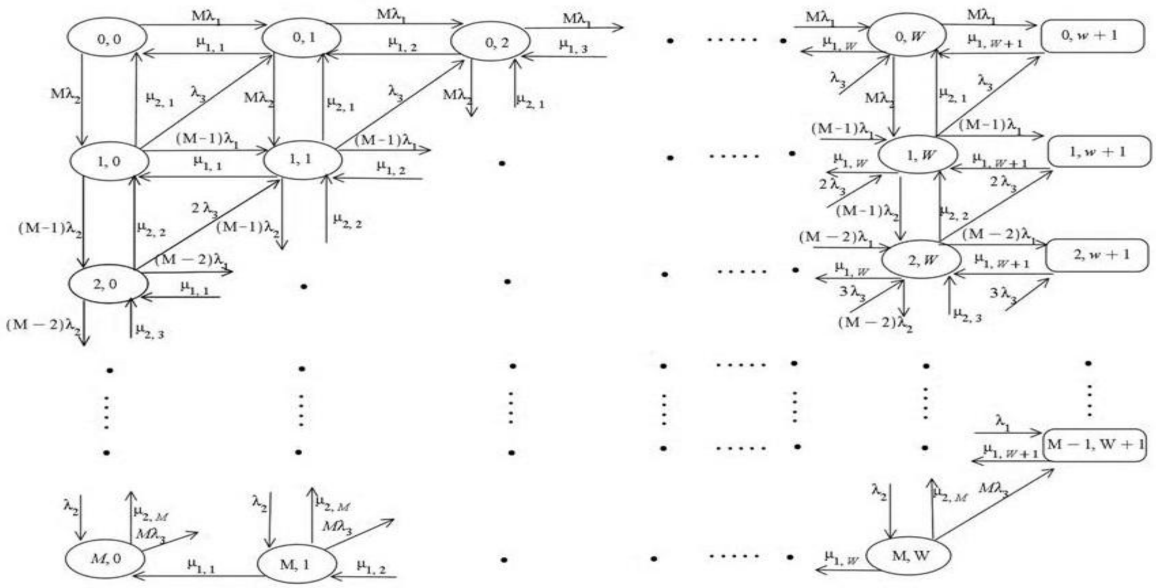

Figure4-1.State-transition-rate diagram

1.2 Availability and Reliability Analysis of the System

At time $t = 0$ the system start operation with no failed units. The availability function under the exponential failure time and exponential repair time distributions can be developed through the birth –death process. Let Y be the random variable representing the time to failure of the system.

The mean repair rate $\mu_{1,n}$ is given by:

$$\mu_{1,n} = \begin{cases} n\mu_1 & , \quad if\ 1 \le \mathrm{n} \le \min(\mathrm{R}_1, \mathrm{W}) \\ R_1\mu_1 & , \quad if\ \mathrm{R}_1 \le \mathrm{n} \le \mathrm{W} \\ 0 & \quad otherwise \end{cases}$$

The mean repair rate $\mu_{2,n}$ is given by:

$$\mu_{2,n} = \begin{cases} n\mu_2 & , \quad if\ 1 \le \mathrm{n} \le \min(\mathrm{R}_2, \mathrm{W}) \\ R_2\mu_2 & , \quad if\ \mathrm{R}_2 \le \mathrm{n} \le \mathrm{W} \\ 0 & \quad otherwise \end{cases}$$

The Laplace transforms of $P_{i,j}(t)$ are defined by:

$$P^*_{i,j}(s) = \int_0^\infty e^{-st} P_{i,j}(t)\,dt,\ i = 0,1,2,...,M \ ,\ j = 0,1,...,W,W+1 \text{ at } j = W+1 \text{ we find } i \ne M$$

State –transition-rate diagram can be obtained in Fig.4-1, and it leads to the following Laplace transform expressions for $P_{i,j}{}^*(t)$:

$$\left(s + M\lambda_1 + M\lambda_2\right)P^*_{0,0}(s) - \mu_{1,1}P^*_{0,1}(s) - \mu_{2,1}P^*_{1,0}(s) = 1 \qquad (4.1a)$$

$$\left(s + M\lambda_1 + M\lambda_2 + \mu_{1,n}\right)P^*_{0,n}(s) - \mu_{2,1}P^*_{1,n}(s) - \lambda_3 P^*_{1,n-1}(s)$$
$$- \mu_{1,n+1}P^*_{0,n+1}(s) - M\lambda_1 P^*_{0,n-1}(s) = 0, \quad 1 \le n \le W \qquad (4.1b)$$

$$\left(s + \mu_{1,W+1}\right)P^*_{0,W+1}(s) - \lambda_3 P^*_{1,W}(s) - M\lambda_1 P^*_{0,W}(s) = 0 \qquad (4.1c)$$

$$\left(s+(M-u)\lambda_1+(M-u)\lambda_2+u\lambda_3+\mu_{2,n}\right)P^*_{u,0}(s)-\mu_{2,u+1}P^*_{u+1,0}(s)$$
$$-(M-u+1)\lambda_2P^*_{u-1,0}(s)-\mu_{1,1}P^*_{u,1}(s)=0,\quad 1\le u\le M-1 \quad (4.1d)$$
$$\left(s+M\lambda_3+\mu_{2,M}\right)P^*_{M,0}(s)-\lambda_2P^*_{M-1,0}(s)-\mu_{1,1}P^*_{M,1}(s)=0 \quad (4.1e)$$
$$\left(s+\mu_{1,W+1}\right)P^*_{u,W+1}(s)-(M-u)\lambda_1P^*_{u,W}(s)-(u+1)\lambda_3P^*_{u+1,W}(s)=0,$$
$$1\le u\le M-1 \quad (4.1f)$$
$$\left(s+\mu_{2,M}+\mu_{1,W}+M\lambda_3\right)P^*_{M,W}(s)-\lambda_2P^*_{M-1,W}(s)=0 \quad (4.1g)$$
$$\left(s+\mu_{2,M}+\mu_{1,u}+M\lambda_3\right)P^*_{M,u}(s)-\lambda_2P^*_{M-1,u}(s)-\mu_{1,u+1}P^*_{M,u+1}(s)=0,$$
$$1\le u\le W-1 \quad (4.1h)$$
$$\left(s+(W-u)\lambda_2+\mu_{2,u}+(M-u)\lambda_1+\mu_{1,n}+u\lambda_3\right)P^*_{u,n}(s)$$
$$-(M-u)\lambda_1P^*_{u,n-1}(s)-\mu_{2,u+1}P^*_{u+1,n}(s)-\mu_{1,n+1}P^*_{u,n+1}(s)$$
$$-(M-u+1)\lambda_2P^*_{u-1,n}(s)-(u+1)\lambda_3P^*_{u+1,n-1}(s)=0,$$
$$1\le u\le M-1,\quad 1\le n\le W-1 \quad (4.1i)$$
$$\left(s+(W-u)\lambda_2+\mu_{2,u}+(M-u)\lambda_1+\mu_{1,W}+u\lambda_3\right)P^*_{u,W}(s)$$
$$-\mu_{1,W+1}P^*_{u,n+1}(s)-\mu_{2,u+1}P^*_{u+1,W}(s)-(M-u)\lambda_1P^*_{u,W-1}(s)$$
$$-(M-u+1)\lambda_2P^*_{u-1,W}(s)-(u+1)\lambda_3P^*_{u+1,W-1}(s)=0,$$
$$1\le u\le M-1,\quad 1\le n\le W-1 \quad (4.1j)$$

By solving equations $(4.1a-4.1j)$ and taking inverse Laplace transforms (using maple program).We obtain the availability function as follows:

$$A_Y(t)=L^{-1}\left(\sum_{i+j=0}^{M+W}P^*_{i,j}(s)\right)=\left(\sum_{i+j=0}^{M+W}P^*_{i,j}(t)\right) \quad (4.2)$$

Where $i=0,1,2,...,M$ & $j=0,1,...,W$.

To obtain the reliability function of model $(4.1a-4.1j)$, we assume that all failed states of the system are absorbing states and set all transition rates from these states equal to zero. We also consider that $P_{i,j}(t)\to\hat{P}_{i,j}(t)$ the reliability function can be obtained by taking the inverse of Laplace transform as follows.

$$R_Y(t)=L^{-1}\left(\sum_{i+j=0}^{M+W}\hat{P}^*_{i,j}(s)\right)=\left(\sum_{i+j=0}^{M+W}\hat{P}^*_{i,j}(t)\right) \quad (4.3)$$

The mean time to failure $MTTF$ can be obtained from the following relation.

$$MTTF=\lim_{s\to 0}R^*_Y(s)=\lim_{s\to 0}\left\{\sum_{i+j=0}^{M+W}\hat{P}^*_{i,j}(s)\right\}=\left\{\sum_{i+j=0}^{M+W}\hat{P}^*_{i,j}(0)\right\} \quad (4.4)$$

We perform a sensitivity analysis for changes in the reliability of the system $R_Y(t)$ from changes in system parameters $\lambda_1,\lambda_2,\lambda_3,\mu_1$ and μ_2 .by differentiating equation (12) with respect to λ_1 we obtain

$$\frac{\partial R_Y(t)}{\partial\lambda_1}=\frac{\partial}{\partial\lambda_1}\left\{\sum_{i+j=0}^{M+W}\hat{P}_{i,j}(t)\right\}=\left\{\sum_{i+j=0}^{M+W}\frac{\partial}{\partial\lambda_1}\hat{P}_{i,j}(t)\right\} \quad (4.5)$$

We use the same procedure to get $\frac{\partial R_Y(t)}{\partial\lambda_2},\frac{\partial R_Y(t)}{\partial\lambda_3},\frac{\partial R_Y(t)}{\partial\mu_1},\frac{\partial R_Y(t)}{\partial\mu_2}$

2. NUMERICAL RESULTS

In this section, we use MAPLE computer program to provide the numerical results of the effects of various parameters on system reliability, system availability and $MTTF$. We choose $\lambda_1 = 0.001,\ \lambda_2 = 0.003, \lambda_3 = 0.005$ and fix $\mu_1 = 0.01, \mu_2 = 0.03$.The following cases are analyzed graphically to study the effect of various parameters on system reliability.

Case 1:Fix $M = 2, R_1 = 1, R_2 = 2$, and choose $W = 1,2,3.$

Case 2: Fix $M = 3, W = 3, R_2 = 2$, and choose $R_1 = 1,2,3.$

Case 3: Fix $M = 3, W = 3, R_1 = 3$, and choose $R_2 = 1,2,3.$

It can be observed from Fig.4-2, Fig.4-3 that the system reliability increases as W increases or R_1 increase. It is also noticed from the Fig.4-4 that increasing of R_2 rarely effect on system reliability $R_Y(t)$ when number of repairman more than one.

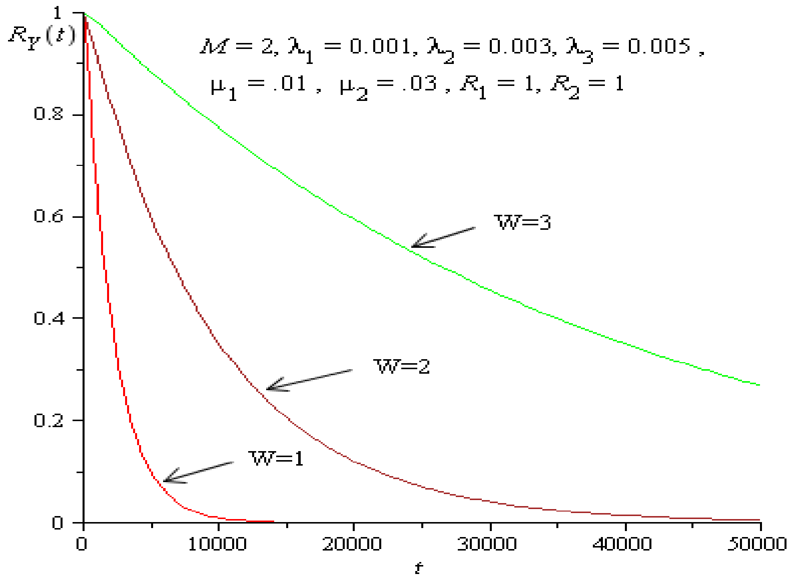

Figure4-2.System reliability for different numbers of standby units.

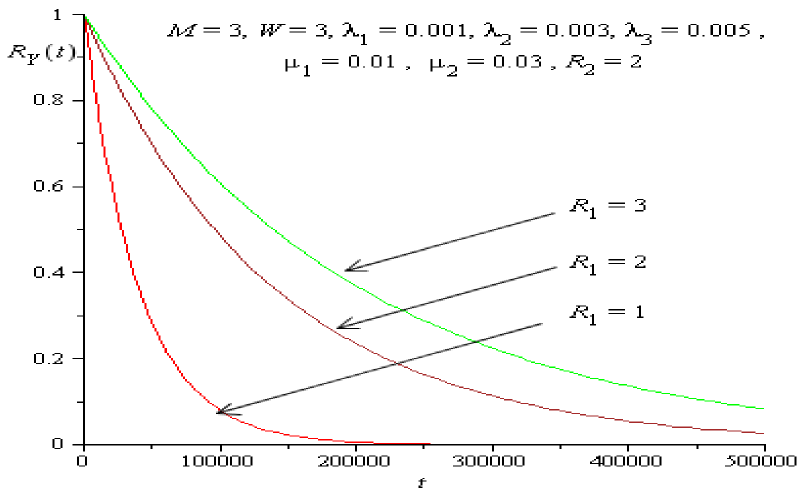

Figure4-3.System reliability for different numbers of repairmen in first service line.

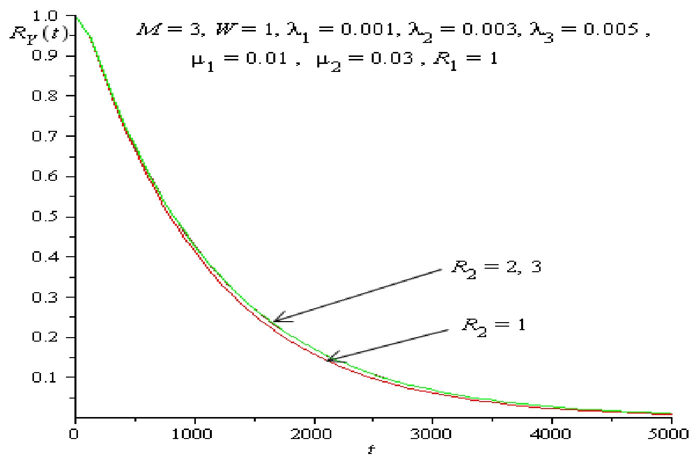

Figure4-4.System reliability for different numbers of repairmen in second service line.

Next we analyze the impact of partial failure λ_2 on the availability and $MTTF$ as presented in Fig.4-5.We find that $A_Y(t)$ decrease as partial failure λ_2 increase. From Fig.4-6, we observe that the $MTTF$ decrease as partial failure λ_2 increase.

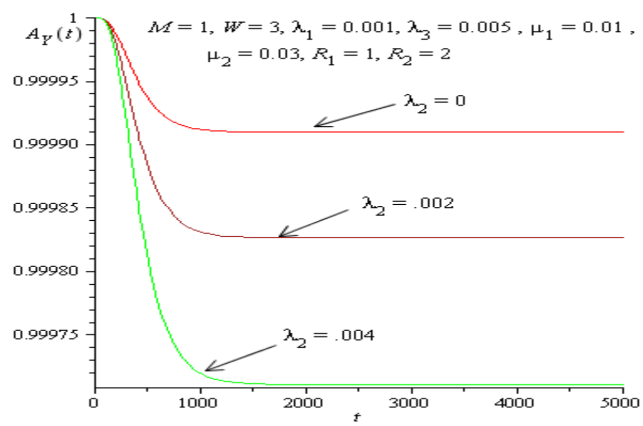

Figure4-5.System availability for different partial failure rates.

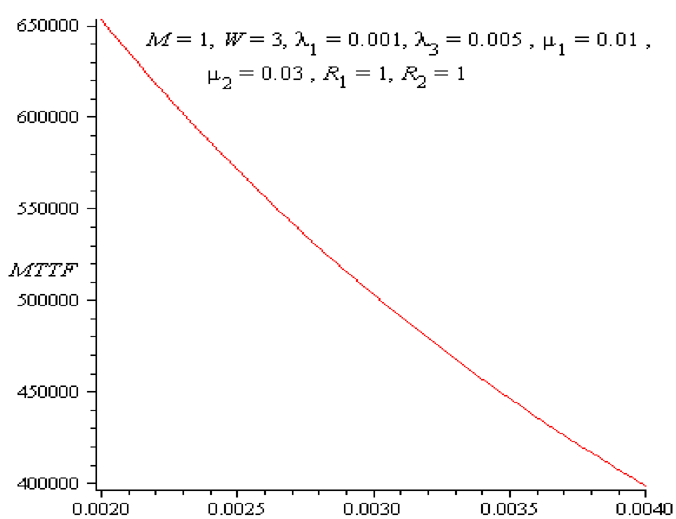

Figure4-6. *MTTF* versus partial failure λ_2

Finally we perform sensitivity analysis for system reliability $R_Y(t)$ with respect to system parameters $\lambda_1, \lambda_2, \lambda_3, \mu_1$ and μ_2. In Fig.4-7 we can easily observe that the biggest impact almost happened at the same time for all system parameters. Moreover, we find λ_1 is the most prominent parameter, μ_1 and λ_2 are the second and the third in magnitude, and the effect of λ_3 is weak. The sensitivities of μ_2 on the $R_Y(t)$ are almost equal to zero.

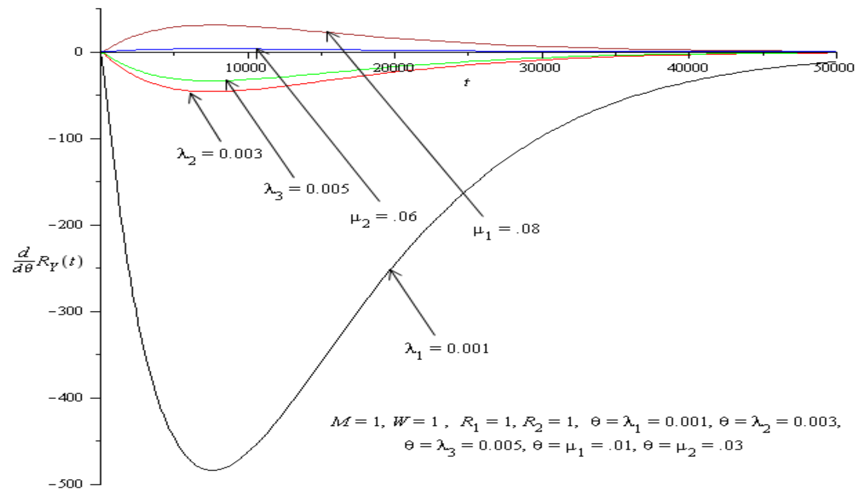

Figure4-7.Sensitivity of system reliability with respect to system parameters.

3. CONCLUSIONS

In this paper, a mathematical model was constructed for a repairable system subject to degradation. Availability, reliability and mean time to system failure in addition to sensitivity analysis for the system reliability were obtained and the results were shown graphically by the aid of MAPLE program. Results indicate that the reliability of the system reliability increases by increasing of cold standby units, R_1 and it decreases by increasing of partial failure rates.

4. REFERENCES

[1] Gupta, R., Tyagi, P.K. and Kishan, R. A two-unit system with correlated failures and repairs, and random appearance and disappearance of repairman. International Journal of Systems Sciences, 27(6), 561-566, (1996).

[2] Lam, Y. Calculating the rate of occurrence of failures for continuous time Markov chains with application to a two-component parallel system. Journal of Operational Research Society, 46, 528-536, (1995).

[3] Sim, S.H. and Endrehyi, J. A failure repair model with minimal and major maintenance. Journal of IEEE Trans. on Reliability, 55(1), 134-140, (2006).

[4] Galikowsky, C., Sivazlian, B. D. and Chaovalitwongse, P. Optimal redundancies for reliability and availability of series systems. Microelectronics and Reliability, 36, 1537-1546 ,(1996).

[5] Wang, K. H. and Sivazlian, B. D.Reliability of a system with warm standbys and repairmen, Microelectronics and Reliability, 29, 849-860, (1989).

[6] ke JB, Chen JW,Wang KH. Reliability Measures of a Repairable System with Standby Switching Failures and Reboot Delay. Quality Technology &Quantitative Management Vol. 8, No. 1, pp. 15-26, (2011).

[7] Wang KH, Ke JB,Ke JC.Profit analysis of the M/M/R machine repair problem with balking ,reneging, and standby switching failures .Computers and Operations Research 34:835–47,(2007).

[8] Ke JC, Hsu YL, Liu TH, Zhang ZG. Computational analysis of machine repair problem with unreliable multi-repairmen. Computers &Operations Research 40:848–855,(2013).

[9] Wang KH, Chen WL,Yang DY. Optimal management of the machine repair problem with working vacation :New ton's method . Computational and Applied Mathematics 233:449–58, (2009).

[10] Hsu YL, Ke JC and Liu TH. Standby system with general repair, reboot delay, switching failure and unreliable repair facility A statistical standpoint. Mathematics and Computers in Simulation 81 (2011) 2400–2413.

[11] Jain M, Rakhee and Singh M . Bilevel control of degraded machining system with warm standbys, setup and vacation. Applied Mathematical Modeling 28 (2004) 1015–1026.

[12] Wang KH, Dong WL and Ke JB. Comparison of reliability and the availability between four systems with warm standby components and standby switching failures. Applied Mathematics and Computation 183 (2006) 1310–1322.