MODELING A RANDOM YIELD IN- HOUSE PRODUCTION SET UP IN A NEWSVENDOR PROBLEM

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ABSTRACT

In supply uncertainty caused by random yield, the firm receives a random portion of an order placed with a supplier. Yield uncertainty exists in many industries, including semiconductors, electronic fabrication and assembly, food processing, bio-pharmaceuticals, and resource based industries such a mining and agriculture. The existence of yield / supply process uncertainty often results in lower supply chain performance. We consider this issue in the context of a newsboy problem where before the selling season; the firm makes joint pricing and inventory decisions. We consider the in-house production case in this model and derive analytical expressions therein.

1. INTRODUCTION

Our objective is to investigate the effect of supply uncertainty on optimal decisions and expected profit. A fundamental problem in supply chain management is how to match supply and demand so that the system operates effectively. However, manufacturers and retailers often suffer supply process uncertainty in addition to demand uncertainty. When the supply process is random, the quantity received may be different from the quantity ordered. We consider a price-setting newsvendor model in which a firm needs to make joint inventory and pricing decisions before the selling season. The supply process is uncertain such that the received quantity is the product of the order quantity and a random yield rate.It is well known that in the classical newsvendor problem without a random yield, the optimal price in the stochastic system is greater than the optimal price in the deterministic system if the demand model is in the form of multiplicative. We prove that this is also true in the random yield model.

2. LITERATURE REVIEW

This paper is related to several streams of literature. The first stream is on the price – setting newsvendor problem. For a comprehensive review, see the work of Petruzzi and Dada (1999), Yano and Gilbert (2003), and Yao et al.(2006) and the references therein. Additive and multiplicative forms are two commonly used demand models in the newsvendor pricing literature (see, e.g., Petruzzi and Dada 1999, Agrawal and Seshadri 2000, Chen et al.2009).We assume that the demand model is multiplicative, which can be represented as the product of a deterministic, price-dependent demand function and a random noise term (see, e.g., Granot and Yin 2005 and 2008, Song et al.2009, Chen and Bell 2009).Studies in this stream do not usually consider supply uncertainty.

The second stream cares optimal inventory decisions under supply uncertainty due to random yield. Yano and Lee (1995) provide an excellent review of this literature. Recent works include those of Bollapragada and Morton (1999), Khouja(1999),Gupta and Cooper(2005),and Guler and Bilgic (2009).The stochastically proportional yield model is the most widely studied (see,e.g.,Shih 1980,Henig and Gerchak 1990,Li and Zheng 2006,and Liu et al.2010),and is also used in this paper. For other classes of yield models, please refer to Dada et al. (2007).Most papers in this line of research focus on determining optimal lot size or characterizing the structure of the optimal inventory policy.

In the literature on production planning under supply uncertainty, selling price is commonly assumed to be exogenously given. However, there are a few exceptions. Tang and Yin (2007) study the pricing policy under supply uncertainty and examine the impact of that uncertainty on optimal decisions with a known, price-dependent demand. Kazaz (2004) studies production planning for olive oil with random yield and stochastic demand where both the selling price and purchasing cost of fruit are yield dependent. Recently, Kazaz and Webster (2011) consider an agricultural firm that operates under supply uncertainty and yield-dependent trading costs. They extend Kazaz (2004) by considering deterministic, price-dependent demand and offering a risk-averse analysis, and considering the influence of fruit futures. Kazaz and Webster (2011) focus on the impact of the yield-dependent trading cost on the optimal selling price and production quantity. Kazaz (2008) studies a similar model with two model settings: an early pricing model and a postponed pricing model. Kazaz (2008) focuses on identifying the optimal selling price and production decisions for the two models. Our paper is close to those of Kazaz and Webster (2011) and Kazaz (2008) because we also model the problem by using an emergent purchasing option and price-dependent stochastic demand. However, our focus is on the impact of supply uncertainty on both optimal decisions and expected profits.

In this paper, we assume that the firm has an emergency order/production option for excessive demand. With the emergency supply option, in additional to a "normal order" before the start of the selling season, the inventory manager may use an "emergency order" to fulfil shortages after demand realization, usually at a higher production/purchase cost. The existence of an emergency supply option may have more advantages in enhancing supply chain performance, especially in random yield settings.

3. MATHEMATICAL MODEL

In the In-house production model a firm has to pay for the quantity input instead of quantity received. Before the selling season the firm decides the production quantity q and the unit selling price p to maximize its expected profit. During the selling season demand is realized and satisfied by firm's on-hand inventory. Here ϵ is the random yield rate with a support $[L_\epsilon, U_\epsilon](U_\epsilon > L_\epsilon \geq 0)$and a mean μ.Then $q\epsilon$ is the yield from production. If the realized demand exceeds the quantity received,$q\epsilon$,then remaining demand is satisfied by emergency purchase from spot market at unit price e. Salvaged value of leftover inventory is denoted as unit price $s(s\geq 0)$.The unit order cost is denoted as c. We assume that $s<c<e$.

Let $D(p,\delta)$be the random demand.It is a function of the unit selling price p and market noise δ that is independent of p. We use multiplicative demand model in which demand is represented as the product of a deterministic, price – dependent demand function and a random noise term, $D(p,\delta) = d(p)\,\delta$, where δ is defined on $[L_\epsilon, U_\epsilon](U_\epsilon > L_\epsilon \geq 0)$. We assume that $d(p)$ is the continuous, strictly decreasing, nonnegative, twice differentiable function defined on $[c, \bar{p}]$, where $\bar{p}$ is the maximum admissible price .

We can take $\bar{p}$ = min {p|d(p) = 0}.If $d(p)> 0$ for all $p> 0$, we define $\bar{p} = +\infty$.

$d(p)$ has an IPE, i.e. $d\eta(p)/dp \geq 0$, where $\eta(p) = -pd'(p)/d(p)$.

The Profit function

The firm's expected profit as a function of input quantity q and unit selling price p can be written as

$$\pi(q,p) = pE_\delta[\mathrm{D}(\mathrm{p},\delta)] + sE_{\epsilon,\delta}\left[\left(q\epsilon - \mathrm{D}(\mathrm{p},\delta)\right)^+\right] - eE_{\epsilon,\delta}[(\mathrm{D}(\mathrm{p},\delta) - q\epsilon)^+] - c \qquad (1)$$

$$=(p-e)E_\delta[\mathrm{D}(\mathrm{p},\delta)] - (e-s)E_{\epsilon,\delta}\left[\left(q\epsilon - \mathrm{D}(\mathrm{p},\delta)\right)^+\right] + (e\mu - c)q \quad (2)$$

$$=(p-s)E_\delta[\mathrm{D}(\mathrm{p},\delta)] - (e-s)E_{\epsilon,\delta}[(\mathrm{D}(\mathrm{p},\delta) - q\epsilon)^+] - (c - s\mu)q \quad (3)$$

4. ANALYTICAL RESULTS

***Lemma* 1** : $If\ d(p)\ has\ IPE, then\ \pi_d(p) is\ quasi$ –Concave in p in the interval$[c/\mu, \bar{p}]$. Hence the optimal price p_d can be uniquely determined by $d\pi_d(p)/dp = 0$,

i.e.,$(p_d - c/\mu)d'(p_d)$+ d(p_d) = 0

Proof: Let z = q/dp be the stocking factor from (2) and (3);

$$\pi(z,p) = (p-e)d(p) + (e\text{?} - c)\,z\,d(p) - (e-s)\int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{z_u} F(x)dx\,dG(u)d(p) \qquad (4)$$

(∵ $E_\delta[\mathrm{D}(\mathrm{p},\delta)]$ = d (p) for δ = 1 and replace q = z d(p))

$$= \mathrm{d}\,(\mathrm{p})\left[(p-e) + (e\text{?} - c)\,z - (e-s)\int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{z_u} F(x)dx\,dG(u)\right]$$

Replacing q = z d (p) and $E_\delta[\mathrm{D}(\mathrm{p},\delta)]$ = d (p) in (3) ;

$$\pi(z,p) = \mathrm{d}\,(\mathrm{p})\left[(p-s) - (c - s\text{?})\,z - (e-s)z\int_{L_\delta}^{U_\delta}\int_{L_\epsilon}^{x/z} dF(x)\,G(u)\,du\right] \qquad (5)$$

For fixed price p the following lemma gives the optimal stocking factor.

Lemma 2: Suppose that $e\text{?} \geq c$. Then for any given p , $\pi(z,p)$ is concave in z and the optimal stocking factor $z^*(p)$ satisfy

$$\mu - \int_{L_\epsilon}^{U_\epsilon} F(zu)u\,d\,G(u) = E_\epsilon[(1 - F(z\epsilon))\epsilon] = \frac{c - s\mu}{e-s} \quad (6) \quad \text{or}$$

$$\int_{L_\delta}^{U_\delta}\int_{L_\epsilon}^{x/z} u dG(u)\,dF(x)\,du = \frac{c - s\mu}{e-s} = \text{?} - \frac{e\mu - c}{e-s} \quad (7)$$

Hence, the corresponding production input quantity is $q^*(p) = d(p)z^*(p)$.

Proof: Taking first and second derivatives of $\pi(z,p)$ w.r.t. z in (4) , we have

$$\frac{\partial \pi(z,p)}{\partial z} = d(p)\left[(e\mu - c) - (e-s)\int_{L_\in}^{U_\in} uF(zu)dG(u)\right]$$

$= d(p)\left[-(e-s)\int_{L_\in}^{U_\in} uF(zu)dG(u) + (e\mu - c) + s\mu - s\mu\right]$

$= d(p)\left[-(e-s)\int_{L_\in}^{U_\in} uF(zu)dG(u) + (e-s)\mu - (c - s\mu)\right]$

$= d(p)\left[-(e-s)\int_{L_\in}^{U_\in} uF(zu)dG(u) + (e-s)\int_{L_\in}^{U_\in} u\, d\, G(u) - (c - s\mu)\right]$

$(\because \mu = \int_{L_\in}^{U_\in} u\, d\, G(u))$

$= d(p)\left[(e-s)\left\{\int_{L_\in}^{U_\in} u\big(1 - F(zu)\big)dG(u)\right\} - (c - s\mu)\right]$ (8)

$\frac{\partial^2 \pi(z,p)}{\partial z^2} = -d(p)(e-s)\int_{L_\epsilon}^{U_\epsilon} u\, \frac{\partial}{\partial z}$ (F(zu)) dG(u)

$= -d(p)(e-s)\int_{L_\epsilon}^{U_\epsilon} u^2$ f(zu) dG(u)

< 0

$\therefore\ \pi\ (z,p)$ is concave in z for any given p . Taking (8) equal to zero and rearranging,

$$\int_{L_\epsilon}^{U_\epsilon} u\,\big(1 - F(zu)\big)dG(u) = \frac{c - s\mu}{e - s}$$

$$\int_{L_\epsilon}^{U_\epsilon} u\, dG(u) - \int_{L_\epsilon}^{U_\epsilon} u\, F(zu)dG(u) = \frac{c - s\mu}{e - s}$$

$\mu - \int_{L_\epsilon}^{U_\epsilon} u\, F(zu)dG(u) = \frac{c - s\mu}{e - s}$ $(\because \mu = \int_{L_\in}^{U_\in} u\, d\, G(u))$

on the other hand from (5), we get

$$\pi\ (z,p) = d(p)\left[(p-s) - (c - s\mu)z - (e-s)z\int_{L_\delta}^{U_\delta}\int_{L_\in}^{x/z} G(u)du\, dF(z)\right]$$

$\frac{\partial \pi\,(z,p)}{\partial z} = d(p)\left[-(c - s\mu) - (e-s)\int_{L_\delta}^{U_\delta}\int_{L_\epsilon}^{x/z} G(u)dudF(z) - (e-s)z\int_{L_\delta}^{U_\delta}\frac{d}{dz}\big(\int_{L_\epsilon}^{x/z} G(u)dudF(z)\right]$

$= d(p)\left[-(c - s\mu) - (e-s)\int_{L_\delta}^{U_\delta}\int_{L_\epsilon}^{x/z} G(u)dudF(z) - (e-s)z\int_{L_\delta}^{U_\delta} G\left(\frac{x}{z}\right)d\left(\frac{x}{z}\right)dF(z)\right]$

$= d(p)\left[-(c - s\mu) - (e-s)\int_{L_\delta}^{U_\delta}\int_{L_\epsilon}^{x/z} G(u)dudF(z) - (e-s)z\int_{L_\delta}^{U_\delta} G\left(\frac{x}{z}\right).\left(\frac{-x}{z^2}\right)dF(z)\right]$

$= d(p)\left[-(c - s\mu) - (e-s)\int_{L_\delta}^{U_\delta}\int_{L_\epsilon}^{x/z} G(u)dudF(z) + (e-s)\int_{L_\delta}^{U_\delta} G\left(\frac{x}{z}\right).\left(\frac{x}{z}\right)dF(z)\right]$

$= d(p)\left[-(c - s\mu) + (e-s)\int_{L_\delta}^{U_\delta}\int_{L_\epsilon}^{x/z} u\, dG(u)dF(x)\right]$ (by integration of parts) (9)

Taking (9) equal to zero,

$$\int_{L_\delta}^{U_\delta}\int_{L_\in}^{x/z} udG(u)\, dF(x)\, du = \frac{c - s\mu}{e - s} = \mu - \frac{e\mu - c}{e - s}$$

Lemma 3: The optimal selling price p^* is greater than the optimal riskless price ${p^*}_d$.

Proof:

For (condition) any $c/_\mu < p < {p^*}_d$,

We have (p - $c/_\mu$) d'(p) + d(p) ≥ 0

i.e. d(p) ≥ $-(\mu p - c)d'(p)/_\mu$

we have for any p >$c/_\mu$

$\frac{d\pi\,(z^*,p)}{dp} = d'$(p) $\left[(p-e) + (e-s)\int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{z^*u} xdF(x)dG(u)\right] + d(p)$ (10)

$\geq d'$(p) $\left[(p-e) + (e-s)\int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{z^*u} tdF(t)dG(u)\right] - \frac{\mu p - c}{\mu} d'(p)$

$= \frac{d'(p)}{\mu}\left[\mu(p-e) + \mu\,(e-s)\int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{z^*u} tdF(t)dG(u) - (\mu p - c)\right]$

$$= \frac{d^{'}(\mathrm{p})}{\mu}\left[(-\mu e + +\mu\,(e-s)\int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{z^*u} tdF(t)dG(u)\right]$$

$$= \frac{-(e-s)d^{'}(\mathrm{p})}{\mu}\left[(\frac{\mu e-c}{e-s} - \mu\int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{z^*u} tdF(t)dG(u)\right]$$

$$= \frac{-(e-s)d^{'}(\mathrm{p})}{\mu}\left[\int_{L_\epsilon}^{U_\epsilon} uF(z^*u)dG(u) - \mu\int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{z^*u} tdF(t)dG(u)\right] \quad \text{(by 6)}$$

$$= \frac{-(e-s)d^{'}(\mathrm{p})}{\mu}\int_{L_\epsilon}^{U_\epsilon} uF(z^*u)dG(u)\left[1 - \frac{\mu\int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{z^*u} tdF(t)dG(u)}{\int_{L_\epsilon}^{U_\epsilon} uF(z^*u)dG(u)}\right]$$

Define $A(z) = 1 - \frac{\mu\int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{zu} tdF(t)dG(u)}{\int_{L_\epsilon}^{U_\epsilon} uF(zu)dG(u)}$.

Now we show that A(z) > 0 for all 0 < z < ∞ .

Note that $\lim_{z\to\infty} A(z) = 0$, there fore it suffices to show that $A^{'}(z) < 0$ w. r.t z for all 0 < z <∞.

Differentiating A(z) w.r.t z and rearranging items we get,

$$A^{'}(z) = \frac{-\mu\left[\int_{L_\epsilon}^{U_\epsilon} uF(zu)dG(u).\frac{d}{dz}\left(\int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{zu} tdF(t)dG(u)\right) - \int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{zu} tdF(t)dG(u)\,.\frac{d}{dz}\,(\int_{L_\epsilon}^{U_\epsilon} uF(zu)dG(u))\right]}{\left(\int_{L_\epsilon}^{U_\epsilon} uF(zu)dG(u)\right)^2}$$

$$= \frac{-\mu\left[\int_{L_\epsilon}^{U_\epsilon} uF(zu)dG(u).\left(\int_{L_\epsilon}^{U_\epsilon} zudF(zu)dG(u)\right) - \int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{zu} tdF(t)dG(u)\,.\,(\int_{L_\epsilon}^{U_\epsilon} u\,\frac{d}{dz}F(zu)dG(u))\right]}{\left(\int_{L_\epsilon}^{U_\epsilon} uF(zu)dG(u)\right)^2}$$

$$= \frac{-\mu\left[\int_{L_\epsilon}^{U_\epsilon} uF(zu)dG(u).\left(\int_{L_\epsilon}^{U_\epsilon} zu^2 f(zu)dG(u)\right) - \int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{zu} tdF(t)dG(u)\,.\,(\int_{L_\epsilon}^{U_\epsilon} u^2\,f(zu)dG(u))\right]}{\left(\int_{L_\epsilon}^{U_\epsilon} uF(zu)dG(u)\right)^2}$$

$$= \frac{-\mu\left(\int_{L_\epsilon}^{U_\epsilon} u^2 f(zu)dG(u)\left[(\int_{L_\epsilon}^{U_\epsilon} zuF(zu) - \int_{L_\delta}^{zu} tdF(t))\;dG(u)\right]\right)}{\left(\int_{L_\epsilon}^{U_\epsilon} uF(zu)dG(u)\right)^2}$$

For all z, 0 < z < ∞ ; zu f(zu) - $\int_{L_\delta}^{zu} tdF(t) > 0$

So for all ${}^{c}/_{\mu} < p \;\leq\; p^*{}_d$, we have

$\frac{d\pi(z^*,p)}{dp} > 0$. Since $d^{'}(p) < 0$

So, the optimal selling price p^* is greater than the optimal riskless price $p^*{}_d$.

Theorem: Suppose that the mean demand *d(p)* has IPE. Then $\pi\,(z^*, p)$ is a quasi – concave function of p over the interval (${}^{c}/_{?}$, $\bar{p}$) and hence there exists a unique maximizer of $\pi\,(z^*, p)$.

Proof: To show that $\pi\,(z^*, p)$ is quasi – concave in p over (${}^{c}/_{?}$, $\bar{p}$),

we only need to show that $\frac{d^2\pi\,(z^*,p)}{dp^2}\,|_{\frac{d\pi\,(z^*,p)}{dp}=0} < 0$.

From (10) we know that at $\frac{d\pi\,(z^*,p)}{dp} = 0$.

(p-e) + (e-s) $\int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{z^*u} tdF(t)dG(u) = -\frac{d(p)}{d^{'}(p)}$(11)

With the assumption of IPE property,

$$\frac{d\eta(p)}{d^{'}(p)} \geq 0$$

$$\Rightarrow\; -\frac{\left[\left(pd^{''}(p) + d^{'}(p)\right)d(p) - p\,(d^{'}(p))^2\right]}{d^2(p)} \geq 0$$

$\Rightarrow d^{''}$ (p) ≤ $\frac{d^{'}(p)}{p}\left[\frac{p\,d^{'}(p)}{d(p)} - 1\right]$(12)

From (10) we get that; differentiate (10) w.r.t p;

$$\frac{d^2\pi\,(z^*,p)}{dp^2}\,|_{\frac{d\pi\,(z^*,p)}{dp}=0} = d^{''}(p)\left[(p-e) + (e-s)\int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{z^*u} tdF(t)dG(u)\right] + d^{'}(p) + d(p)$$

$= \quad d''(p)\left[(p-e) + \ (e-s)\int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{z^*u} tdF(t)dG(u)\right] + \ 2d'(p)$

From (11) replace the value;

$= \ -\frac{d(p)}{d'(p)} d''(p) + 2d'(p)$

$\leq \ -\frac{d(p)}{d'(p)} . \frac{d'(p)}{p}\left[\frac{p\ d'(p)}{d(p)} - 1\right] + 2d'(p)$ (replacing from (11));

$= \ -\frac{d(p)}{p} . \frac{p\ d'(p)}{d(p)} + \frac{d(p)}{p} + 2d'(p)$

$= d'(p) + \frac{d(p)}{p}$

Replacing value of *d(p)* from (11);

$\frac{d^2\pi\ (z^*,p)}{dp^2}\Big|_{\frac{d\pi\ (z^*,p)}{dp}=0} \ = \frac{-d'(p)}{p}\left[(p-e) + \ (e-s)\int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{z^*u} tdF(t)dG(u)\right] + \ d'(p)$

$= \frac{-d'(p)}{p}\left[(p-e) + \ (e-s)\int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{z^*u} tdF(t)dG(u) - p\right]$

$= \frac{-d'(p)}{p\ \square}\ (e-s)\left[\frac{-e\square}{e-s} + \ \square\int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{z^*u} tdF(t)dG(u)\right]$

$< \frac{-d'(p)}{p\ \square}\ (e-s)\left[\frac{-(e\square - c)}{e-s} + \ \square\int_{L_\epsilon}^{U_\epsilon}\int_{L_\delta}^{z^*u} tdF(t)dG(u)\right]$ (∵ eμ < -eμ+c)

< 0 (by $A^*(z) > 0$)

5. CONCLUSION AND FUTURE RESEARCH

We consider a price-setting newsvendor model in which a firm needs to make joint inventory and pricing decisions before the selling season. The supply process is uncertain such that the received quantity is the product of the order quantity and a random yield rate.We consider the in-house production case in this model and derive analytical expressions therein. We prove that in the price dependent newsvendor model with random yield (supply), when the in house production set up model is considered, and the demand is modeled in the multiplicative form, the optimal price in the stochastic system is greater than the optimal price in the deterministic system.

For further research we may consider the procurement model wherein the firm pays for the quantity received only, unlike in this case where the firm pays for the order/production quantity. Further, we may consider the additive demand model which is also commonly used in literature.

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