BAYESIAN EQUILIBRIUM STRATEGIES OF n CLASS OF CONSUMERS

Zhang Hailong & Ai Kefeng
University of Shanghai for Science and Technology, Shanghai, 200093, China

ABSTRACT
Nonlinear pricing is a pricing method that the consumers pay the total price of a commodity or service which is not linearly proportional to the purchase of the total quantity. After generalizing the type of consumers' demand and combining with the actual situation, this paper designs a nonlinear price mechanism game. According to detailed simplifying and reasoning the constraints, we derived the Bayesian equilibrium strategy of this game. Under the condition of incomplete information, we know the seller how to obtain the maximum benefit of optimal price plan. Finally we compared it with the Nash equilibrium which is under the complete information.

Keywords: *nonlinear pricing, Bayesian equilibrium, incomplete information, Nash equilibrium*.

1. INTRODUCTION

Nonlinear pricing refers to a pricing method that the total consumer spending and consumption was nonlinear pricing. Compared with linear pricing, nonlinear pricing gives full consideration to the needs of the consumer heterogeneity which is a prominent feature, thereby enhancing the producers and consumers' welfare and realizing the Pareto improvement. Nonlinear pricing is an essentially mechanism design problem as well as a kind of price discrimination. However mechanism design is a special kind of incomplete information game. In recent years, nonlinear pricing has been widespread applications in communication, electricity, civil aviation, postal services, transport and other public utilities pricing strategy. Maskin and Riley (1984) was the first to study the issue of optimal pricing mechanism. When consumers' heterogeneous types obey continuous distribution, they conclude that the optimal monopoly pricing is concave nonlinear continuous curve which is the basic starting point of the optimal pricing. Wilson [1] published in the "Short course on nonlinear pricing" gives the constructor of the two-part pricing packages. Wang Bing[2] studied nonlinear pricing strategies of China Mobile Communications Corporation from the "yield trap" angle. In the single cross assumption, Li Keke[3] derived the optimal nonlinear pricing of the n class of consumers. Under normal circumstances, Ai Kefeng[4] gives the optimal nonlinear pricing of three class of consumers. Meeting individual rationality and incentive compatibility constraint optimization problems, this article gives the perfect Bayesian equilibrium of the n class of consumers' nonlinear pricing issues.

2. CONSUMERS' NONLINEAR PRICING

The seller sells the products of constant marginal cost c to consumers. Consumers paid the seller $T(q)$ for buying the products of the quantity of q. So the benefit of the seller is:

$$u_0(q,T(q)) = T(q) - cq \tag{1}$$

However, these products can bring certain benefits for consumers. Assuming that its gross surplus for the quantity of products of q is $\theta V(q)$ is the common information for buyers and sellers. $V(q)$ satisfies the following condition:

1. $V(0)=0$ No transaction, the consumers' gross surplus is 0;
2. $V'(q)>0$ With the increase in trading volume, consumer gross surplus increases;
3. $V''(q)<0$ With the increase in trading volume, increased speed of consumer gross surplus is getting slower and slower. So the consumer's benefit is:

$$u_1(q,T(q),\theta) = \theta V(q) - T(q) \tag{2}$$

Based on the needs of consumers, we divided consumers into n class. θ presents the strength of consumers' demand which belongs to consumer's private information. The seller does not know or do not know the class of this consumers' demand, so we discuss nonlinear pricing problem at the view of the incomplete information. However, the seller know the probability distribution of θ: $p(\theta=\theta_i)=p_i$ $(i=1,\cdots,n)$ and $\sum_{i=1}^{n} p_i = 1$; $\theta_i < \theta_j\,(1 \le i < j \le n)$。 According to Harsanyi axiom, game progress in the following order: the seller first announced charges (possibly non-linear), then the consumers choose to accept or reject. Assumes that the transaction will occur, of course the request for this price $T(q)$ satisfies the constraints: $\theta V(q) - T(q) \ge 0$.If the seller knows the value of θ, he will offer

$\theta V(q)=T(q)$ so as to maximize the benefits. q must satisfies: $\theta V'(q)=c$. However, the seller only knows the distribution function of θ.Thus the seller formulates $(q_i,T_i),(i=1,\cdots,n)$ scheme to cope with the θ_i class consumers. Then the seller's expected revenue should be:

$$Eu_2=\sum_{i=1}^{n}p_i(T_i-cq_i) \tag{3}$$

Now the seller faces two types of constraints. The first type constraints require that the customer is willing to buy such scheme which is called individual rationality constraints. Individual rationality constraint can be expressed using the formula:

$$\theta_i V(q_i)-T_i\geq 0,\quad i=1,\cdots,n \tag{4}$$

Second type constraints require the seller to design the appropriate solution for every class of consumers. Such constraints called incentive compatibility constraints. Incentive compatibility constraint can be expressed using the formula:

$$\theta_i V(q_i)-T_i\geq \theta_i V(q_j)-T_j \quad \text{where } i,j=1,\cdots,n \text{ and } i\neq j \tag{5}$$

The seller formulates $(q_i,T_i),(i=1,\cdots,n)$ scheme to maximum their own expected revenue must be under the conditions of individual rationality and incentive compatibility constraints.

3. MODEL AND SOLVING

According to the above information, we converted n class of consumers' optimal nonlinear pricing problem to the following constraints optimization problem:

$$\max_{\{(q_i,T_i),i=1,\cdots n\}}\sum_{i=1}^{n}p_i(T_i-cq_i) \tag{6}$$

$$st.\begin{cases}\theta_i V(q_i)-T_i\geq 0 & (i=1,\cdots,n) \quad (IR_i)\\ \theta_i V(q_i)-T_i\geq\theta_i V(q_j)-T_j & (i,j=1,\cdots,n;i\neq j)\quad (IC_i^j)\end{cases} \tag{7}$$

3.1. Simplify the Constraints

First, we assume that $(q_i,T_i),i\geq 1$ is the solution of the optimization problem. From $IR_1,IC_i^1,(i\geq 2)$, we can see $\theta_i V(q_i)-T_i\geq\theta_i V(q_1)-T_1\geq\theta_i V(q_1)-\theta_1 V(q_1)=(\theta_i-\theta_1)V(q_1)\geq 0$. So $IR_i,(i\geq 2)$ must be established. Unless $q_1=0$ (i.e. the company does not sell the product to this type of consumers), or there must be $IR_i(i=2,\cdots,n)$ is strictly greater-than sign. Furthermore IR_1 must be equation, assuming it is strictly greater-than sign, we put $T_i+\varepsilon,(i=1,\cdots,n)$ (where ε is an arbitrary small positive amount, the same below) into (7) .it have been established, so $T_i,(i=1,\cdots,n)$ can be further increased at least a little. Contradicted the assumption, thereby IR_1 is an equation. So the constraint optimization problem (7) simplifies to:

$$st.\begin{cases}\theta_1 V(q_1)-T_1=0 & (IR_1)\\ \theta_i V(q_i)-T_i\geq\theta_i V(q_j)-T_j & (i,j=1,\cdots,n\quad i\neq j)\quad (IC_i^j)\end{cases} \tag{8}$$

3.2. Further Simplify the Constraint

Conclusion 1 Not all $IC_i^1(i\geq 2)$ are strictly greater-than sign.

Assumptions $n-1$ formulas in $IC_i^1(i\geq 2)$ are strictly greater-than sign. Then putting $T_2+\varepsilon\ldots T_n+\varepsilon$ into the formula (8) is still valid. So the seller's asking price $T_i,(i\geq 2)$ could improve at least a little. So it contradicts $(q_i,T_i),(i\geq 1)$ which we assume is the optimal solution.

Conclusion2 If $IC_j^1(j>2)$ is an equation, $IC_i^1(i<j),IC_j^i(i<j)$ are also equations. According to IC_j^i,IC_j^1,IC_i^1,we can easily launch of the above conclusion.

Conclusion 3 If $IC_j^1(j>2)$ is an equation, $IC_m^n(m\leq j,n<m)$ are equations. We can easily deduce conclusion 3 by conclusion 2.

Conclusion 4 $IC_j^{j-1}(j\geq 2)$ must be equations.

If $IC_j^{j-1}(j \geq 2)$ is not an equation, we can deduce $IC_m^k(1 \leq k \leq j-1, j \leq m \leq n)$ are strictly greater-than sign from conclusion 2 and conclusion 3. Otherwise, putting $T_j + \varepsilon \ldots T_n + \varepsilon$ into (8) is still valid. Then it contradicts the assumption.

3.3. Solving Objective Function

By the conclusion 1-4, the n class consumers' optimal nonlinear pricing problem can be described as:

$$\max_{\{(q_i, T_i), i=1,\cdots n\}} \sum_{i=1}^{n} p_i (T_i - c q_i) \tag{9}$$

$$st. \begin{cases} \theta_1 V(q_1) = T_1 \\ \theta_i V(q_i) - T_i = \theta_i V(q_{i-1}) - T_{i-1} \quad i = 2, \cdots, n \end{cases} \tag{10}$$

The constraint constraints (10) are consistent with the literature [3] in a single cross- assumption. From constraints (10), we get: $T_i = \theta_i V(q_i) - \sum_{k=2}^{i} (\theta_k - \theta_{k-1}) V(q_{k-1}), \ (2 \leq i \leq n)$ .Derivation was then put it into the objective function of equation (9) we get:

$$p_i[\theta_i V'(q_i) - c] - \sum_{j=i+1}^{n} p_j (\theta_{i+1} - \theta_i) V'(q_i) = 0 \tag{11}$$

$$\text{Then:} \begin{cases} \theta_i V'(q_i) = \dfrac{c}{1 - \dfrac{\theta_{i+1} - \theta_i}{\theta_i} \displaystyle\sum_{j=i+1}^{n} \dfrac{p_j}{p_i}} \\ \theta_n V'(q_n) = c \end{cases} \tag{12}$$

(12) is the n class of consumers' optimal nonlinear pricing problem optimal solution. However, the seller is able to develop this optimal price scheme need the parameters of consumers' demand to meet certain conditions：

$$\frac{\theta_{i+1} - \theta_i}{\theta_i} \sum_{j=i+1}^{n} \frac{p_j}{p_i} < 1, (i = 1, \ldots, n-1) \tag{13}$$

4. ANALYZE THE MODEL

If it is the case of perfect information (i.e. the seller knows this consumer is what type), the seller knows the Nash equilibrium is $(q_i^*, T_i^*), (i \geq 2)$. Obviously Nash equilibrium meets:

$$\theta_i V'(q_i^*) = c, T^* = \theta_i V(q_i^*), (i = 1, \cdots, n) \tag{14}$$

When the parameters satisfy (13) . By

$$V'(q_i) = \frac{c}{\theta_i - (\theta_{i+1} - \theta_i) \displaystyle\sum_{j=i+1}^{n} \frac{p_j}{p_i}} > c/\theta_i = V'(q_i^*), (1 \leq i < n) \quad \text{and}$$

$V'(q_i)$ which is a decreasing function; we can get $q_i < q_i^*$. Due to incomplete information, the sellers make use of all the information they knows to distinguish between the different types of consumer demand and to develop optimal price scheme (12). Lastly, let the highest type of consumes' purchasing power to reach the seller gains optimal.

5. REFERENCES

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