EXACT TRAVELLING WAVE SOLUTIONS FOR THREE NONLINEAR EVOLUTION EQUATIONS BY A BERNOULLI SUB-ODE METHOD

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ABSTRACT
In this paper, based on a proposed Bernoulli sub-ODE method and with the aid of mathematical software Maple, we derive some new exact travelling wave solutions for the (3+1) dimensional Jimbo-Miva equation, SRLW equation and generalized Burgers equation. These solutions are of new forms so far in the literature.

Keywords: *Bernoulli sub-ODE method, Travelling wave solutions, Exact solution, Evolution equation, (3+1) dimensional Jimbo-Miva equation, SRLW equation, Generalized Burgers equation*

1. INTRODUCTION

The nonlinear phenomena exist in all the fields including either the scientific work or engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics and so on. It is well known that many nonlinear evolution equations (NLEEs) are widely used to describe these complex phenomena. Research on solutions of NLEEs is popular. So, the powerful and efficient methods to find analytic solutions and numerical solutions of nonlinear equations have drawn a lot of interest by a diverse group of scientists. Many efficient methods have been presented so far.

During the past few decades searching for explicit solutions of nonlinear evolution equations by using various different methods have been the main goal for many researchers, and many powerful methods for constructing exact solutions of nonlinear evolution equations have been established and developed such as the inverse scattering transform, the Darboux transform, the tanh-function expansion and its various extension, the Jacobi elliptic function expansion, the homogeneous balance method, the sine-cosine method, the rank analysis method, the exp-function expansion method and so on [1-13].

In this paper, we propose a Bernoulli sub-ODE method to construct exact travelling wave solutions for NLEES.

The rest of the paper is organized as follows. In Section 2, we describe the Bernoulli sub-ODE method for finding travelling wave solutions of nonlinear evolution equations, and give the main steps of the method. In the subsequent sections, we will apply the Bernoulli sub-ODE method to find exact travelling wave solutions for the (3+1) dimensional Jimbo-Miva equation, SRLW equation, and generalized Burgers equation. In the last section, some conclusions are presented.

2. DESCRIPTION OF THE BERNOULLI SUB-ODE METHOD

In this section we present the solutions of the following ODE:

$$G'+\lambda G=\mu G^2 \quad (1)$$

where $\lambda \neq 0, G=G(\xi)$.

When $\mu \neq 0$, Eq. (1) is the type of Bernoulli equation, and we can obtain the solution as

$$G=\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}} \quad (2)$$

where d is an arbitrary constant.

Suppose that a nonlinear equation, say in two or three independent variables x, y and t, is given by

$$P(u,u_t,u_x,u_y,u_{tt},u_{xt},u_{yt},u_{xx},u_{yy}......)=0 \quad (3)$$

where $u=u(x,y,t)$ is an unknown function, P is a polynomial in $u=u(x,y,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved.

Step 1.We suppose that

$$u(x,y,t)=u(\xi), \xi=\xi(x,y,t) \quad (4)$$

The travelling wave variable (4) permits us reducing Eq. (3) to an ODE for $u=u(\xi)$

$$P(u, u', u'', \ldots) = \ldots \quad (5)$$

Step 2. Suppose that the solution of (5) can be expressed by a polynomial in G as follows:

$$u(\xi)=\alpha_m G^m+\alpha_{m-1}G^{m-1}+\ \ldots \tag{6}$$

where $G=G(\xi)$ satisfies Eq. (1), and $\alpha_m,\alpha_{m-1}\ldots$ are constants to be determined later, $\alpha_m\neq 0$. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and non-linear terms appearing in (5).

Step 3. Substituting (6) into (5) and using (1), collecting all terms with the same order of G together, the left-hand side of Eq. (5) is converted into another polynomial in G. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_m,\alpha_{m-1},\ldots\lambda,\mu$.

Step 4. Solving the algebraic equations system in Step 3, and by using the solutions of Eq. (1), we can construct the travelling wave solutions of the nonlinear evolution equation (5).

In the subsequent sections we will illustrate the validity of the proposed method by applying it to solve several nonlinear evolution equations.

3. APPLICATION OF BERNOULLI SUB-ODE METHOD FOR THE (3+1) DIMENSIONAL JIMBO-MIVA EQUATION

In this section, we consider the following (3+1) dimensional Jimbo-Miva equation:

$$u_{xxxy}+3u_y u_{xx}+3u_x u_{xy}+2u_{yt}-3u_{xz}=0 \tag{7}$$

Suppose that

$$u(x,y,z,t)=u(\xi),\xi=kx+my+rz+\omega t \tag{8}$$

where k,m,r,ω are constants that to be determined later.

By (8), (7) is converted into an ODE

$$k^3mu^{(4)}+6mk^2u'u''+(2m\omega-3kr)u''=0 \tag{9}$$

Integrating (9) once we obtain

$$k^3mu'''+3mk^2(u')^2+(2m\omega-3kr)u'=g \tag{10}$$

where g is the integration constant to be determined later.

Suppose that the solution of (10) can be expressed by a polynomial in G as follows:

$$u(\xi)=\sum_{i=0}^{m}a_iG^i \tag{11}$$

where a_i are constants, and $G=G(\xi)$ satisfies Eq. (1).

Balancing the order of u''' and $(u')^2$ in Eq. (10), we have $m+3=2m+2\Rightarrow m=1$. So Eq. (11) can be rewritten as

$$u(\xi)=a_1G+a_0,\ a_1\neq \tag{12}$$

where a_1,a_0 are constants to be determined later.

Substituting (12) into (10) and collecting all the terms with the same power of G together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$G^0:-g=0$$

$$G^1:-2a_1m\omega\lambda+3kr\lambda a_1-k^3ma_1\lambda^3=0$$

$$G^2:-3a_1kr\mu+7k^3ma_1\mu\lambda^2+3mk^2a_1^{\,2}\lambda^2+2a_1m\omega\mu=0$$

$$G^3:-12k^3ma_1\lambda\mu^2-6k^2a_1^{\,2}m\mu\lambda=0$$

$$G^4:6k^3\mu^3ma_1+3mk^2a_1^{\,2}\mu^2=0$$

Solving the algebraic equations above, yields:

$$a_1=-2k\mu, a_0=a_0, k=k, r=r, m=m, g=0, \omega=\frac{1}{2}\frac{k(3r-mk^2\lambda^2)}{m} \tag{13}$$

where k,r,m are arbitrary nonzero constants.

Substituting (13) into (12), we get that

$$u(\xi)=-2k\mu G+a_0, \quad \xi=kx+ry+mz+\frac{1}{2}\frac{k(3r-mk^2\lambda^2)}{m}t \tag{14}$$

Combining with Eq. (1.2), we can obtain the travelling wave solutions of (7) as follows:

$$u(\xi)=-2k\mu(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}})+a_0 \tag{15}$$

where $\xi=kx+ry+mz+\frac{1}{2}\frac{k(3r-mk^2\lambda^2)}{m}t$, and μ,λ are arbitrary nonzero constants.

Remark 1. Our result (15) is new exact travelling wave solutions for the (3+1) dimensional Jimbo-Miva equation (7).

4. APPLICATION OF BERNOULLI SUB-ODE METHOD FOR SRLW EQUATION

In this section, we will consider the following SRLW equation:

$$u_{xxtt}+u_{tt}+u_{xx}+uu_{xt}+u_xu_t=0 \tag{16}$$

Suppose that

$$u(x,t)=u(\xi), \xi=kx+\omega t \tag{17}$$

where k,ω are constants that to be determined later.

By (17), (16) is converted into an ODE

$$(k^2+\omega^2)u''+k\omega uu''+k\omega(u')^2+k^2\omega^2u^{(4)}=0 \tag{18}$$

Integrating (18) once we obtain

$$(k^2+\omega^2)u'+k\omega uu'+k^2\omega^2u'''=g \tag{19}$$

where g is the integration constant to be determined later..

Suppose that the solution of (19) can be expressed by a polynomial in G as follows:

$$u(\xi)=\sum_{i=0}^{m}a_iG^i \tag{20}$$

where a_i are constants, and $G=G(\xi)$ satisfies Eq. (1).

Balancing the order of u''' and uu' in Eq. (19), we have $m+3=2m+1\Rightarrow m=2$. So Eq. (20) can be rewritten as

$$u(\xi)=a_2G^2+a_1G+a_0, a_2\neq 0 \tag{21}$$

where a_2,a_1,a_0 are constants to be determined later.

Substituting (21) into (19) and collecting all the terms with the same power of G together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$G^0:-g=0$$

$$G^1:-ka_1a_0\omega\lambda-\omega^2\lambda a_1-k^2\lambda a_1-k^2\omega^2a_1\lambda^3=0$$

$$G^2:\omega^2a_1\mu-2\omega^2a_2\lambda+7k^2\omega^2a_1\mu\lambda^2+k\omega\mu a_1a_0-ka_1^2\omega\lambda-2k\omega a_0a_2\lambda-8k^2\omega^2a_2\lambda^3-2k^2a_2\lambda+k^2\mu a_1=0$$

$$G^3:2k^2a_2\mu-3k\omega a_1a_2\lambda+k\omega\mu a_1^2-12k^2\omega^2\mu^2a_1\lambda+38k^2\omega^2\lambda^2a_2\mu+2\omega^2a_2\mu+2k\omega a_0a_2\mu=0$$

$$G^4:-54k^2\mu^2\omega^2\lambda a_2+6k^2a_1\omega^2\mu^3-2k\omega\lambda a_2^2+3kk\omega a_1a_2\mu=0$$

$$G^5: 24k^2\mu^3\omega^2 a_2 + 2ka_2{}^2\omega\mu = 0$$

Solving the algebraic equations above, yields:

$$a_2 = -12k\omega\mu^2, a_1 = 12k\omega\mu\lambda, a_0 = -\frac{\omega^2 + k^2 + \omega^2 k^2\lambda^2}{k\omega}, k = k, g = 0, \omega = \omega \tag{22}$$

where k, ω are arbitrary nonzero constants.

Substituting (22) into (21), we get that

$$u(\xi) = -12k\omega\mu^2 G^2 + 12k\omega\mu\lambda G - \frac{\omega^2 + k^2 + \omega^2 k^2\lambda^2}{k\omega}, \qquad \xi = kx + \omega t \tag{23}$$

Combining with Eq. (1.2), we can obtain the travelling wave solutions of (16) as follows:

$$u(\xi) = -12k\omega\mu^2 \left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right)^2 + 12k\omega\mu\lambda\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right) - \frac{\omega^2 + k^2 + \omega^2 k^2\lambda^2}{k\omega} \tag{24}$$

where $\xi = kx + \omega t$, and μ, λ are arbitrary nonzero constants.

Remark 2. The solution denoted in (24) is new exact travelling wave solutions for the SRLW equation (16).

5. APPLICATION OF BERNOULLI SUB-ODE METHOD FOR GENERALIZED BURGERS EQUATION

In this section, we will consider the following generalized Burgers equation:

$$u_t + \alpha u_{xx} + u^m u_x = 0 \tag{25}$$

Suppose that

$$u(x,t) = u(\xi), \xi = x - ct \tag{26}$$

where c is a constant that to be determined later.

By (26), (25) is converted into an ODE

$$-cu' + \alpha u'' + u^m u' = 0 \tag{27}$$

Suppose that the solution of (27) can be expressed by a polynomial in G as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i G^i \tag{28}$$

where a_i are constants, and $G = G(\xi)$ satisfies Eq. (1).

Balancing the order of $u^m u'$ and u'' in Eq. (27), we have $mn + n + 1 = n + 2 \Rightarrow n = \frac{1}{m}$. So we make a variable $u = v^{1/m}$, and (27) is converted into

$$-cmvv' + \alpha(1-m)(v')^2 + \alpha mvv'' + mv^2 v' = 0 \tag{29}$$

Suppose that the solution of (29) can be expressed by a polynomial in G as follows:

$$v(\xi) = \sum_{i=0}^{l} b_i G^i \tag{30}$$

where b_i are constants, and $G = G(\xi)$ satisfies Eq. (1.1).

Balancing the order of $v^2 v'$ and vv'' in Eq. (29), we have $2l + l + 1 = l + l + 2 \Rightarrow l = 1$. So Eq. (30) can be rewritten as

$$v(\xi) = b_1 G + b_0, b_1 \neq 0 \tag{31}$$

Substituting (31) into (29) and collecting all the terms with the same power of G together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$G^1: -ma_1a_0^{\ 2}\lambda + cm\lambda a_1a_0 + \alpha ma_1a_0\lambda^2 = 0$$

$$G^2: -ma_1^{\ 2}a_0\lambda + ma_1a_0^{\ 2}\mu - cma_1a_0\mu + \alpha a_1^{\ 2}\lambda^2 + cma_1^{\ 2}\lambda - 3\alpha ma_1a_0\mu\lambda = 0$$

$$G^3: -ma_1^{\ 3}\lambda - 2\alpha a_1^{\ 2}\mu\lambda - cma_1^{\ 2}\mu - \alpha a_1^{\ 2}m\mu\lambda + 2\alpha a_1a_0\mu^2 m + 2ma_1^{\ 2}a_0\mu = 0$$

$$G^4: \alpha a_1^{\ 2}\mu^2 m + ma_1^{\ 3}\mu + \alpha a_1^{\ 2}\mu^2 = 0$$

Solving the algebraic equations above, yields:

Case 1:

$$a_1 = -\frac{\alpha\mu(m+1)}{m}, a_0 = 0, c = -\frac{\alpha\lambda}{m} \tag{32}$$

Substituting (31) into (30), we get that

$$u(\xi) = -\frac{\alpha\mu(m+1)}{m}G, \qquad \xi = x + \frac{\alpha\lambda}{m}t \tag{33}$$

Combining with Eq. (1.2), we can obtain the travelling wave solutions of (25) as follows:

$$u(\xi) = -\frac{\alpha\mu(m+1)}{m}\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right) \tag{34}$$

where μ, λ are arbitrary nonzero constants. Since $\xi = x + \frac{\alpha\lambda}{m}t$, then furthermore we have

$$u(x,y,t) = -\frac{\alpha\mu(m+1)}{m}\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda(x+\frac{\alpha\lambda}{m}t)}}\right) \tag{35}$$

Case 2:

$$a_1 = -\frac{\alpha\mu(m+1)}{m}, a_0 = \frac{\alpha\lambda(m+1)}{m}, c = \frac{\alpha\lambda}{m} \tag{36}$$

Substituting (31) into (30), we get that

$$u(\xi) = -\frac{\alpha\mu(m+1)}{m}G + \frac{\alpha\lambda(m+1)}{m}, \quad \xi = x - \frac{\alpha\lambda}{m}t \tag{37}$$

Combining with Eq. (1.2), we can obtain the travelling wave solutions of (25) as follows:

$$u(\xi) = -\frac{\alpha\mu(m+1)}{m}\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right) + \frac{\alpha\lambda(m+1)}{m} \tag{38}$$

where μ, λ are arbitrary nonzero constants. Since $\xi = x - \frac{\alpha\lambda}{m}t$, then furthermore we have

$$u(x,y,t) = -\frac{\alpha\mu(m+1)}{m}\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda(x-\frac{\alpha\lambda}{m}t)}}\right) + \frac{\alpha\lambda(m+1)}{m} \tag{39}$$

Remark 3. Our results (35) and (39) are new exact travelling wave solutions for the generalized Burgers equation (25).

6. CONCLUSIONS

We have seen that some new travelling wave solution of the (3+1) dimensional Jimbo-Miva equation, SRLW equation, and generalized Burgers equation are successfully found by using the Bernoulli sub-ODE method. As one can see, this method is concise, powerful, and effective. Also this method can be generalized to solve many other nonlinear evolution equations.

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