RESULTS ON DELAY-DEPENDENT STABILITY CRITERIA FOR NEUTRAL SYSTEMS WITH MIXED TIME-VARYING DELAYS AND NONLINEAR PERTURBATIONS

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ABSTRACT

In this paper, the problem of delay-dependent stability criteria for neutral systems with mixed time-varying delays and nonlinear perturbations is considered. Based on the new Lyapunov-Krasovskii functional, Leibniz-Newton formula, decomposition technique of coefficient matrix, utilization of zero equations, model transformation and linear matrix inequality, new delay-dependent stability criteria are established in terms of linear matrix inequalities (LMIs). Numerical examples show that the proposed criteria improve the existing results significantly with much less computational effort.

Keywords: *Neutral system, Lyapunov-Krasovskii functional, Delay-dependent stability criteria, Leibniz-Newton formula, Nonlinear perturbations.*

1. INTRODUCTION

1cm During the past several years, the problem of stability for neutral differential systems, which have delays in both its state and the derivatives of its states, has been widely investigated by many researchers, especially in the last decade. Because neutral systems with delays can be found in such places as population ecology, distributed networks containing lossless transmission lines, heat exchangers, robots in contact with rigid environments, etc (e.g., [12]-[14],[22]). It is well known that time delay and nonlinearities, as time delays, may cause instability and poor performance of practical systems such as engineering, biology, economics, and so on [5]. Stability criteria for delay systems can be classified into two categories: delay-independent and delay-dependent criteria. Delay-independent criteria do not employ any information on the size of the delay; while delay-dependent criteria make use of such information at different levels. Delay-dependent stability conditions are generally less conservative than delay-independent one especially when the delay is small [32].

Recently, many researchers have been studied the problem of stability for neutral time-delay systems with nonlinear perturbations such as [36] considers the delay-dependent robust stability of neutral systems with mixed delays and nonlinear perturbations. In [31], novel delay-dependent asymptotical stability of neutral systems with nonlinear perturbations is studied. In [29], he proposed on exponential stability of neutral delay differential system with nonlinear uncertainties by using Lyapunov method. The problem of the robust exponential stability of uncertain neutral systems with time-varying delays and nonlinear perturbations has been studied in [4]. In [26], they have been studied delay-dependent robust stability problem for neutral system with mixed time-varying delays. The uncertainties under consideration are nonlinear time-varying parameter perturbations and norm-bounded uncertainties, respectively.

In this paper, the problem of delay-dependent stability criteria for neutral systems with mixed time-varying delays and nonlinear perturbations is studied. Based on combination of Leibniz-Newton formula, decomposition technique of coefficient matrix, linear matrix inequality and the use of suitable Lyapunov-Krasovskii functional, new delay-dependent exponential stability criteria will be obtained in terms of LMIs. Finally, numerical examples will be given to show the effectiveness of the obtained results.

2. PROBLEM STATEMENT AND PRELIMINARIES

1cm We introduce some notations and lemmas that will be used throughout the paper. R^+ denotes the set of all real non-negative numbers; R^n denotes the n-dimensional space with the vector norm $\|\cdot\|$; $\|x\|$ denotes the Euclidean vector norm of $x \in R^n$; $\|A\|$ denotes the spectral norm of matrix A; $R^{n\times r}$ denotes the set of $n \times r$ real matrices; A^T denotes the transpose of the matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{Re\,\lambda : \lambda \in \lambda(A)\}$;

$\lambda_{\min}(A) = \min\{Re\lambda : \lambda \in \lambda(A)\}$; matrix A is called semi-positive definite ($A \geq 0$) if $x^T Ax \geq 0$, for all $x \in R^n$; A is positive definite ($A > 0$) if $x^T Ax > 0$ for all $x \neq 0$; matrix B is called semi-negative definite ($B \leq 0$) if $x^T Bx \leq 0$, for all $x \in R^n$; B is negative definite ($B < 0$) if $x^T Bx < 0$ for all $x \neq 0$; $A > B$ means $A - B > 0$ $(B - A < 0)$; $A \geq B$ means $A - B \geq 0$ $(B - A \leq 0)$; $C([-b,0], R^n)$ denotes the space of all continuous vector functions mapping $[-b,0]$ into R^n where $b = \max[h, r]$.

Consider system described by the following state equations of the form

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-h(t)) + C\dot{x}(t-r(t)) + f(t,x(t)) & \\ \qquad + g(t, x(t-h(t))) + w(t, \dot{x}(t-r(t))), & t > 0; \\ x(t) = \phi(t), \quad \dot{x}(t) = \psi(t), & t \in [-b,0], \end{cases} \tag{2.1}$$

where $x(t) \in R^n$ is the state, A, B and C are given constant matrices of appropriate dimensions. $h(t)$ and $r(t)$ are discrete and neutral time-varying delays, respectively,

$$0 \leq h(t) \leq h, \quad \dot{h}(t) \leq h_d, \tag{2.2}$$

$$0 \leq r(t) \leq r, \quad \dot{r}(t) \leq r_d, \tag{2.3}$$

where h and r are given positive real constants representing upper bounds of discrete and neutral delays, respectively, h_d, r_d are given positive real constants. Consider the initial functions $\phi(t), \psi(t) \in C([-b,0], R^n)$ with the norm $\|\phi\| = \sup_{t\in[-b,0]}\|\phi(t)\|$ and $\|\psi\| = \sup_{t\in[-b,0]}\|\psi(t)\|$. The uncertainties $f(.), g(.)$ and $w(.)$ represent the nonlinear parameter perturbations with respect to the current state $x(t)$, discrete delayed state $x(t-h(t))$ and neutral delayed state $\dot{x}(t-r(t))$, respectively, and are bounded in magnitude:

$$f^T(t, x(t)) f(t, x(t)) \leq \eta^2 x^T(t) x(t), \tag{2.4}$$

$$g^T(t, x(t-h(t))) g(t, x(t-h(t))) \leq \rho^2 x^T(t-h(t)) x(t-h(t)), \tag{2.5}$$

$$w^T(t, \dot{x}(t-r(t))) w(t, \dot{x}(t-r(t))) \leq \gamma^2 \dot{x}^T(t-r(t)) \dot{x}(t-r(t)), \tag{2.6}$$

where η, ρ, γ are given positive real constants. In order to improve the bound of the discrete delayed $h(t)$ in system (2.1), let us decompose the constant matrix B as

$$B = B_1 + B_2, \tag{2.7}$$

where $B_1, B_2 \in \mathrm{R}^{n\times n}$ are constant matrices. By Leibniz-Newton formula, we have

$$x(t) - x(t-h(t)) - \int_{t-h(t)}^{t} \dot{x}(s)ds = 0. \tag{2.8}$$

By utilizing the following zero equation, we obtain

$$Ex(t) - Ex(t-\alpha h(t)) - E\int_{t-\alpha h(t)}^{t} \dot{x}(s)ds = 0, \tag{2.9}$$

where α is given positive real constant and $E \in \mathbb{R}^{n\times n}$ will be chosen to guarantee the asymptotic stability of system (2.1). By (2.7)-(2.9), system (2.1) can be represented in the form of a neutral system with discrete and distributed delays and nonlinear perturbations

$$\begin{aligned} \dot{x}(t) = {} & [A + B_1 + E]x(t) + B_2 x(t-h(t)) - Ex(t-\alpha h(t)) + f(t, x(t)) + g(t, x(t-h(t))) \\ & + w(t, \dot{x}(t-r(t))) + C\dot{x}(t-r(t)) - B_1 \int_{t-h(t)}^{t} \dot{x}(s)ds - E\int_{t-\alpha h(t)}^{t} \dot{x}(s)ds. \end{aligned} \tag{2.10}$$

Lemma 2.1 (Jensen's inequality) *[5] For any constant matrix* $Q \in R^{n\times n}, Q = Q^T > 0$, *scalar* $h > 0$, *vector function* $\dot{x} : [0, h] \to R^n$ *such that the integrations concerned are well defined, then*

$$-h\int_{-h}^{0}\dot{x}^T(s+t)Q\dot{x}(s+t)ds \le -\left(\int_{-h}^{0}\dot{x}(s+t)ds\right)^T Q\left(\int_{-h}^{0}\dot{x}(s+t)ds\right).$$

Rearranging the term $\int_{-h}^{0}\dot{x}(s+t)ds$ with $x(t)-x(t-h)$, we can yield the following inequality:

$$-h\int_{-h}^{0}\dot{x}^T(s+t)Q\dot{x}(s+t)ds \le \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} -Q & Q \\ Q & -Q \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}.$$

Lemma 2.2 *[34] Let $x(t)\in R^n$ be a vector-valued function with first-order continuous-derivative entries. Then, the following integral inequality holds for any matrices $X, M_i \in R^{n\times n}, i=1,2,\ldots,5$ and a scalar function* $h := h(t) \ge 0$:

$$-\int_{t-h}^{t}\dot{x}^T(s)X\dot{x}(s)ds \le \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} M_1^T+M_1 & -M_1^T+M_2 \\ -M_1+M_2^T & -M_2^T-M_2 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}$$

$$+h\begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} M_3 & M_4 \\ M_4^T & M_5 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix},$$

where

$$\begin{bmatrix} X & M_1 & M_2 \\ M_1^T & M_3 & M_4 \\ M_2^T & M_4^T & M_5 \end{bmatrix} \ge 0.$$

3. MAIN RESULTS

In this section, we present the stability conditions dependent on time-varying delays of neutral system (2.1) via linear matrix inequality (LMI) approach. We introduce the following notations for later use,

$$\sum = [\Sigma_{i,j}]_{12\times 12}, \tag{3.1}$$

where $\Sigma_{i,j} = \Sigma_{j,i}^T, i, j = 1,2,3,...,12$ and $W_1 = P_1 E,$

$$\Sigma_{1,1} = P_1A + A^TP_1 + P_1B_1 + B_1^TP_1 + W_1 + W_1^T + Q_1^T + Q_1 + Q_5^T + Q_5 + Q_9^TA + A^TQ_9 + P_2$$
$$+P_3 - P_5 + hM_1^T + hM_1 + h^2M_3 - P_8 + \alpha hM_6^T + \alpha hM_6 + \alpha^2h^2M_8 - 4h^2P_{10} - 4\alpha^2h^2P_{11} + h^2P_{12}$$
$$+\alpha^2h^2P_{13} + \varepsilon_1\eta^2 I + N_1^T + N_1 + K_1^T + K_1 + L_1^TA + A^TL_1, \quad \Sigma_{1,2} = P_1B_2 - Q_1^T + Q_2 + Q_6 + Q_9^T$$
$$+A^TQ_{10} + P_5 - hM_1^T + hM_2 + h^2M_4 + N_2 - N_1^T + K_2 + A^TL_2 + L_1^TB, \quad \Sigma_{1,3} = -W_1 + Q_3 + Q_5^T + Q_7$$
$$+A^TQ_{11} + P_8 - \alpha hM_6^T + \alpha hM_7 + \alpha^2h^2M_9 + N_3 + K_3 - K_1^T + A^TL_3, \quad \Sigma_{1,4} = Q_4 + Q_8 - Q_9^T + A^TQ_{12}$$
$$+N_4 + K_4 + A^TL_4 - L_1^T, \quad \Sigma_{1,5} = 4hP_{10} + N_5 + K_5 + A^TL_5, \quad \Sigma_{1,6} = 4\alpha hP_{11} + N_6 + K_6 + A^TL_6,$$
$$\Sigma_{1,7} = -P_1B_1 - Q_1^T + N_7 - N_1^T + K_7 + A^TL_7, \quad \Sigma_{1,8} = -W_1 - Q_5^T + N_8 + K_8 - K_1^T + A^TL_8,$$
$$\Sigma_{1,9} = P_1 + Q_9^T + N_9 + K_9 + A^TL_9 + L_1^T, \quad \Sigma_{1,10} = P_1 + Q_9^T + N_{10} + K_{10} + A^TL_{10} + L_1^T,$$
$$\Sigma_{1,11} = P_1C + Q_9^TC + N_{11} + K_{11} + A^TL_{11} + L_1^TC, \quad \Sigma_{1,12} = P_1 + Q_9^T + N_{12} + K_{12} + A^TL_{12} + L_1^T,$$

$$\Sigma_{2,2} = -Q_2^T - Q_2 + Q_{10}^T B + B^T Q_{10} - P_2 + h_d P_2 - P_5 - hM_2^T - hM_2 + h^2 M_5 + \varepsilon_2 \rho I - N_2^T - N_2$$
$$+ L_2^T B + B^T L_2, \quad \Sigma_{2,3} = -Q_3 - Q_6^T + B^T Q_{11} - N_3 - K_2^T + B^T L_3, \quad \Sigma_{2,4} = -Q_4 - Q_{10}^T$$
$$+ B^T Q_{12} - N_4 + B^T L_4 - L_2^T, \quad \Sigma_{2,5} = -N_5 + B^T L_5, \quad \Sigma_{2,6} = -N_6 + B^T L_6, \quad \Sigma_{2,7} = -Q_2^T - N_7$$
$$- N_2^T + B^T L_7, \quad \Sigma_{2,8} = -Q_6^T - N_8 - K_2^T + B^T L_8, \quad \Sigma_{2,9} = Q_{10}^T - N_9 + B^T L_9 + L_2^T, \quad \Sigma_{2,10} = Q_{10}^T$$
$$- N_{10} + B^T L_{10} + L_2^T, \quad \Sigma_{2,11} = Q_{10}^T C - N_{11} + B^T L_{11} + L_2^T C, \quad \Sigma_{2,12} = Q_{10}^T - N_{12} + B^T L_{12} + L_2^T,$$
$$\Sigma_{3,3} = -Q_7^T - Q_7 - P_3 + \alpha h_d P_3 - P_8 - \alpha h M_7^T - \alpha h M_7 + \alpha^2 h^2 M_{10} - K_3^T - K_3, \quad \Sigma_{3,4} = -Q_8 - Q_{11}^T$$
$$- K_4 - L_3^T, \quad \Sigma_{3,5} = -K_5, \quad \Sigma_{3,6} = -K_6, \quad \Sigma_{3,7} = -Q_3^T - N_3^T - K_7, \quad \Sigma_{3,8} = -Q_7^T - K_8 - K_3^T,$$
$$\Sigma_{3,9} = Q_{11}^T - K_9 + L_3^T, \quad \Sigma_{3,10} = Q_{11}^T - K_{10} + L_3^T, \quad \Sigma_{3,11} = Q_{11}^T C - K_{11} + L_3^T C,$$
$$\Sigma_{3,12} = Q_{11}^T - K_{12} + L_3^T, \quad \Sigma_{4,4} = -Q_{12}^T - Q_{12} + h^2 P_4 + h^2 P_5 + h^2 P_6 + \alpha^2 h^2 P_7 + \alpha^2 h^2 P_8$$
$$+ \alpha^2 h^2 P_9 + h^4 P_{10} + h^4 P_{11} + P_{14} - L_4^T - L_4, \quad \Sigma_{4,5} = -L_5, \quad \Sigma_{4,6} = -L_6, \quad \Sigma_{4,7} = -Q_4^T - N_4^T - L_7,$$
$$\Sigma_{4,8} = -Q_8^T - K_4^T - L_8, \quad \Sigma_{4,9} = Q_{12}^T + L_4^T - L_9, \quad \Sigma_{4,10} = Q_{12}^T + L_4^T - L_{10},$$
$$\Sigma_{4,11} = Q_{12}^T C + L_4^T C - L_{11}, \Sigma_{4,12} = Q_{12}^T$$
$$+ L_4^T - L_{12}, \quad \Sigma_{5,5} = -4P_{10} - P_{12}, \quad \Sigma_{5,6} = 0, \quad \Sigma_{5,7} = -N_5^T, \quad \Sigma_{5,8} = -K_5^T, \quad \Sigma_{5,9} = L_5^T, \quad \Sigma_{5,10} = L_5^T,$$
$$\Sigma_{5,11} = L_5^T C, \quad \Sigma_{5,12} = L_5^T,$$
$$\Sigma_{6,6} = -4P_{11} - P_{13}, \quad \Sigma_{6,7} = -N_6^T, \quad \Sigma_{6,8} = -K_6^T,$$
$$\Sigma_{6,9} = L_6^T, \quad \Sigma_{6,10} = L_6^T, \quad \Sigma_{6,11} = L_6^T C, \quad \Sigma_{6,12} = L_6^T;$$
$$\Sigma_{7,7} = -P_4 - N_7^T - N_7, \quad \Sigma_{7,8} = -N_8 - K_7^T, \quad \Sigma_{7,9} = -N_9$$
$$+ L_7^T, \quad \Sigma_{7,10} = -N_{10} + L_7^T, \quad \Sigma_{7,11} = -N_{11} + L_7^T C, \quad \Sigma_{7,12} = -N_{12} + L_7^T,$$
$$\Sigma_{8,8} = -P_7 - K_8^T$$
$$- K_8, \quad \Sigma_{8,9} = -K_9 + L_8^T, \quad \Sigma_{8,10} = -K_{10} + L_8^T, \quad \Sigma_{8,11} = -K_{11} + L_8^T C, \quad \Sigma_{8,12} = -K_{12} + L_8^T,$$
$$\Sigma_{9,9} = -\varepsilon_1 I + L_9^T + L_9, \quad \Sigma_{9,10} = L_{10} + L_9^T, \quad \Sigma_{9,11} = L_{11} + L_9^T C, \quad \Sigma_{9,12} = L_{12} + L_9^T,$$
$$\Sigma_{10,10} = -\varepsilon_2 I + L_{10}^T + L_{10}, \quad \Sigma_{10,11} = L_{11} + L_{10}^T C, \quad \Sigma_{10,12} = L_{12} + L_{10}^T,$$
$$\Sigma_{11,11} = -P_{14} + r_d P_{14} + \varepsilon_3 \gamma^2 I + L_{11}^T C + C^T L_{11}, \quad \Sigma_{11,12} = L_{11}^T + C^T L_{12},$$
$$\Sigma_{12,12} = \varepsilon_3 I + L_{12}^T + L_{12},$$

and

$$x(t - h(t)) = x_{-h(t)}\ , x(t - \alpha h(t)) = x_{-\alpha h(t)}.$$

Theorem 3.1 *For* $\|C\| + \gamma < 1$ *and given positive real constants* $\alpha, h, h_d, r, r_d, \eta, \rho$ *and* γ*, the system (2.1) is asymptotically stable, if there exist positive definite symmetric matrices* P_s*,* $s = 1,2,\ldots,14$*, any appropriate dimensional matrices* $Q_i, N_i, K_i, L_i, M_j, i = 1,2,\ldots,12,\ \ j = 1,2,\ldots,10$ *and positive real constants* $\varepsilon_1, \varepsilon_2$ *and* ε_3 *such that the following symmetric linear matrix inequalities hold:*

$$\begin{bmatrix} P_6 & M_1 & M_2 \\ M_1^T & M_3 & M_4 \\ M_2^T & M_4^T & M_5 \end{bmatrix} \geq 0, \tag{3.2}$$

$$\begin{bmatrix} P_9 & M_6 & M_7 \\ M_6^T & M_8 & M_9 \\ M_7^T & M_9^T & M_{10} \end{bmatrix} \geq 0, \tag{3.3}$$

$$\sum < 0. \tag{3.4}$$

Proof. Construct a Lyapunov-Krasovskii functional candidate for the system (2.10) of the form

$$V(t) = \sum_{i=1}^{9} V_i(t), \tag{3.5}$$

where

$$V_1(t) = x^T(t)P_1x(t)$$

$$= \begin{bmatrix} x(t) \\ x_{-h(t)} \\ x_{-\alpha h(t)} \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_1 & 0 & 0 & 0 \\ Q_1 & Q_2 & Q_3 & Q_4 \\ Q_5 & Q_6 & Q_7 & Q_8 \\ Q_9 & Q_{10} & Q_{11} & Q_{12} \end{bmatrix} \begin{bmatrix} x(t) \\ x_{-h(t)} \\ x_{-\alpha h(t)} \\ \dot{x}(t) \end{bmatrix},$$

$$V_2(t) = \int_{t-h(t)}^{t} x^T(s)P_2x(s)ds - \int_{t-\alpha h(t)}^{t} x^T(s)P_3x(s)ds,$$

$$V_3(t) = h\int_{-h}^{0}\int_{t+s}^{t} \dot{x}^T(\theta)[P_4 + P_5 + P_6]\dot{x}(\theta)d\theta ds,$$

$$V_4(t) = \alpha h\int_{-\alpha h}^{0}\int_{t+s}^{t} \dot{x}^T(\theta)[P_7 + P_8 + P_9]\dot{x}(\theta)d\theta ds,$$

$$V_5(t) = 2h^2\int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{x}^T(s)P_{10}\dot{x}(s)ds d\lambda d\theta,$$

$$V_6(t) = 2(\alpha h)^2\int_{-\alpha h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{x}^T(s)P_{11}\dot{x}(s)ds d\lambda d\theta,$$

$$V_7(t) = h\int_{-h}^{0}\int_{t+s}^{t} x^T(\theta)P_{12}x(\theta)d\theta ds,$$

$$V_8(t) = \alpha h\int_{-\alpha h}^{0}\int_{t+s}^{t} x^T(\theta)P_{13}x(\theta)d\theta ds,$$

$$V_9(t) = \int_{t-r(t)}^{t} \dot{x}^T(s)P_{14}\dot{x}(s)ds.$$

Calculating the time derivatives of $V_i(t)$, $i = 1,2,3,...,9$ along the trajectory of (2.10) yields

$$\dot{V}_1(t) = 2\begin{bmatrix} x(t) \\ x_{-h(t)} \\ x_{-\alpha h(t)} \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} P_1 & Q_1^T & Q_5^T & Q_9^T \\ 0 & Q_2^T & Q_6^T & Q_{10}^T \\ 0 & Q_3^T & Q_7^T & Q_{11}^T \\ 0 & Q_4^T & Q_8^T & Q_{12}^T \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= 2\begin{bmatrix} x(t) \\ x_{-h(t)} \\ x_{-\alpha h(t)} \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} P_1 & Q_1^T & Q_5^T & Q_9^T \\ 0 & Q_2^T & Q_6^T & Q_{10}^T \\ 0 & Q_3^T & Q_7^T & Q_{11}^T \\ 0 & Q_4^T & Q_8^T & Q_{12}^T \end{bmatrix} \begin{bmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \\ \Omega_4 \end{bmatrix}, \tag{3.6}$$

where

$$\Omega_1 = [A + B_1 + E]x(t) + B_2 x_{-h(t)} - E x_{-\alpha h(t)} + f(t, x(t)) + g(t, x_{-h(t)})$$
$$+ w(t, \dot{x}(t - r(t))) + C\dot{x}(t - r(t)) - B_1 \int_{t-h(t)}^{t} \dot{x}(s)ds - E\int_{t-\alpha h(t)}^{t} \dot{x}(s)ds,$$

$$\Omega_2 = x(t) - x_{-h(t)} - \int_{t-h(t)}^{t} \dot{x}(s)ds,$$

$$\Omega_3 = x(t) - x_{-\alpha h(t)} - \int_{t-\alpha h(t)}^{t} \dot{x}(s)ds,$$

$$\Omega_4 = Ax(t) + Bx_{-h(t)} + f(t, x(t)) + g(t, x_{-h(t)}) + \omega(t, \dot{x}(t - r(t))) + C\dot{x}(t - r(t)) - \dot{x}(t).$$

By Lemma 2.1 and 2.2 with (3.2)-(3.3), we obtain

$$\dot{V}_2(t) = x^T(t)P_2 x(t) - (1 - \dot{h}(t))x_{-h(t)}^T P_2 x_{-h(t)} + x^T(t)P_3 x(t) - (1 - \alpha\dot{h}(t))x_{-\alpha h(t)}^T P_3 x_{-\alpha h(t)}$$
$$\leq x^T(t)P_2 x(t) - x_{-h(t)}^T P_2 x_{-h(t)} + h_d x_{-h(t)}^T P_2 x_{-h(t)} + x^T(t)P_3 x(t) - x_{-\alpha h(t)}^T P_3 x_{-\alpha h(t)}$$
$$+ \alpha h_d x_{-\alpha h(t)}^T P_3 x_{-\alpha h(t)}, \tag{3.7}$$

$$\dot{V}_3(t) = h^2 \dot{x}^T(t)[P_4 + P_5 + P_6]\dot{x}(t) - h\int_{t-h}^{t} \dot{x}^T(s)[P_4 + P_5 + P_6]\dot{x}(s)ds$$
$$\leq h^2 \dot{x}^T(t)[P_4 + P_5 + P_6]\dot{x}(t) - \int_{t-h(t)}^{t} \dot{x}^T(s)dsP_4 \int_{t-h(t)}^{t} \dot{x}(s)ds$$
$$+ \begin{bmatrix} x(t) \\ x_{-h(t)} \end{bmatrix}^T \begin{bmatrix} -P_5 & P_5 \\ P_5 & -P_5 \end{bmatrix} \begin{bmatrix} x(t) \\ x_{-h(t)} \end{bmatrix} + h^2 \begin{bmatrix} x(t) \\ x_{-h(t)} \end{bmatrix} \begin{bmatrix} M_3 & M_4 \\ M_4^T & M_5 \end{bmatrix} \begin{bmatrix} x(t) \\ x_{-h(t)} \end{bmatrix}$$
$$+ h\begin{bmatrix} x(t) \\ x_{-h(t)} \end{bmatrix} \begin{bmatrix} M_1^T + M_1 & -M_1^T + M_2 \\ -M_1 + M_2^T & -M_2^T - M_2 \end{bmatrix} \begin{bmatrix} x(t) \\ x_{-h(t)} \end{bmatrix}, \tag{3.8}$$

$$\dot{V}_4(t) = (\alpha h)^2 \dot{x}^T(t)[P_7 + P_8 + P_9]\dot{x}(t) - \alpha h\int_{t-\alpha h}^{t} \dot{x}^T(s)[P_7 + P_8 + P_9]\dot{x}(s)$$
$$\leq (\alpha h)^2 \dot{x}^T(t)[P_7 + P_8 + P_9]\dot{x}(t) - \int_{t-\alpha h(t)}^{t} \dot{x}^T(s)dsP_7 \int_{t-\alpha h(t)}^{t} \dot{x}(s)ds$$

$$+\begin{bmatrix} x(t) \\ x_{-\alpha h(t)} \end{bmatrix}^T \begin{bmatrix} -P_8 & P_8 \\ P_8 & -P_8 \end{bmatrix} \begin{bmatrix} x(t) \\ x_{-\alpha h(t)} \end{bmatrix} + (\alpha h)^2 \begin{bmatrix} x(t) \\ x_{-\alpha h(t)} \end{bmatrix} \begin{bmatrix} M_8 & M_9 \\ M_9^T & M_{10} \end{bmatrix} \begin{bmatrix} x(t) \\ x_{-\alpha h(t)} \end{bmatrix}$$

$$+\alpha h \begin{bmatrix} x(t) \\ x_{-\alpha h(t)} \end{bmatrix} \begin{bmatrix} M_6^T + M_6 & -M_6^T + M_7 \\ -M_6 + M_7^T & -M_7^T - M_7 \end{bmatrix} \begin{bmatrix} x(t) \\ x_{-\alpha h(t)} \end{bmatrix}, \tag{3.9}$$

$$\dot{V}_5(t) = h^4 \dot{x}^T(t) P_{10} \dot{x}(t) - 2h^2 \int_{-h}^{0} \int_{t-\theta}^{t} \dot{x}^T(s) P_{10} \dot{x}(s) ds d\theta$$

$$\leq h^4 \dot{x}^T(t) P_{10} \dot{x}(t) - 4[(hx(t) - \int_{t-h}^{t} x^T(s) ds)^T P_{10} (hx(t) - \int_{t-h}^{t} x(s) ds)], \tag{3.10}$$

$$\dot{V}_6(t) = (\alpha h)^4 \dot{x}^T(t) P_{11} \dot{x}(t) - 2(\alpha h)^2 \int_{-\alpha h}^{0} \int_{t-\theta}^{t} \dot{x}^T(s) P_{11} \dot{x}(s) ds d\theta$$

$$\leq h^4 \dot{x}^T(t) P_{11} \dot{x}(t) - 4[(\alpha h x(t) - \int_{t-\alpha h}^{t} x(s) ds)^T P_{11} (\alpha h x(t) - \int_{t-\alpha h}^{t} x(s) ds)], \tag{3.11}$$

$$\dot{V}_7(t) = h^2 x^T(t) P_{12} x(t) - h \int_{-h}^{0} x^T(t+s) P_{12} x(t+s) ds$$

$$\leq h^2 x^T(t) P_{12} x(t) - \int_{t-h}^{t} x^T(s) ds P_{12} \int_{t-h}^{t} x(s) ds, \tag{3.12}$$

$$\dot{V}_8(t) = (\alpha h)^2 x^T(t) P_{13} x(t) - \alpha h \int_{-h}^{0} x^T(t+s) P_{13} x(t+s) ds$$

$$\leq (\alpha h)^2 x^T(t) P_{13} x(t) - \int_{t-\alpha h}^{t} x^T(s) ds P_{13} \int_{t-\alpha h}^{t} x(s) ds, \tag{3.13}$$

$$\dot{V}_9(t) = \dot{x}^T(t) P_{14} \dot{x}(t) - (1 - \dot{r}(t)) \dot{x}^T(t - r(t)) P_{14} \dot{x}(t - r(t))$$

$$\leq \dot{x}^T(t) P_{14} \dot{x}(t) - \dot{x}^T(t - r(t)) P_{14} \dot{x}(t - r(t)) + r_d \dot{x}^T(t - r(t)) P_{14} \dot{x}(t - r(t)). \tag{3.14}$$

From the Leibniz-Newton formula, the following equations are true for any real matrices $N_i, K_i,\ i = 1,2,...,12$ with appropriate dimensions

$$2[x^T(t) N_1^T + x_{-h(t)}^T N_2^T + x_{-\alpha h(t)}^T N_3^T + \dot{x}(t) N_4^T + \int_{t-h}^{t} x^T(s) ds N_5^T + \int_{t-\alpha h}^{t} x^T(s) ds N_6^T$$

$$+ \int_{t-h(t)}^{t} \dot{x}^T(s) ds N_7^T + \int_{t-\alpha h(t)}^{t} \dot{x}^T(s) ds N_8^T + f^T(t, x(t)) N_9^T + g^T(t, x_{-h(t)}) N_{10}^T$$

$$+ \dot{x}(t - r(t)) N_{11}^T + w^T(t, \dot{x}(t - r(t))) N_{12}^T] \times [x(t) - x_{-h(t)} - \int_{t-h(t)}^{t} \dot{x}(s) ds] = 0, \tag{3.15}$$

$$2[x^T(t) K_1^T + x_{-h(t)}^T K_2^T + x_{-\alpha h(t)}^T K_3^T + \dot{x}(t) K_4^T + \int_{t-h}^{t} x^T(s) ds K_5^T + \int_{t-\alpha h}^{t} x^T(s) ds K_6^T$$

$$+ \int_{t-h(t)}^{t} \dot{x}^T(s) ds K_7^T + \int_{t-\alpha h(t)}^{t} \dot{x}^T(s) ds K_8^T + f^T(t, x(t)) K_9^T + g^T(t, x_{-h(t)}) K_{10}^T$$

$$+ \dot{x}(t - r(t)) K_{11}^T + w^T(t, \dot{x}(t - r(t))) K_{12}^T] \times [x(t) - x_{-\alpha h(t)} - \int_{t-\alpha h(t)}^{t} \dot{x}(s) ds] = 0. \tag{3.16}$$

From the utilization of zero equation, the following equation is true for any real matrices $L_i,\ i = 1,2,...,12$ with appropriate dimensions

$$2[x^T(t) L_1^T + x_{-h(t)}^T L_2^T + x_{-\alpha h(t)}^T L_3^T + \dot{x}(t) L_4^T + \int_{t-h}^{t} x^T(s) ds L_5^T + \int_{t-\alpha h}^{t} x^T(s) ds L_6^T$$

$$+ \int_{t-h(t)}^{t} \dot{x}^T(s) ds L_7^T + \int_{t-\alpha h(t)}^{t} \dot{x}^T(s) ds L_8^T + f^T(t, x(t)) L_9^T + g^T(t, x_{-h(t)}) L_{10}^T$$

$$+\dot{x}(t-r(t))L_{11}^T+w^T(t,\dot{x}(t-r(t)))L_{12}^T]\times[Ax(t)+Bx_{-h(t)}+f(t,x(t))$$
$$+g(t,x_{-h(t)})+w(t,\dot{x}(t-r(t)))+C\dot{x}(t-r(t))-\dot{x}(t)]=0. \tag{3.17}$$

From (2.4)-(2.6), we obtain for any positive real constants $\varepsilon_1>0$, $\varepsilon_2>0$ and $\varepsilon_3>0$,

$$0\le\varepsilon_1\eta^2x^T(t)x(t)-\varepsilon_1f^T(t,x(t))f(t,x(t)), \tag{3.18}$$
$$0\le\varepsilon_2\rho^2x_{-h(t)}^Tx_{-h(t)}-\varepsilon_2g^T(t,x_{-h(t)})g(t,x_{-h(t)}), \tag{3.19}$$
$$0\le\varepsilon_3\gamma^2\dot{x}^T(t-r(t))\dot{x}(t-r(t))-\varepsilon_3w^T(t,\dot{x}(t-r(t)))w(t,\dot{x}(t-r(t))). \tag{3.20}$$

According to (??)-(3.20), it is straightforward to see that

$$\dot{V}(t)\le\zeta^T(t)\sum\zeta(t), \tag{3.21}$$

where $\sum$ is defined in (3.1) and

$$\zeta^T(t)=[x^T(t),x_{-h(t)}^T,x_{-\alpha h(t)}^T,\dot{x}^T(t),\int_{t-h}^t x^T(s)ds,\int_{t-\alpha h}^t x^T(s)ds,\int_{t-h(t)}^t\dot{x}^T(s)ds,\int_{t-\alpha h(t)}^t\dot{x}^T(s)ds,f^T(t,x(t)),g^T(t,x_{-h(t)}),\dot{x}^T(t-r(t)),\dot{x}^T(t-r(t)),w^T(t,\dot{x}(t-r(t)))].$$

It is fact that if inequality (3.4) holds, i.e., $\sum<0$, then $\dot{V}(t)<-\varepsilon\|x\|^2,\varepsilon>0$. This means that the system (2.1) is asymptotically stable. The proof of the theorem is complete.

If $w(t,\dot{x}(t-r(t)))=0$, then system (2.1) reduces to the following system:

$$\begin{cases}\dot{x}(t)-C\dot{x}(t-h(t))=Ax(t)+Bx(t-h(t))+f(t,x(t))+g(t,x(t-h(t))),t>0;\\ x(t)=\phi(t),\quad \dot{x}(t)=\psi(t),\quad t\in[-h,0].\end{cases} \tag{3.22}$$

According to method of Theorem 3.1, we can obtain the delay-dependent asymptotic stability criteria for neutral system (3.22) with the following Corollary 3.2.

Corollary 3.2 *For $\|C\|<1$ and given positive real constants $\alpha,h,h_d,r,r_d,\eta,\rho$ and γ, system (3.22) is asymptotically stable, if there exist positive definite symmetric matrices P_s, $s=1,2,\ldots,14$, any appropriate dimensional matrices Q_i,N_i,K_i,L_i, M_j, $i=1,2,\ldots,11$, $j=1,2,\ldots,10$ and positive real constants ε_1, ε_2 and $\varepsilon_3=0$ such that the following symmetric linear matrix inequalities hold:*

$$\begin{bmatrix}P_6 & M_1 & M_2\\ M_1^T & M_3 & M_4\\ M_2^T & M_4^T & M_5\\ & & \end{bmatrix}\ge0, \tag{3.23}$$

$$\begin{bmatrix}P_9 & M_6 & M_7\\ M_6^T & M_8 & M_9\\ M_7^T & M_9^T & M_{10}\\ & & \end{bmatrix}\ge0, \tag{3.24}$$

$$\prod=[\Sigma_{i,j}]_{11\times11}<0. \tag{3.25}$$

4. NUMERICAL EXAMPLES

In order to show the effectiveness of the approaches presented in Section 3, two numerical examples are provided.

Example 4.1 Consider the delay-dependent stability criteria of neutral system (2.1) with mixed time-varying delays and nonlinear perturbations with

$$A = \begin{bmatrix} -1.2 & 0.1 \\ -0.1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{bmatrix}, \quad C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad \eta \geq 0, \quad \rho \geq 0, \quad \gamma \geq 0. \tag{4.1}$$

Decompose the matrix $B = B_1 + B_2$, where

$$B_1 = \begin{bmatrix} -0.3 & 0.35 \\ -0.5 & -0.4 \end{bmatrix}, B_2 = \begin{bmatrix} -0.3 & 0.35 \\ -0.5 & -0.4 \end{bmatrix}. \tag{4.2}$$

By Theorem 3.1 to the system (2.1) with (4.1) and (4.2), we can obtain the maximum upper bounds of the discrete time-varying delay under different values of η, ρ and γ as shown in Table 1. From Table 1, it is easy to see that our results (Theorem 3.1) give a much less conservative result than those results in [7], [28], [26], [27] and [36]. The numerical solutions $x_1(t)$ and $x_2(t)$ of system (2.1) with (4.1) and

$$f(t, x(t)) = \begin{bmatrix} 0.1\sin(t)x_1(t) \\ 0.1\cos(t)x_2(t) \end{bmatrix}, \quad g(t, x(t-h(t))) = \begin{bmatrix} 0.1\cos(t)^2 x_1(t-h(t)) \\ 0.1\sin(t)^2 x_2(t-h(t)) \end{bmatrix},$$

$$w(t, \dot{x}(t-r(t))) = \begin{bmatrix} 0.1\cos(t)\dot{x}_1(t-r(t)) \\ 0.1\sin(t)\dot{x}_2(t-r(t)) \end{bmatrix}, \quad h(t) = 2\sin^2(\frac{t}{4}), \quad r(t) = \cos^2(\frac{t}{4}),$$

are plotted in Figure 1.

Table 1 Maximum allowable time delay upper bound for h with $\alpha = 0.1, r_d = 0, h_d = 0.5$, $\eta = 0, \rho = 0.1$ and $\eta = 0.1, \rho = 0.1$, respectively, and different values of γ.

Methods	$\eta = 0, \rho = 0.1$			$\eta = 0.1, \rho = 0.1$		
γ	0	0.1	0.2	0	0.1	0.2
Han and Yu [7]	0.9328	0.7402	0.5637	0.8148	0.6439	0.4864
Zhang and Yu [36]	0.9488	0.7695	0.6087	0.8408	0.6841	0.5420
Qiu et al. [26]	0.9839	0.8024	0.6392	0.8752	0.7166	0.5727
Rakkiyappan et al. [27]	1.4886	1.2437	0.9921	1.3244	1.0901	0.8475
Rakkiyappan et al. [28]	1.6325	1.3386	1.0816	1.4440	1.1950	0.9734
Our results	3.1790	2.2812	1.4217	3.0902	2.2464	1.4001

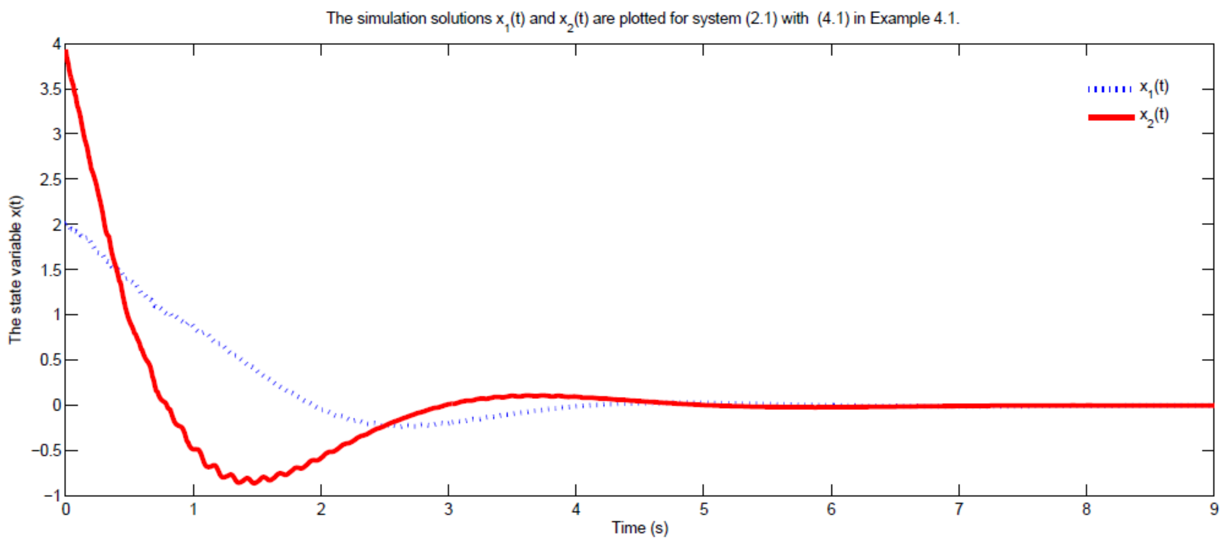

Figure 1: The simulation solutions $x_1(t)$ and $x_2(t)$ are presented for system (2.1) with (4.1) in Example 4.1 and initial conditions $x_1(t) = 2 + 3\sin(t)$, $x_2(t) = 2 + 2\cos(t)$, $t \in [-2,0]$, by using the Runge-Kutta 4th order method with Matlab.

Example 4.2 Consider the delay-dependent stability criteria of neutral system (3.22) with time-varying delays and nonlinear perturbations with

$$A = \begin{bmatrix} -2 & 0.5 \\ 0 & -0.9 \end{bmatrix}, B = \begin{bmatrix} 1 & 0.4 \\ 0.4 & -1 \end{bmatrix}, C = \begin{bmatrix} 0.2 & 1 \\ 0 & 0.2 \end{bmatrix}, \quad \eta = 0.2, \rho = 0.1, h(t) = r(t) = h. \tag{4.3}$$

Table 2 lists the comparison of the upper bounds delay h for delay-dependent stability criteria of system (3.22) with (4.3) by different methods in [24] and [31]. It is clear from Table 2 that our results (Corollary 3.2) are superior to those in [31] and [24]. The numerical solutions $x_1(t)$ and $x_2(t)$ of system (3.22) with (4.3),

$$f(t, x(t)) = \begin{bmatrix} 0.2\sin(t)x_1(t) \\ 0.2\cos(t)x_2(t) \end{bmatrix}, \quad g(t, x(t-h(t))) = \begin{bmatrix} 0.1\cos(t)^2 x_1(t-h(t)) \\ 0.1\sin(t)^2 x_2(t-h(t)) \end{bmatrix}, \quad h = 2,$$

are plotted in Figure 2.

Table 2 Upper bound of time delay h in Example 4.2 for $\eta = 0.2, \rho = 0.1$.

Methods	$\eta = 0.2, \rho = 0.1$
Xiong et al. [31]	0.583
Park [24]	1.7043
Our result	3.3021

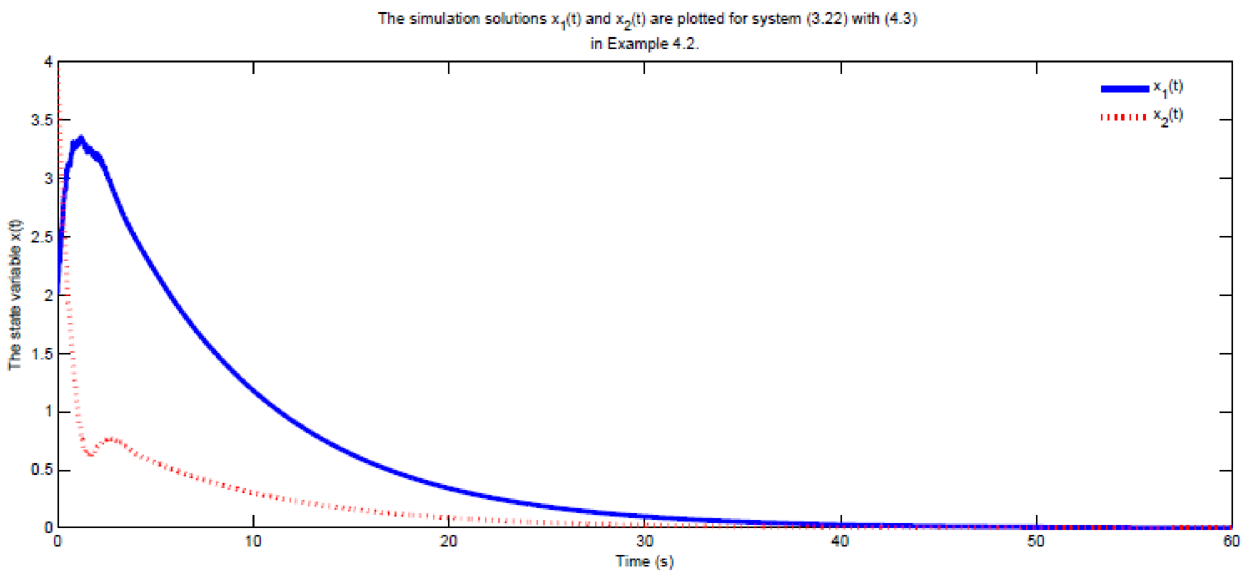

Figure 2: The simulation solutions $x_1(t)$ and $x_2(t)$ are presented for system (3.22) with (4.3) in Example 4.2 and initial conditions $x_1(t) = 2 + 3\sin(t)$, $x_2(t) = 2 + 2\cos(t)$, $t \in [-2,0]$, by using the Runge-Kutta 4th order method with Matlab.

5. CONCLUSIONS

1cm The problem of delay-dependent stability criteria of neutral systems with mixed time-varying delays and nonlinear perturbations has been investigated. Based on the combination of model transformation, decomposition technique of coefficient matrix, utilization of zero equation and new Lyapunov functional with triple integral terms, sufficient conditions for asymptotically stability have been obtained and formulated in term of linear matrix inequalities (LMIs). Numerical examples have shown significant improvements over some existing results.

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