COMMON FIXED POINT THEOREMS IN FUZZY METRIC SPACE USING COMPATIBLE MAPPINGS OF TYPE (A-1) OR (A-2)

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ABSTRACT
In this paper, the concept of compatible maps of type (A-1) or type (A-2) in fuzzy metric space has been introduced to prove common fixed point theorems which generalize the results of Cho [1]. We also cited an example in support of our result.

Keywords: *Common fixed points, fuzzy metric space, compatible maps, compatible maps of type (A-1), compatible mappings of type (A-2).*
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1. INTRODUCTION
The concept of Fuzzy sets was initially investigated by Zadeh [13] as a new way to represent vagueness in everyday life. Subsequently, it was developed by many authors and used in various fields. To use this concept in Topology and Analysis, several researchers have defined Fuzzy metric space in various ways. In this paper we deal with the Fuzzy metric space defined by Kramosil and Michalek [8] and modified by George and Veeramani [4]. Recently, Grabiec [5] has proved fixed point results for Fuzzy metric space. In the sequel, Singh and Chauhan [10] introduced the concept of compatible mappings in Fuzzy metric space and proved the common fixed point theorem. Jungck et. al. [6] introduced the concept of compatible maps of type (A) in metric space and proved fixed point theorems. Cho [2, 3] introduced the concept of compatible maps of type (α) and compatible maps of type (β) in fuzzy metric space. In 2011, using the concept of compatible maps of type (A) and type (β), Singh et. al. [11, 12] proved fixed point theorems in a fuzzy metric space.
In this paper, a fixed point theorem for six self maps has been established using the concept of compatible maps of type (A-1) or (A-2) which generalizes the result of Cho [1].
For the sake of completeness, we recall some definitions and known results in Fuzzy metric space.

2. PRELIMINARIES
Definition 2.1. [9] A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is called a *t-norm* if $([0, 1], *)$ is an abelian topological monoid with unit 1 such that $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for $a, b, c, d \in [0, 1]$.
Examples of t-norms are $a * b = ab$ and $a * b = \min\{a, b\}$.

Definition 2.2. [9] The 3-tuple (X, M, *) is said to be a *Fuzzy metric space* if X is an arbitrary set, * is a continuous t-norm and M is a Fuzzy set in $X^2 \times [0, \infty)$ satisfying the following conditions :
for all $x, y, z \in X$ and $s, t > 0$.
(FM-1) $M(x, y, 0) = 0$,
(FM-2) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$,
(FM-3) $M(x, y, t) = M(y, x, t)$,
(FM-4) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$,
(FM-5) $M(x, y, .) : [0, \infty) \to [0, 1]$ is left continuous,
(FM-6) $\lim_{t\to\infty} M(x, y, t) = 1$.
Note that M(x, y, t) can be considered as the degree of nearness between x and y with respect to t. We identify x = y with M(x, y, t) = 1 for all t > 0. The following example shows that every metric space induces a Fuzzy metric space.

Example 2.1. [9] Let (X, d) be a metric space. Define $a * b = \min\{a, b\}$ and $M(x, y, t) = \frac{t}{t + d(x, y)}$ for all $x, y \in X$ and all $t > 0$. Then (X, M, *) is a Fuzzy metric space. It is called the Fuzzy metric space induced by d.

Definition 2.3. [9] A sequence $\{x_n\}$ in a Fuzzy metric space (X, M, *) is said to be a *Cauchy sequence* if and only if for each $\varepsilon > 0$, $t > 0$, there exists $n_0 \in N$ such that $M(x_n, x_m, t) > 1 - \varepsilon$ for all $n, m \geq n_0$.
The sequence $\{x_n\}$ is said to *converge* to a point x in X if and only if for each $\varepsilon > 0$, $t > 0$ there exists $n_0 \in N$ such that $M(x_n, x, t) > 1 - \varepsilon$ for all $n \geq n_0$.
A Fuzzy metric space (X, M, *) is said to be *complete* if every Cauchy sequence in it converges to a point in it.

Definition 2.4. [10] Self mappings A and S of a Fuzzy metric space (X, M, *) are said to be *compatible* if and only if $M(ASx_n, SAx_n, t) \to 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $Sx_n, Ax_n \to p$ for some p in X as $n \to \infty$.

Definition 2.5. [12] Self maps A and B of a Fuzzy metric space (X, M, *) are said to be *compatible maps of type (A)* if $M(ABx_n, BBx_n, t) \to 1$ and $M(BAx_n, AAx_n, t) \to 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $Ax_n, Bx_n \to z$ for some z in X as $n \to \infty$.

Definition 2.6. Self maps A and B of a Fuzzy metric space (X, M, *) are said to be *compatible maps of type (A-1)* if $M(ABx_n, BBx_n, t) \to 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $Ax_n, Bx_n \to z$ for some z in X as $n \to \infty$.

Definition 2.7. Self maps A and B of a Fuzzy metric space (X, M, *) are said to be *compatible maps of type (A-2)* if $M(BAx_n, AAx_n, t) \to 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $Ax_n, Bx_n \to z$ for some z in X as $n \to \infty$.

Definition 2.8. [3] Self maps A and B of a Fuzzy metric space (X, M, *) are said to be *compatible maps of type β* if $M(AAx_n, BBx_n, t) \to 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $Ax_n, Bx_n \to z$ for some z in X as $n \to \infty$.

Example 2.2. Let (X, M, *) be a Fuzzy metric space where X = [0, 2]. t-norm is defined by a * b = min {a, b} for all $a, b \in [0, 1]$ and $M(x, y, t) = \dfrac{t}{t + |x - y|}$ for all $x, y \in X$. Define self maps A and S on X as follows :

$$Ax = \begin{cases} 2 - x & \text{if} \quad 0 \leq x < 1 \\ 2 & \text{if} \quad 1 \leq x \leq 2 \end{cases} \quad \text{and} \quad Sx = \begin{cases} x & \text{if} \quad 0 \leq x < 1 \\ 2 & \text{if} \quad 1 \leq x \leq 2 \end{cases}.$$

Taking $x_n = 1 - \dfrac{1}{n}$.

Then $Ax_n, Sx_n \to 1$ as $n \to \infty$.

Now, $\lim_{n\to\infty} M(ASx_n, SAx_n, t) \neq 1$.

Hence the pair (A, S) is not compatible.

Also, $\lim_{n\to\infty} M(AAx_n, SSx_n, t) \neq 1$, hence the pair (A, S) is not compatible of type (β).

But, $\lim_{n\to\infty} M(ASx_n, SSx_n, t) = 1$, hence the pair (A, S) is compatible of type (A-1). Similarly, the pair (A, S) is compatible of type (A-2).

Remark 2.1. From the above example, it is clear that the compatible mappings of type (β) is compatible mapping of type (A-1) or (A-2) but converse is not true in general.

Remark 2.2. Clearly, if a pair of mapping (A, S) is compatible of type (A-1) then the pair (S, A) is compatible of type (A-2). Further if A and S are compatible mappings of type A then the pair (A, S) is compatible of type (A-1) as well as (A-2).

Proposition 2.1. Suppose A and B be self maps of a Fuzzy metric space (X, M, *) with continuous t-norm * and $t * t \geq t$ for all $t \in [0, 1]$.

(i) B is continuous then the pair (A, B) is compatible of type (A-1) if and only if A and B are compatible.
(ii) A is continuous then the pair (A, B) is compatible of type (A-2) if and only if A and B are compatible.

Proof. (i) Let $\{x_n\}$ be a sequence in X such that $Ax_n, Bx_n \to z$ for some z in X as $n \to \infty$ and suppose the pair (A, B) is compatible of type (A-1). Since B is continuous, we have $BAx_n \to Bz$ and $BBx_n \to Bz$ then by (FM-4), we have

$$M(ABx_n, BAx_n, t) \geq M(ABx_n, BBx_n, t/2) * M(BBx_n, BAx_n, t/2)$$
$$\to 1 * 1 \geq 1 \text{ as } n \to \infty.$$

Hence, the mappings A and B are compatible.
Now, suppose A and B are compatible mappings. Therefore, using continuity of B, we have

$$M(ABx_n, BBx_n, t) \geq M(ABx_n, BAx_n, t/2) * M(BAx_n, BBx_n, t/2)$$
$$\to 1 * 1 \geq 1 \text{ as } n \to \infty.$$

Hence, the mappings A and B are compatible of type (A-1).
(ii) It is similar to proof of (i).
Now, we give some properties of compatible mappings of type (A-1) and type (A-2) which will be used in our main theorem.

Proposition 2.2. Let A and B be self maps of a Fuzzy metric space (X, M, *). If the pair (A, B) is compatible of type (A-1) and Az = Bz for some z in X then ABz = BBz.

Proof. Let $\{x_n\}$ be a sequence in X defined by $x_n = z$ for n = 1, 2, 3, ... and let Az = Bz then we have $Ax_n \to Az$ and $Bx_n \to Bz$. Since the pair (A, B) is compatible of type (A-1), we have

$$M(ABz, BBz, t) = M(ABx_n, BBx_n, t) \to 1 \text{ as } n \to \infty.$$

Hence, ABz = BBz.

Proposition 2.3. Let A and B be self maps of a Fuzzy metric space (X, M, *). If the pair (A, B) is compatible of type (A-2) and Az = Bz for some z in X then BAz = AAz.

Proof. It is similar to the proof of proposition 2.2.

Proposition 2.4. Let A and B be self maps of a Fuzzy metric space (X, M, *) with continuous t-norm * and $t * t \geq t$ for all $t \in [0, 1]$. If the pair (A, B) is compatible of type (A-1) and $\{x_n\}$ is a sequence in X such that $Ax_n, Bx_n \to z$ for some z in X as $n \to \infty$ then $BBx_n \to Az$ if A is continuous at z.

Proof. Since A is continuous at z and the pair (A, B) is compatible of type (A-1), we have

$$ABx_n \to Az \quad \text{and} \quad M(ABx_n, BBx_n, t) \to 1 \text{ as } n \to \infty.$$

Therefore,

$$M(Az, BBx_n, t) \geq M(Az, ABx_n, t/2) * M(ABx_n, BBx_n, t/2)$$
$$\to 1 * 1 \geq 1 \text{ as } n \to \infty.$$

Hence, $BBx_n \to Az$ as $n \to \infty$.

Proposition 2.5. Let A and B be self maps of a Fuzzy metric space (X, M, *) with continuous t-norm * and $t * t \geq t$ for all $t \in [0, 1]$. If the pair (A, B) is compatible of type (A-2) and $\{x_n\}$ is a sequence in X such that $Ax_n, Bx_n \to z$ for some z in X as $n \to \infty$ then $AAx_n \to Bz$ if B is continuous at z.

Proof. It is similar to the proof of proposition 2.4.

Proposition 2.6. [12] In a fuzzy metric space (X, M, *) limit of a sequence is unique.

Lemma 2.1. [5] Let (X, M, *) be a fuzzy metric space. Then for all $x, y \in X$, M(x, y, .) is a non-decreasing function.

Lemma 2.2. [1] Let (X, M, *) be a fuzzy metric space. If there exists $k \in (0, 1)$ such that for all $x, y \in X$

$$M(x, y, kt) \geq M(x, y, t) \ \forall \ t > 0$$

then $x = y$.

Lemma 2.3. [12] Let $\{x_n\}$ be a sequence in a fuzzy metric space (X, M, *). If there exists a number $k \in (0, 1)$ such that

$$M(x_{n+2}, x_{n+1}, kt) \geq M(x_{n+1}, x_n, t) \ \forall \ t > 0 \text{ and } n \in N.$$

Then $\{x_n\}$ is a Cauchy sequence in X.

Lemma 2.4.[7] The only t-norm * satisfying $r * r \geq r$ for all $r \in [0, 1]$ is the minimum t-norm, that is

$$a * b = \min\{a, b\} \text{ for all } a, b \in [0, 1].$$

3. MAIN RESULT

Theorem 3.1. Let (X, M, *) be a complete Fuzzy metric space with continuous t-norm * and $t * t \geq t$, for all $t \in [0, 1]$ and let A, B, S, T, P and Q be mappings from X into itself such that the following conditions are satisfied:

(a) $P(X) \subset ST(X)$, $Q(X) \subset AB(X)$;

(b) AB = BA, ST = TS, PB = BP, QT = TQ;

(c) either P or AB is continuous;

(d) (P, AB) and (Q, ST) are compatible of type (A-1) or type (A-2);

(e) there exists $q \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$

$$M(Px, Qy, qt) \geq M(ABx, STy, t) * M(Px, ABx, t) * M(Qy, STy, t) * M(Px, STy, t).$$

Then A, B, S, T, P and Q have a unique common fixed point in X.

Proof : Let $x_0 \in X$. From (a) there exist $x_1, x_2 \in X$ such that

$$Px_0 = STx_1 \quad \text{and} \quad Qx_1 = ABx_2 .$$

Inductively, we can construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$Px_{2n-2} = STx_{2n-1} = y_{2n-1} \quad \text{and}$$
$$Qx_{2n-1} = ABx_{2n} = y_{2n} \quad \text{for } n = 1, 2, 3, \ldots .$$

Step 1. Put $x = x_{2n}$ and $y = x_{2n+1}$ in (e), we get

$$\begin{aligned} M(Px_{2n}, Qx_{2n+1}, qt) &\geq M(ABx_{2n}, STx_{2n+1}, t) * M(Px_{2n}, ABx_{2n}, t) \\ &\quad * M(Qx_{2n+1}, STx_{2n+1}, t) * M(Px_{2n}, STx_{2n+1}, t). \\ &= M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n}, t) \\ &\quad * M(y_{2n+2}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+1}, t)\} \\ &\geq M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, t). \end{aligned}$$

From lemma 2.4, we have

$$M(y_{2n+1}, y_{2n+2}, qt) \geq M(y_{2n}, y_{2n+1}, t).$$

Similarly, we have

$$M(y_{2n+2}, y_{2n+3}, qt) \geq M(y_{2n+1}, y_{2n+2}, t).$$

Thus, we have

$$\begin{aligned} M(y_{n+1}, y_{n+2}, qt) &\geq M(y_n, y_{n+1}, t) \text{ for } n = 1, 2, \ldots \\ M(y_n, y_{n+1}, dt) &\geq M(y_n, y_{n+1}, t/q) \\ &\geq M(y_{n-2}, y_{n-1}, t/q^2) \\ &\quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ &\geq M(y_1, y_2, t/q^n) \to 1 \text{ as } n \to \infty, \end{aligned}$$

and hence $M(y_n, y_{n+1}, t) \to 1$ as $n \to \infty$ for any $t > 0$.

For each $\varepsilon > 0$ and $t > 0$, we can choose $n_0 \in N$ such that

$$M(y_n, y_{n+1}, t) > 1 - \varepsilon \quad \text{for all } n > n_0.$$

For $m, n \in N$, we suppose $m \geq n$. Then we have

$$M(y_n, y_m, t) \geq M(y_n, y_{n+1}, t/m\text{-}n) * M(y_{n+1}, y_{n+2}, t/m\text{-}n) * \ldots * M(y_{m-1}, y_m, t/m\text{-}n)$$
$$\geq (1 - \varepsilon) * (1 - \varepsilon) * \ldots * (1 - \varepsilon) \ (m - n) \text{ times}$$
$$\geq (1 - \varepsilon)$$

and hence $\{y_n\}$ is a Cauchy sequence in X.

Since (X, M, *) is complete, $\{y_n\}$ converges to some point $z \in X$. Also its subsequences converges to the same point i.e. $z \in X$

i.e., $\{Qx_{2n+1}\} \to z$ and $\{STx_{2n+1}\} \to z$ (1)

$\{Px_{2n}\} \to z$ and $\{ABx_{2n}\} \to z$ (2)

Case I. Suppose AB is continuous.

Since AB is continuous, we have

$$(AB)^2x_{2n} \to ABz \text{ and}$$
$$ABPx_{2n} \to ABz.$$

As (P, AB) is compatible pair of type (A-1), then by proposition (2.4) we have

$$PPx_{2n} \to ABz.$$

Step 2. Put $x = Px_{2n}$ and $y = x_{2n+1}$ in (e), we get

$$M(PPx_{2n}, Qx_{2n+1}, qt) \geq M(ABPx_{2n}, STx_{2n+1}, t) * M(PPx_{2n}, ABPx_{2n}, t)$$
$$* M(Qy, STx_{2n+1}, t) * M(PPx_{2n}, STx_{2n+1}, t).$$

Taking $n \to \infty$, we get

$$M(ABz, z, qt) \geq M(ABz, z, t) * M(ABz, ABz, t) * M(z, z, t) * M(ABz, z, t)$$

i.e. $M(ABz, z, qt) \geq M(ABz, z, t)$.

Therefore, by using lemma 2.2, we get

$$ABz = z. \quad (3)$$

Step 3. Put $x = z$ and $y = x_{2n+1}$ in (e), we have

$$M(Pz, Qx_{2n+1}, qt) \geq M(ABz, STx_{2n+1}, t) * M(Pz, ABz, t) * M(Qx_{2n+1}, STx_{2n+1}, t) * M(Pz, STx_{2n+1}, t).$$

Taking $n \to \infty$ and using equation (1), we get

$$M(Pz, z, qt) \geq M(z, z, t) * M(Pz, z, t) * M(z, z, t) * M(Pz, z, t)$$
$$\geq M(Pz, z, t) * M(Pz, z, t)$$

i.e. $M(Pz, z, qt) \geq M(Pz, z, t)$.

Therefore, by using lemma 2.2, we get

$$Pz = z.$$

Therefore, $ABz = Pz = z$.

Step 4. Putting $x = Bz$ and $y = x_{2n+1}$ in condition (e), we get

$$M(PBz, Qx_{2n+1}, qt) \geq M(ABBz, STx_{2n+1}, t) * M(PBz, ABBz, t) * M(Qx_{2n+1}, STx_{2n+1}, t) * M(PBz, STx_{2n+1}, t)$$

As BP = PB, AB = BA, so we have

$$P(Bz) = B(Pz) = Bz \text{ and } (AB)(Bz) = (BA)(Bz) = B(ABz) = Bz.$$

Taking $n \to \infty$ and using (1), we get

$$M(Bz, z, qt) \geq M(Bz, z, t) * M(Bz, Bz, t) * M(z, z, t) * M(Bz, z, t)$$
$$\geq M(Bz, z, t) * M(Bz, z, t)$$

i.e. $M(Bz, z, qt) \geq M(Bz, z, t)$.

Therefore, by using lemma 2.2, we get

$$Bz = z$$

and also we have

$ABz = z$
$\Rightarrow \quad Az = z.$
Therefore, $Az = Bz = Pz = z.$ (4)

Step 5. As $P(X) \subset ST(X)$, there exists $u \in X$ such that
$$z = Pz = STu.$$

Putting $x = x_{2n}$ and $y = u$ in (e), we get
$M(Px_{2n}, Qu, qt) \geq M(ABx_{2n}, STu, t) * M(Px_{2n}, ABx_{2n}, t) * M(Qu, STu, t) * M(Px_{2n}, STu, t).$
Taking $n \to \infty$ and using (1) and (2), we get
$$M(z, Qu, qt) \geq M(z, z, t) * M(z, z, t) * M(Qu, z, t) * M(z, z, t)$$
$$\geq M(Qu, z, t)$$
i.e. $M(z, Qu, qt) \geq M(z, Qu, t).$
Therefore, by using lemma 2.2, we get
$Qu = z.$
Hence $STu = z = Qu.$
Since (Q, ST) is compatible of type (A-1), then by proposition (2.2), we have
$$Q(ST)u = ST(ST)u.$$
Thus $Qz = STz.$

Step 6. Putting $x = x_{2n}$ and $y = z$ in (e), we get
$$M(Px_{2n}, Qz, qt) \geq M(ABx_{2n}, STz, t) * M(Px_{2n}, ABx_{2n}, t) * M(Qz, STz, t) * M(Px_{2n}, STz, t).$$
Taking $n \to \infty$ and using (2) and step 5, we get
$$M(z, Qz, qt) \geq M(z, Qz, t) * M(z, z, t) * M(Qz, Qz, t) * M(z, Qz, t)$$
$$\geq M(z, Qz, t) * M(z, Qz, t)$$
i.e. $M(z, Qz, qt) \geq M(z, Qz, t).$
Therefore, by using lemma 2.2, we get
$Qz = z.$

Step 7. Putting $x = x_{2n}$ and $y = Tz$ in (e), we get
$$M(Px_{2n}, QTz, qt) \geq M(ABx_{2n}, STTz, t) * M(Px_{2n}, ABx_{2n}, t) * M(QTz, STTz, t) * M(Px_{2n}, STTz, t).$$
As $QT = TQ$ and $ST = TS$, we have
$$QTz = TQz = Tz \quad \text{and} \quad ST(Tz) = T(STz) = TQz = Tz.$$
Taking $n \to \infty$, we get
$$M(z, Tz, qt) \geq M(z, Tz, t) * M(z, z, t) * M(Tz, Tz, t) * M(z, Tz, t)$$
$$\geq M(z, Tz, t) * M(z, Tz, t)$$
i.e. $M(z, Tz, qt) \geq M(z, Tz, t).$
Therefore, by using lemma 2.2, we get
$Tz = z.$
Now $STz = Tz = z$ implies $Sz = z.$
Hence $Sz = Tz = Qz = z.$ (5)
Combining (4) and (5), we get
$$Az = Bz = Pz = Qz = Tz = Sz = z.$$
Hence, z is the common fixed point of A, B, S, T, P and Q.
"Similarly, it is clear that z is also the common fixed point of A, B, S, T, P and Q when AB is continuous and (P, AB) and (Q, ST) are compatible of type (A-2)."

Case II. Suppose P is continuous.
As P is continuous,
$$P^2x_{2n} \to Pz \quad \text{and} \quad P(AB)x_{2n} \to Pz.$$
As (P, AB) is compatible pair of type (A-1), then by proposition (2.4), we have
$$AB(AB)x_{2n} \to Pz.$$

Step 8. Putting $x = ABx_{2n}$ and $y = x_{2n+1}$ in condition (e), we have

$$M(PABx_{2n}, Qx_{2n+1}, qt) \geq M(ABABx_{2n}, STx_{2n+1}, t) * M(PABx_{2n}, ABABx_{2n}, t) * M(Qx_{2n+1}, STx_{2n+1}, t) * M(PABx_{2n}, STx_{2n+1}, t).$$

Taking $n \to \infty$, we get

$$M(Pz, z, qt) \geq M(Pz, z, t) * M(Pz, Pz, t) * M(z, z, t) * M(Pz, z, t)$$

i.e. $M(Pz, z, qt) \geq M(Pz, z, t)$.

Therefore by using lemma 2.2, we have

$$Pz = z.$$

Further, using steps 5, 6, 7, we get

$$z = Qz = STz = Sz = Tz.$$

Step 9. As $Q(X) \subset AB(X)$, there exists $u \in X$ such that

$$z = Qz = ABu.$$

Putting $x = u$ and $y = x_{2n+1}$ in (e), we get

$$M(Pu, Qx_{2n+1}, qt) \geq M(ABu, STx_{2n+1}, t) * M(Pu, ABu, t) * M(Qx_{2n+1}, STx_{2n+1}, t) * M(Pu, STx_{2n+1}, t).$$

Taking $n \to \infty$, we get

$$M(Pu, z, qt) \geq M(z, z, t) * M(Pu, z, t) * M(z, z, t) * M(Pu, z, t)$$

i.e. $M(Pu, z, qt) \geq M(Pu, z, t)$.

Therefore, by using lemma 2.2, we get

$$Pu = z.$$

Since $z = Qz = ABu$, so $Pu = ABu$.

Since (P, AB) is compatible of type (A-1), so by proposition (2.2), we have

$$Pz = ABz.$$

Also, $z = Bz$ follows from step 4. Thus, $z = Az = Bz = Pz$.

Therefore, $z = Az = Bz = Pz = Qz = Sz = Tz$,

i.e. z is the common fixed point of the six maps A, B, S, T, P and Q in this case also.

"Similarly, it is clear that z is also the common fixed point of A, B, S, T, P and Q when P is continuous and (P, AB) and (Q, ST) are compatible of type (A-2)."

Uniqueness : Let u be another common fixed point of A, B, S, T, P and Q.

Then $Au = Bu = Pu = Qu = Su = Tu = u$.

Put $x = z$ and $y = u$ in (e), we get

$$M(Pz, Qu, qt) \geq M(ABz, STu, t) * M(Pz, ABz, t) * M(Qu, STu, t) * M(Pz, STu, t)$$

Taking $n \to \infty$, we get

$$M(z, u, qt) \geq M(z, u, t) * M(z, z, t) * M(u, u, t) * M(z, u, t) \geq M(z, u, t) * M(z, u, t)$$

i.e. $M(z, u, qt) \geq M(z, u, t)$.

Therefore by using lemma 2.2, we get

$$z = u.$$

Therefore z is the unique common fixed point of self maps A, B, S, T, P and Q.

Remark 3.1. If we take $B = T = I$, the identity map on X in theorem 3.1, then condition (b) is satisfied trivially and we get

Corollary 3.1. Let (X, M, *) be a complete Fuzzy metric space with continuous t-norm * and $t * t \geq t$, for all $t \in [0, 1]$ and let A, S, P and Q be mappings from X into itself such that the following conditions are satisfied:

(a) $P(X) \subset S(X)$, $Q(X) \subset A(X)$;

(b) either A or P is continuous;

(c) (P, A) and (Q, S) are compatible of type (A-1) or (A-2);

(d) there exists $q \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$

$$M(Px, Qy, qt) \geq M(Ax, Sy, t) * M(Px, Ax, t) * M(Qy, Sy, t) * M(Px, Sy, t).$$

Then A, S, P and Q have a unique common fixed point in X.

Remark 3.2. In view of remark 3.1, corollary 3.1 is a generalization of the result of Cho [1] in the sense that condition of compatibility of the pairs of self maps has been restricted to compatibility of type (A-1) or (A-2) and only one map of the first pair is needed to be continuous.
Now we give an example in support of our result.

Example 3.1. Let (X, M, *) be a complete Fuzzy metric space where X = [0, 1] and t-norm is defined by a*b = min{a, b} for all a, b $\in$ X and $M(x,y,t) = \frac{t}{t+|x-y|}$ for all x, y $\in$ X and t > 0. Let A, B, P, Q, S and T be mappings from X into itself is defined as :

$$Ax = \frac{x}{5},\quad Bx = \frac{x}{3},\quad Px = \frac{x}{6},\ Qx = 0,\ Sx = x,\ Tx = \frac{x}{2} \quad \text{for all } x \in X.$$

Clearly, conditions (a), (b), (c) and (e) of theorem 3.1 are satisfied.
Infact, if $\lim_{n\to\infty} x_n = 0$, where $\{x_n\}$ is a sequence in X.
Also, (P, AB) and (Q, ST) are compatible of type (A-1) or type (A-2).

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