ANALYSIS OF MULTI CYLINDRICAL SHELLS ADAPTED WITH RETAINING WALLS

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ABSTRACT

Retaining walls are used to retain earth or other materials which have the tendency to slide and repose at a particular inclination. They provide lateral support to the embankment or other materials in order to hold them in a vertical position. As against equal forces shells are less thick in comparison with flat slabs, their application is economical regarding consumptive materials, so in this research in order to reduce consumptive materials, multi cylindrical shells are adapted with retaining walls and calculation and analysis of one kind of new lightened retaining wall is proposed. To analyze shell structures, there are different theories. The application of these theories depends on geometrical shape, shell quality, shell usage, boundary condition and enforced loads. To analyze and calculate lightened shell retaining wall, equations are required which should both present the mechanical behavior of the shell with required exactness and also be solvable mathematically. To attain this goal, the method proposed in this research can be applied. In solving the problem formed of multi cylindrical shell with slabs, engineering mechanics combined principles are used and the equations of bending theory of cylindrical shell are adapted with retaining walls. Generally, in this paper the analysis method of lightened retaining walls consisted of multi cylindrical shells are presented.

Keywords: *Multi cylindrical shells, Lightened retaining wall, Shell retaining wall.*

1. INTRODUCTION

Retaining walls are structures used to provide stability for earth or other material where conditions disallow the mass to assume its natural slope and are commonly used to hold back or support soil banks coal or ore piles, and water. Retaining walls also have applications in buildings and bridges such as basement, foundation wall, bridge abutment etc. Retaining walls must be of adequate proportions to resist overturning (or excessive tilting) and sliding as well as being structurally adequate. The analysis and computation of a new kind of structural design of lightened retaining wall is presented in this paper based on design factors resulted from theoretical researches about the previous retaining walls and their lightening methods. As figure 1 illustrates the retaining walls resulted will be considerably economical regarding consumption of material. In other words, for front bearing element of retaining wall, multi cylindrical shells and for base element (foundation), flat slab are taken into consideration. As mentioned, for front bearing element against soil pressure, multi cylindrical concrete shell is used instead of flat concrete slab because shells are more suitable than flat slabs since they are thinner than flat slabs when facing equal pressures, so their use is more economical regarding material consumption.

Using shells instead of flat slabs leads to 15 to 30 percent saving in material usage [1, 2]. Analyzing the structure of shells is often based on two distinct theories which are widely used. First: membrane theory, a membrane either flat or bent is considered as a body that has the same shell shape or two-dimension flexible shell and can resist pressure too. Second: bending theory including flexure effects [3, 4]. Membrane theory can't present an appropriate solution in all real and practical conditions of changing shape. This theory can not either predict stress state in borders and also in other specific regions. By applying bending theory, considering membrane forces, shearing forces, and enforced moments to shell structure these weaknesses can be compensated [5, 6]. In this essay we will calculate and adapt bending theories of cylindrical shells with retaining walls. The materials and results of this research will be limited to cylindrical shells the profile of which is semicircle or circular arch.

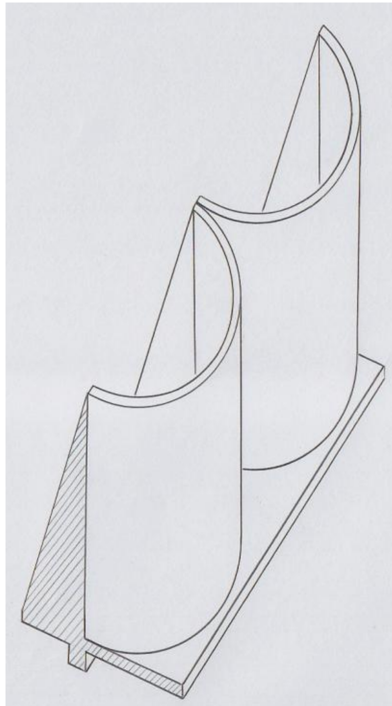

Figure 1. Multi cylindrical shell retaining wall.

2. ANALYZE AND CALCULATE LIGHTENED SHELL RETAINING WALL

One of the main procedures related to designing of shell structure is the selection of geometrical shape and force transfer system design by shell structure. Considering the generalities of structure and the relation of main shell with other elements is one of the important considerations in shell structure designing. This is important because of behavior, balance and general stability of structure specifically the performance of structure against forces. These points are also considered in proposed structure for retaining wall. According to soil lateral force theories, wall shell will be influenced by a triangular load resulted from soil active pressure (Figure 2). Also general proportions (Measures) of proposed retaining wall is regarded the same as figure 3. The connection of wall shell with base slab is assumed rigid. Also the base slab is considered far thicker than cylindrical shells. Further the rigidity of base slab flexure is considered much. The linkage place of cylindrical shell with base slab against torsion has much rigidity. Considering all these factors, the linkage place of base slab with cylindrical shells is rigid and restrained.

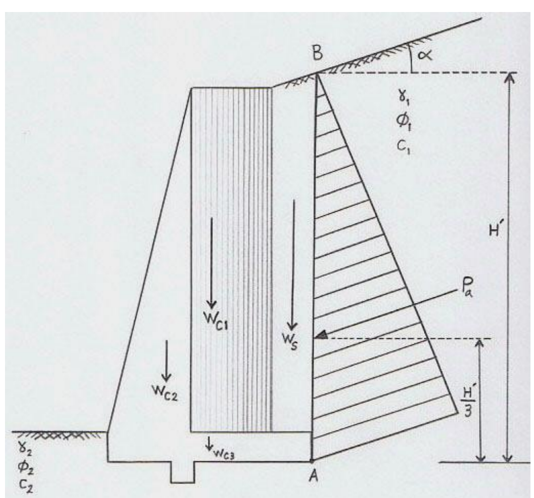

Figure 2. Soil active pressure.

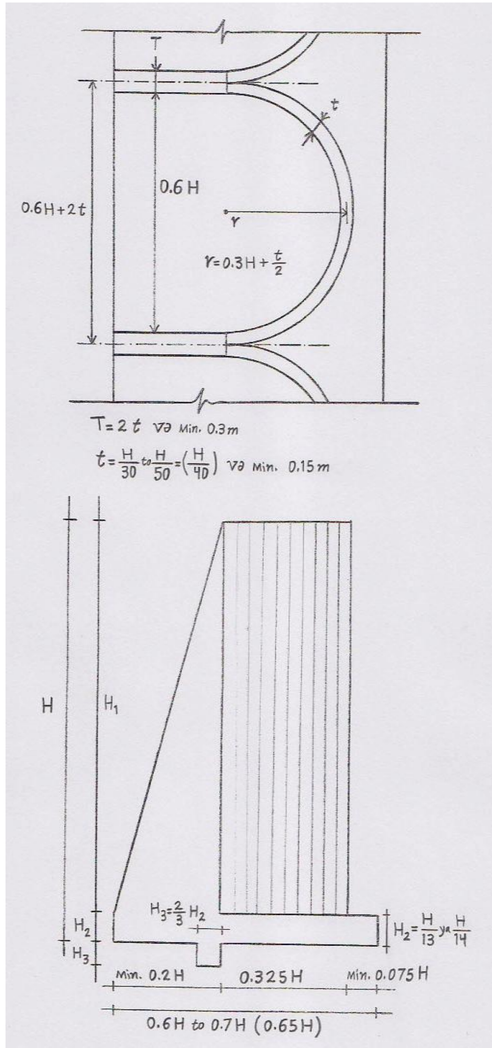

Figure 3. Tentative design dimensions for a multi cylindrical shell retaining wall.

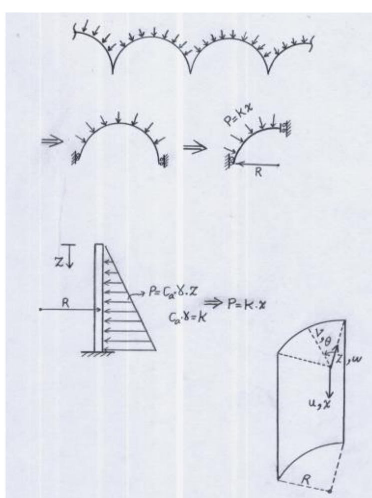

Figure 4. Parameters of multi cylindrical shell retaining wall.

In this section, the suggested cylindrical shell retaining wall without counterfort in classical way calculated, in the next part, both forms, with and without counterfort shell retaining walls, have been calculated through finite element method. Were also applied for the moment analysis of cylindrical shells. It is convenient to write this system of equations in the matrix form:

$$\overline{L}\,\overline{U} = \overline{g} \tag{1}$$

Where:

$$\overline{L} = \begin{bmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & l_{23} \\ l_{31} & l_{32} & l_{33} \end{bmatrix}, \qquad \overline{U} = \begin{Bmatrix} u \\ V \\ w \end{Bmatrix}, \qquad \overline{g} = \frac{1-\nu^2}{Eh} R^2 \begin{Bmatrix} -P_1 \\ -P_2 \\ P_3 \end{Bmatrix}$$

And elements of the differential matrix $\overline{L}$ are:

$$l_{11} = \frac{\partial^2}{\partial \xi^2} + \frac{1-\nu}{2}\frac{\partial^2}{\partial \theta^2}, \qquad l_{12} = l_{21} = \frac{1+\nu}{2}\frac{\partial^2}{\partial \xi \partial \theta}, \qquad l_{13} = l_{31} = -\nu \frac{\partial}{\partial \xi}$$

$$l_{22} = \frac{1-\nu}{2}\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \theta^2} + a^2\left[2(1-\nu)\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \theta^2}\right], \quad l_{23} = l_{32} = -\frac{\partial}{\partial \theta} + a^2\left[(2-\nu)\frac{\partial^3}{\partial \xi^2 \partial \theta} + \frac{\partial^3}{\partial \theta^3}\right]$$

$$l_{33} = 1 + a^2\left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \theta^2}\right)^2$$

Where:

$$\xi = \frac{x}{R}, \qquad a^2 = \frac{h^2}{12R^2}$$

We obtain the following stress resultant-and stress couples-displacements relations for cylindrical shells:

$$N_1 = \frac{Eh}{1-\nu^2}\left[\frac{\partial u}{\partial x} + \frac{\nu}{R}\left(\frac{\partial V}{\partial \theta} - w\right)\right], \qquad N_2 = \frac{Eh}{1-\nu^2}\left[\frac{1}{R}\left(\frac{\partial V}{\partial \theta} - w\right) + \nu\frac{\partial u}{\partial x}\right] \tag{2}$$

$$M_1 = -D\left[\frac{\partial^2 w}{\partial x^2} + \frac{\nu}{R^2}\left(\frac{\partial V}{\partial \theta} + \frac{\partial^2 w}{\partial \theta^2}\right)\right], \quad M_2 = -D\left[\frac{1}{R^2}\left(\frac{\partial V}{\partial \theta} + \frac{\partial^2 w}{\partial \theta^2}\right) + \nu\frac{\partial^2 w}{\partial x^2}\right] \tag{3}$$

Where:

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

Boundary condition is regarded as follows:

$$\begin{aligned} & u = 0 \\ & x = 0 \qquad N_1 = N_2 = M_1 = M_2 = 0 \\ & \theta = 0 \qquad \frac{\partial V}{\partial \theta} = 0, \quad V = 0, \quad \frac{\partial V}{\partial x} = 0 \\ & \theta = \frac{\pi}{2} \qquad \frac{\partial V}{\partial \theta} = 0, \quad w = 0, \quad \frac{\partial w}{\partial x} = 0 \\ & P_1 = P_2 = 0 \end{aligned} \tag{4}$$

$$P_3 = kx$$

$$x = H \qquad \frac{\partial w}{\partial x} = 0 \;,\; \frac{\partial V}{\partial x} = 0 \;,\; \frac{\partial w}{\partial \theta} = 0 \;,\; V = 0 \;,\; w = 0$$

Also:

$$\begin{bmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & l_{23} \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{Bmatrix} 0 \\ V \\ w \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ kx \end{Bmatrix} \qquad \Rightarrow \left| \begin{array}{l} l_{12}V + l_{13}w = 0 \\ l_{22}V + l_{23}w = 0 \\ l_{32}V + l_{33}w = kx \end{array} \right. \tag{5}$$

Therefore the computation of elements of retaining wall by applying the following differential equations and considering border and linkage condition is done:

$$\frac{1+\nu}{2}\frac{\partial^2 V}{\partial \xi \partial \theta} - \nu \frac{\partial w}{\partial \xi} = 0 \tag{6}$$

$$\frac{1-\nu}{2}\frac{\partial^2 V}{\partial \xi^2} + \frac{\partial^2 V}{\partial \theta^2} + a^2\left[2(1-\nu)\frac{\partial^2 V}{\partial \xi^2} + \frac{\partial^2 V}{\partial \theta^2}\right] - \frac{\partial\ w}{\partial \theta} + a^2\left[(2-\nu)\frac{\partial^3 w}{\partial \xi^2 \partial \theta} + \frac{\partial^3 w}{\partial \theta^3}\right] = 0 \tag{7}$$

$$-\frac{\partial\ V}{\partial \theta} + a^2\left[(2-\nu)\frac{\partial^3 V}{\partial \xi^2 \partial \theta} + \frac{\partial^3 V}{\partial \theta^3}\right] + w + a^2\left(\frac{\partial^2 w}{\partial \xi^2} + \frac{\partial^2 w}{\partial \theta^2}\right)^2 = kx \tag{8}$$

The solving of main differential equation of the bending theory of multi cylindrical shells by considering accepted border conditions is selected in the next part through finite element method. So the problem of combining base slab with multi cylindrical shells are solved.

In the next part, displacement and internal forces created in cylindrical shells are calculated through finite element method by using specific instructions [7, 8, 9, 10, 11].

3. NUMERICAL METHOD OF SOLVING PROBLEM

Numerical examples of solving problem by choosing numerical values for physical and geometrical parameters of cylindrical shells are made. The numerical methods, usually the finite element method, discussed in this section (ANSYS computer program as shown in Figure 5). Computations are represented for the following figure geometrical quantities (Figure 6). The shells thickness are determined as follows in case cylindrical shells are thin:

$$t = \frac{H}{50} \text{ to } \frac{H}{30}, \qquad t_{min.} = 150 \text{ mm}$$

Figure 5. Structure elements for the finite element method (with and without counterfort shell retaining walls).

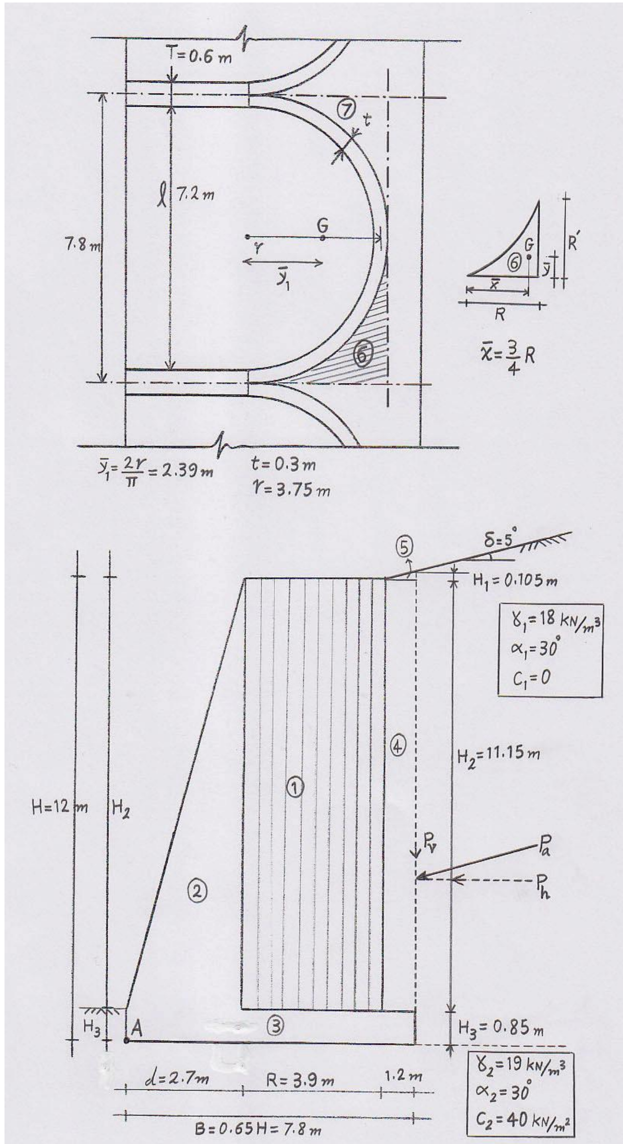

Figure 6. Geometrical parameters of cylindrical shell retaining wall.

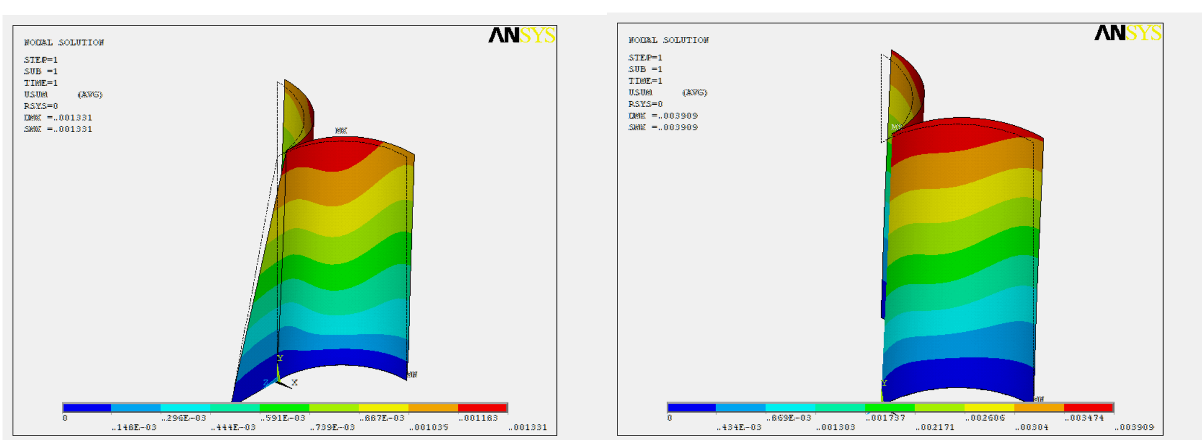

Figure 7. Displacement of cylindrical shell retaining wall by the method of finite element(with and without counterfort shell retaining walls).

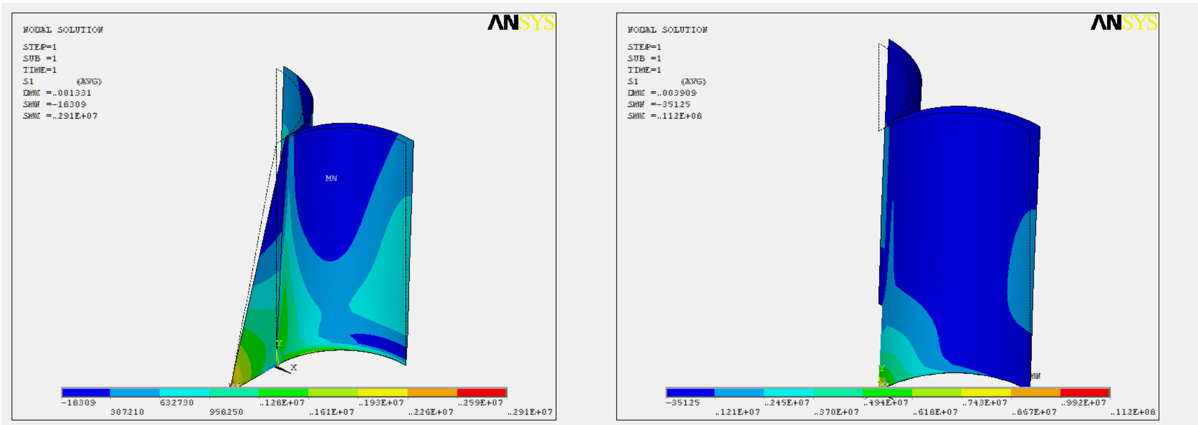

Figure 8. Stress of cylindrical shell retaining wall by the method of finite element (with and without counterfort shell retaining walls).

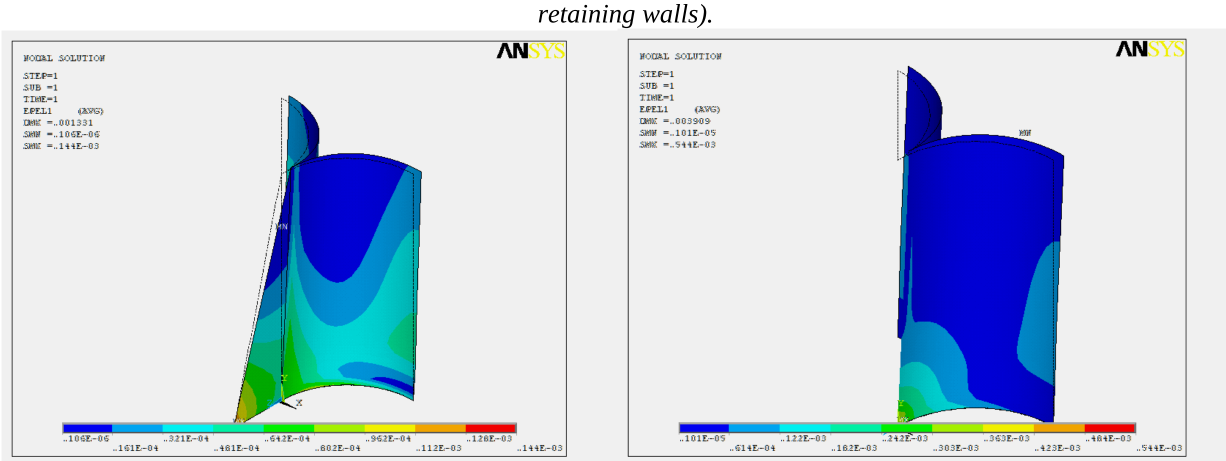

Figure 9. Deflection of cylindrical shell retaining wall by the method of finite element (with and without counterfort shell retaining walls).

Based on this point the amount of bending moment (M_Y) and normal forces (N_Θ) created in the following table is presented:

Table 1. Normal forces and bending moment of counterfort shell retaining wall.

Y (m)	N_Θ (Kg/m)	M_Y (Kg-m/m)
0.00	0.00	2037.7979
1.00	15610.6544	575.2258
2.00	18611.0129	162.20103
3.00	17890.7735	45.68599
4.00	16073.743	12.85288
5.00	13933.6601	3.61142
6.00	11698.5007	1.01340
7.00	9435.3813	0.28397
8.00	7164.0456	0.07945
9.00	4890.2973	0.02219
10.00	2615.8408	0.00618
11.15	-0.02958	0.00142

In this way the forces resulted from lightened retaining walls are calculated.
The reactions resulted from earth filling of retaining wall is also calculated on the base of specific relations resulted from material strength [12, 13, 14]. Here *M* is the bending moment resulted from executed loads effect on retaining

wall that is from center of gravity of retaining wall; G_1 is the soil gravity, q_0 the effect of effective active soil on retaining wall, b the height of retaining wall, b_1 the width of base slab, A the area of base slab to the with of 1 meter, W_x resistant moment [12, 13, 14].

$$P_{\frac{max}{min}} = -\frac{G_1}{A_x} \pm \frac{M}{W_x} \tag{9}$$

$$M = -\frac{q_0 b^2}{6} + \frac{G_1 . b_1}{6} \tag{10}$$

$$A = 1.b_1 \quad , \quad W_x = \frac{1.b_1^2}{6} \tag{11}$$

The general stability of lightened shell retaining walls are studied.

5. CONCLUSION

The results of this research are:

1. Studying the amounts resulted for consumptive concrete volume of gravity retaining walls and lightened shell retaining wall show that the use of counterfort shell retaining walls are considerably economical in consumption of concrete and lead to about 70-75% in comparison with concrete gravity retaining walls. Also, the use of counterfort shell retaining walls are considerably economical in consumption of concrete and lead to about 27-30% in comparison with concrete cantilever retaining walls. Also, the use of counterfort shell retaining walls are considerably economical in consumption of concrete and lead to about 14-16% in comparison with concrete counterfort retaining walls. So it is more economical to use cylindrical shell retaining walls.
2. The research the results of which are presented in this paper, for the first time brings about a method to calculate and analyze lightened shell retaining wall. Also, for the first time cylindrical shells are adapted with retaining walls in the proposed shape and form and numerical example is stated as well.
3. The resulted equations for shell retaining wall with required exactness represent the mechanical behavior of the shell, and it will show itself in the results of stresses, deflections functions, and also in forces and bending moments calculation.
4. The use of shells in front bearing element of lightened retaining wall instead of level slabs will be more economical from consumptive point of view, as the shell curve increases bearing rigidity and capacity of the shell and some of materials will go far away from neutral axel, so the flexural rigidity of shell will increase a lot. Also through increasing base slab thickness, the computation about it will be considered the same as complete rigid slab which is completely obvious in the early computations.

6. ACKNOWLEDGEMENTS

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