EFFECTS OF CHEMICAL REACTION WITH HEAT AND MASS TRANSFER ON PERISTALTIC MOTION OF POWER-LAW FLUID IN AN ASYMMETRIC CHANNEL WITH WALL'S PROPERTIES

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ABSTRACT
In this work, we investigated the peristaltic motion of a non-Newtonian fluid which obeying power-law model, with heat and mass transfer through an asymmetric channel. The wall properties and chemical reaction are considered. This phenomena is modeled mathematically by a system of governing equations which are continuity, momentum with heat and mass. These equations are solved analytically under the conditions of low Reynolds number and long wave length. The solutions of these equations which are stream function, temperature and concentration are obtained as functions of physical parameters of the problem. The effects of these parameters on those solutions are discussed numerically and illustrated graphically through a set of figures.

keywords: *Peristaltic transport, power-law fluid, heat and mass transfer, wall's properties*

1. INTRODUCTION
Peristalsis is defined as a wave of relaxation or contraction (expansion) of the walls of a flexible conduit, which pumping the enclosed material inside or outside the conduit [1,2] . Peristalsis is now well-known to the physiologists to be one of the major mechanisms of fluid transport in many biological systems,e.g. swallowing food through the esophagus, movement of chyme in the gastrointestinal tract, urine transport from the kidney to the bladder through the ureter, transport of spermatozoa, movement of ova in the fallopian tube, vasomotion of small blood vessels such as venules and capillaries and blood flow in arteries, and in many other glandular ducts [1- 6]. There are many industrial applications for peristaltic transport, like blood pumps in heart lung machine, and there are many applications in biomechanical and engineering sciences for peristaltic motion [5,6]. peristaltic motion results physiologically from neuron muscular properties of the tubular smooth muscles [7]. Recently, some studies have proven that the intra-uterine fluid flow due to myometrial contraction is a peristaltic motion [8,9,10].

Several mathematical and experimental models have been developed to understand the fluid mechanical aspects of peristaltic flows with considerations of the nature of the fluid, geometry of the channel, propagating waves and inclusion of other physical effects such as magnetic fields, porous media, chemical reactions, heat and mass transfer [10 - 22]. To the best of our knowledge, no investigation has been made yet to investigate the effects of chemical reaction, heat and mass transfer on peristaltic transport of a power-law fluid.

This work is an extend to the work of Hayat and Javed [19], they studied the velocity of the fluid obeyed power-law, but in our present work we take in our consideration the temperature and concentration of the fluid as well as the chemical reaction.

2. BASIC EQUATIONS
The basic equations which describe the motion of non-Newtonian fluid with heat and mass transfer with chemical reaction can be written as

Continuity equation

$$\nabla \cdot V = 0, \tag{1}$$

Momentum equation

$$\rho \frac{dV}{dt} = -\nabla P + \nabla \cdot \tau + F \tag{2}$$

Energy equation

$$\rho c_p \frac{dT}{dt} = k\nabla^2 T + \tau \cdot \nabla V, \tag{3}$$

Concentration equation

$$\frac{dC}{dt} = D\nabla^2 C - k_1(C - C_0), \tag{4}$$

where V is fluid velocity vector, ρ is the density of the fluid, P is the pressure, F is the external force vector, $\tau = (\tau_{ij})$ is stress tensor, T is temperature, c_p is the specific heat at constant pressure, k is the thermal conductivity, C is the concentration, D is the mass diffusivity, k_1 is chemical reaction parameter, $\frac{d}{dt}$ denotes the material time derivative and ∇^2 is the Laplace operator in second order.

The constitutive equation for power-law fluid is given by

$$\tau_{ij} = 2\mu(4e_{lm}e_{lm})^n e_{ij}; \qquad e_{ij} = \frac{1}{2}\left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i}\right), \tag{5}$$

where μ is coefficient of the viscosity of the fluid, and n is the power-law index. For $n = 1$ relation (5) represents a Newtonian fluid, for $0 < n < 1$ relation (5) describes a diagram with the shape of a pseudoplastic fluid and for $n > 1$ relation (5) represents a dilatant's behavior.

3. MATHEMATICAL FORMULATION

Consider a cartesian coordinates in two-dimensions (x, y) where x -axis is taken in motion direction while y -axis is perpendicular on it as seen in figure (1) and (u, v) are the velocity components in x and y directions respectively. Neglecting the body force, the system of equations (1- 5) are become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{6}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}, \tag{7}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y}, \tag{8}$$

$$\rho c_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \tau_{xx}\frac{\partial u}{\partial x} + \tau_{xy}\frac{\partial u}{\partial y} + \tau_{yx}\frac{\partial v}{\partial x} + \tau_{yy}\frac{\partial v}{\partial y}, \tag{9}$$

$$\left(\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y}\right) = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) - k_1(C - C_0), \tag{10}$$

$$\tau_{xx} = 2\mu\left(4\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2 + 4\left(\frac{\partial v}{\partial y}\right)^2\right)^n \frac{\partial u}{\partial x}, \tag{11}$$

$$\tau_{xy} = \tau_{yx} = \mu\left(4\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2 + 4\left(\frac{\partial v}{\partial y}\right)^2\right)^n \left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right), \tag{12}$$

$$\tau_{yy} = 2\mu\left(4\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2 + 4\left(\frac{\partial v}{\partial y}\right)^2\right)^n \frac{\partial v}{\partial y}. \tag{13}$$

The channel asymmetry is produced by choosing the peristaltic wave train on the walls to have different amplitudes and phase, consider the upper and lower channel walls are

$$y = H_1 = d_1 + a_1\cos\frac{2\pi}{\lambda}(x - c\,t) \tag{14}$$

$$y = H_2 = -d_2 - b_1\cos\left[\frac{2\pi}{\lambda}(x - c\,t) + \vartheta\right] \tag{15}$$

where a_1 and b_1 are the amplitudes of the waves, λ is the wave length, $(d_1 + d_2)$ is the width of the channel and ϑ is the phase difference that varies in the range $0 \le \vartheta \le \pi$. Further, a_1, b_1, d_1, d_2 and ϑ satisfy the condition $a_1^2 + b_1^2 + 2a_1b_1\cos\vartheta \le (d_1 + d_2)^2$.

The appropriate boundary conditions are

$$u = 0, \qquad at \;\; y = H_1, H_2, \tag{16}$$

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial x}\left(\overline{m}\frac{\partial^2}{\partial t^2} + \xi\frac{\partial}{\partial t} + B\frac{\partial^4}{\partial x^4} - \zeta\frac{\partial^2}{\partial x^2} + K\right)y, \quad at \;\; y = H_1, H_2, \tag{17}$$

$$T = T_w, \quad C = C_w \qquad at \;\; y = H_1, \tag{18}$$

$$T = T_o, \quad C = C_0 \qquad at \;\; y = H_2, \tag{19}$$

where $\overline{m}$ is the mass per unit area, ξ is the coefficient of viscous damping, B is the flexural rigidity of the plate, ζ is the elastic tension in membrane, K is the spring stiffness coefficient.

The first condition is the no slip boundary condition, the second is coming from the equation of elasticity (elastic movement), the third and fourth are the conditions of energy and concentration at boundaries.

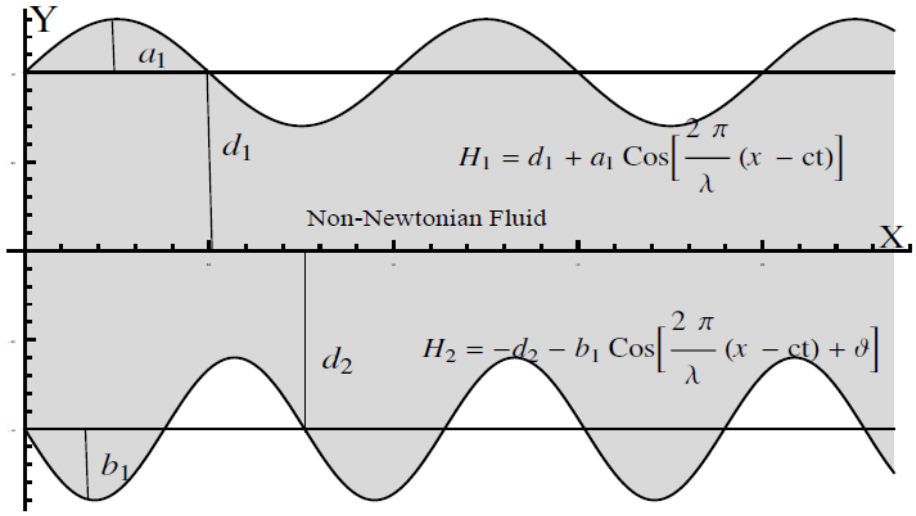

Figure 1. Geometry of the problem.

Introducing the following non-dimensional variables

$$\bar{x} = \frac{x}{\lambda}, \quad \bar{y} = \frac{y}{d_1}, \quad \delta = \frac{d_1}{\lambda}, \quad \bar{t} = \frac{ct}{\lambda}, \quad a = \frac{a_1}{d_1}, \quad b = \frac{b_1}{d_1},$$

$$d = \frac{d_2}{d_1}, \quad h_1 = \frac{H_1}{d_1}, \quad h_2 = \frac{H_2}{d_1}, \quad \bar{\psi} = \frac{\psi}{cd_1}, \quad \Theta = \frac{T - T_0}{T_w - T_0}, \quad \Phi = \frac{C - C_0}{C_w - C_0},$$

$$R = \frac{c\, d_1}{\mu c \lambda}\left(\frac{d_1^2}{2c^2}\right)^n, \quad \bar{\tau} = \frac{d_1 \tau}{\mu c}\left(\frac{d_1^2}{2c^2}\right)^n, \quad E_1 = \frac{\overline{m} c d_1^3}{\mu \lambda^3}\left(\frac{d_1^2}{2c^2}\right)^n, \quad E_2 = \frac{\xi d_1^3}{\mu \lambda^2}\left(\frac{d_1^2}{2c^2}\right)^n,$$

$$E_3 = \frac{B d_1^2}{\mu c \lambda^5}\left(\frac{d_1^2}{2c^2}\right)^n, \quad E_4 = \frac{\zeta d_1^3}{\mu c \lambda^3}\left(\frac{d_1^2}{2c^2}\right)^n, \quad E_5 = \frac{K d_1^3}{\mu c \lambda}\left(\frac{d_1^2}{2c^2}\right)^n, \quad P_r = \frac{\mu c_p}{k}\left(\frac{2c^2}{d_1^2}\right)^n,$$

$$E_c = \frac{\mu c^2}{k \Delta T} = \frac{c^2}{c_p \Delta T}, \quad S_c = \frac{\nu}{D}, \quad \gamma = \frac{k_1 d_1^2}{\nu}, \tag{20}$$

where δ is the wave number, ψ is the stream function, R is the Reynolds number, P_r is the Prandtl number, E_c is the Eckert number, S_c is the Schmidt number, γ is the dimensionless chemical reaction parameter, ν is the kinematic viscosity ($\nu = \frac{\mu}{\rho}$).

Using (20),under the assumptions of long wave length, $\delta = 1$ and low Reynolds number and after dropping bars, the equations (11-13) in terms of the stream function, will take the form

$$\left.\begin{aligned}\tau_{xx} &= 0,\\ \tau_{yy} &= 0,\end{aligned}\right\} \quad (21)$$

$$\tau_{xy} = \left(\frac{\partial^2\psi}{\partial y^2}\right)^{2n+1} \quad (22)$$

After eliminating the pressure between equations (7,8), using (20), introducing the values in the equation (21) and after dropping bars we have

$$\frac{\partial^2\tau_{xy}}{\partial y^2} = 0 \quad (23)$$

and then equations (9,10) will be

$$\frac{\partial^2\Theta}{\partial y^2} + E_c\left(\frac{\partial^2\psi}{\partial y^2}\right)^{2n+2} = 0, \quad (24)$$

$$\frac{1}{S_c}\frac{\partial^2\Phi}{\partial y^2} - \gamma\,\Phi = 0 \quad (25)$$

Also, the upper and lower channel walls will be

$$y = h_1 = 1 + a\cos 2\pi(x-t), \quad (26)$$

$$y = h_2 = -d - b\,(\cos 2\pi(x-t) + \vartheta). \quad (27)$$

The non-dimensional boundary conditions are

$$\frac{\partial\psi}{\partial y} = 0, \qquad at \;\; y = h_1, h_2$$

$$\frac{\partial\tau_{xy}}{\partial y} = \frac{\partial}{\partial x}\left(E_1\frac{\partial^2}{\partial t^2} + E_2\frac{\partial}{\partial t} + E_3\frac{\partial^4}{\partial x^4} - E_4\frac{\partial^2}{\partial x^2} + E_5\right)y, \quad at \;\; y = h_1, h_2$$

$$\Theta = 1, \quad \Phi = 1 \;\text{ at }\; y = h_1$$

$$\Theta = 0, \quad \Phi = 0 \;\text{ at }\; y = h_2$$

Introducing these boundary conditions with the system of equations (22-25), then we have the following solutions

$$\psi = 8a\pi^3\left(\frac{2n+1}{2n+2}\right)\left[\left(\frac{2n+1}{4n+3}\right)\left(y + \frac{h_2-h_1}{2}\right)^{\left(\frac{2n+2}{2n+1}\right)} - y\left(\frac{h_2+h_1}{2}\right)^{\left(\frac{2n+2}{2n+1}\right)}\right]\times$$

$$\left[E_1 - E_4 - 4\pi^2 E_3 - \frac{E_5}{4\pi^2})\sin 2\pi(x-t) + \frac{E_2}{2\pi}\cos 2\pi(x-t)\right]^{\left(\frac{1}{2n+1}\right)}, \tag{28}$$

$$\Theta = \frac{1}{8(1+n)(3+4n)(h_1-h_2)}[E_c c_3 c_5 c_7 (h_1-h_2)(2y-h_1-h_2)^2$$

$$+(y-h_2)(E_c c_1(c_5 c_8 h_1 h_2 + (4c_4 + Ac_6)(h_1^2+h_2^2)) + 8(1+n)(3+4n))$$

$$-E_c c_2 (y-h_1)\left(4c_4(h_1^2-9h_2^2) + Ac_6(h_1^2 - 6(1+2n)^2 h_1 h_2 + 9h_2^2)\right)], \tag{29}$$

$$\Phi = \csc\left((h_1-h_2)\sqrt{\gamma S_c}\right)\sinh\left((y-h_2)\sqrt{\gamma S_c}\right), \tag{30}$$

where

$$c_1 = (B(h_2 - \frac{1}{2}(h_2-h_1))^{\frac{1}{1+2n}}, 0.5cm c_2 = (B(h_1 - \frac{1}{2}(h_2-h_1))^{\frac{1}{1+2n}},$$
$$c_3 = (B(2y-h_1+h_2))^{\frac{1}{1+2n}}, 0.5cm c_4 = n(1+n)(2\pi)^{\frac{3}{1+2n}}, 0.5cm c_5 = A(1+2n)^2,$$
$$c_6 = 8^{\frac{1}{1+2n}}, 0.5cm c_7 = 4^{\frac{1}{1+2n}}, 0.5cm c_8 = 2^{\frac{4+2n}{1+2n}}, 0.5cm A = (\pi)^{\frac{3}{1+2n}},$$
$$B = a\left(\sin 2\pi(x-t)(E_1 - 4\pi^2 E_3 - E_4 - \frac{E_5}{4\pi^2}) + \frac{E_2\cos(2\pi(x-t))}{2\pi}\right)$$

4. RESULTS AND DISCUSSION

In the present study, analytical solutions have been obtained for the problem of two dimensional peristaltic flow of a non-Newtonian power-law fluid in the presence of heat and mass transfer with chemical reaction, through asymmetric channel. The equations governing this motion have been solved under the conditions of low Reynolds number and long wave length, subject to a set of appropriate boundary conditions. The expressions of temperature and concentration have been evaluated for different parameters and have been shown graphically through a set of figures.

Figures (2 - 4) explained the effect of the Eckert number E_c on the temperature Θ, it is observed that the value of Θ increases with the value of Eckert number E_c. Figures (5 - 7) explained the effect of wall parameters on temperature Θ, we observe that Θ increases with the wall parameters.

The effects of the power-low indexes n on the temperature Θ are illustrated in figures (8) and (9). It is observed that the temperature decreases with the power-law index. The effect of the phase angle ϑ on the temperature Θ is explained in figures (10 - 12). It is shown that Θ increases with ϑ.

In figures (13) and (14) the effect of the the Schmidt number S_c is illustrated on the concentration Φ we have observed that Φ decreases with S_c. The concentration Φ decreases with the chemical reaction parameter γ as shown in figures 15) and (16) explained . In figure (17) we showed that the concentration Φ increases with the phase angle ϑ.

5. CONCLUSION

In the present study, the problem of two dimensional peristaltic flow of a non-Newtonian power-law fluid through asymmetric channel has been investigated. The equations governing the fluid flow with heat and mass transfer in the presence of chemical reaction, subjected to a set of appropriate boundary conditions, have been solved analytically under the conditions of low Reynolds number and long wave length. The solutions of these equations are obtained as functions of the physical parameters of the problem, the effects of these parameters of the problem on these solutions have been shown graphically. It is observed that Θ increases with Eckert number E_c, ϑ and wall parameters, but it decreases with the power-law index. Also, we observed that Φ decreases with S_c and the chemical reaction parameter γ, but it increases with the phase angle ϑ.

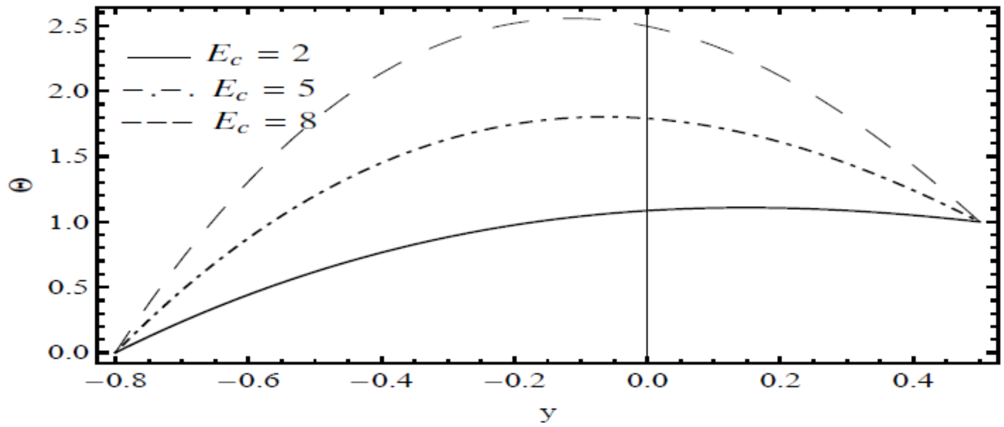

Figure 2. The temperature distribution Θ is plotted against y for different values of Eckert number when $n = 0.6$, $E_1 = 0.3$, $E_2 = 0.1$, $E_3 = 0.1$, $E_4 = 0.1$, $E_5 = 0.1$, $\vartheta = \pi$, $a = 0.5$, $b = 0.3$, $d = 0.5$, $x = 1$, $t = 0.5$.

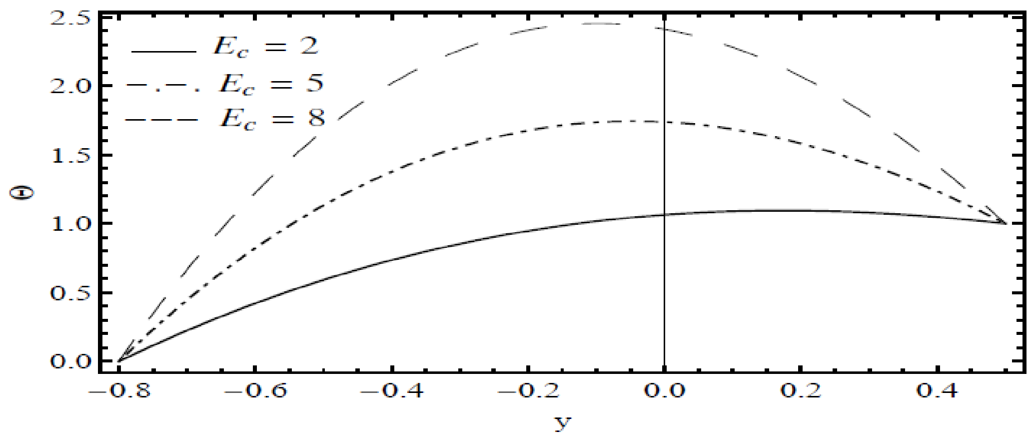

Figure 3. The temperature distribution Θ is plotted against y for different values of Eckert number when $n = 1.0$, $E_1 = 0.3$, $E_2 = 0.1$, $E_3 = 0.1$, $E_4 = 0.1$, $E_5 = 0.1$, $\vartheta = \pi$, $a = 0.5$, $b = 0.3$, $d = 0.5$, $x = 1$, $t = 0.5$.

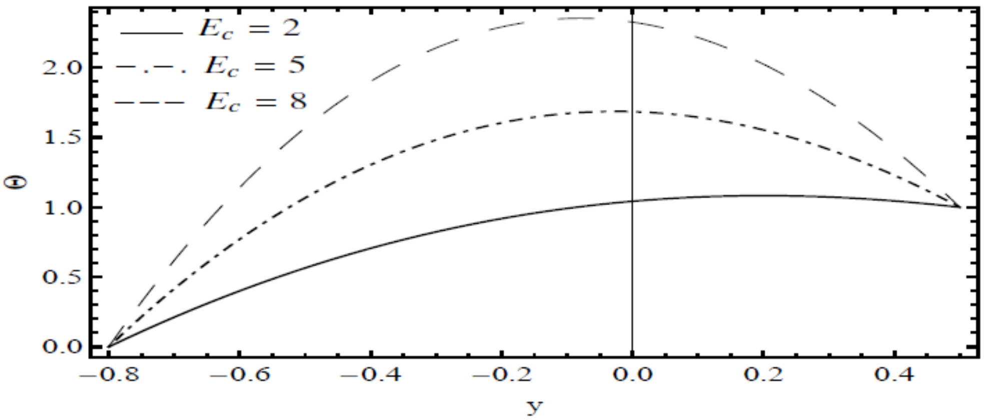

Figure 4. The temperature distribution Θ *is plotted against* y *for different values of Eckert number when* $n = 2.0$, $E_1 = 0.3$, $E_2 = 0.1$, $E_3 = 0.1$, $E_4 = 0.1$, $E_5 = 0.1$, $\vartheta = \pi$, $a = 0.5$, $b = 0.3$, $d = 0.5$, $x = 1$, $t = 0.5$.

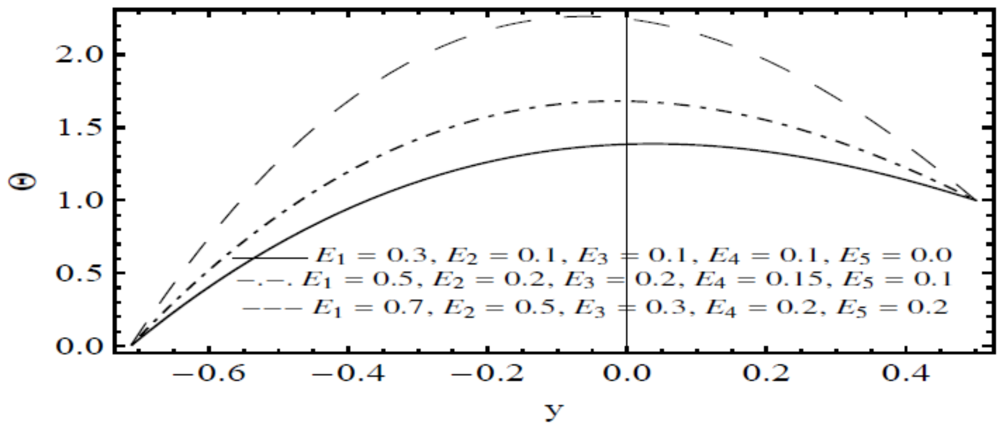

Figure 5. The temperature distribution Θ *is plotted against* y *for different values of wall parameters* E_1, E_2, E_3, E_4, E_5, when $n = 0.6$, $E_c = 4$, $a = 0.5$, $b = 0.3$, $d = 0.5$, $x = 1$, $t = 0.5$, $\vartheta = \frac{3\pi}{4}$.

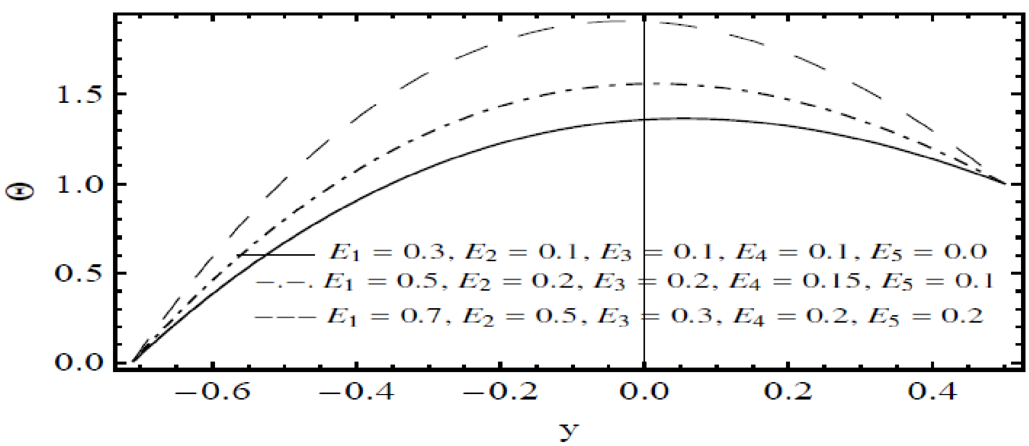

Figure 6. The temperature distribution Θ *is plotted against* y *for different values of wall parameters* E_1, E_2, E_3, E_4, E_5, when $n = 1$, $E_c = 4$, $a = 0.5$, $b = 0.3$, $d = 0.5$, $x = 1$, $t = 0.5$, $\vartheta = \frac{3\pi}{4}$.

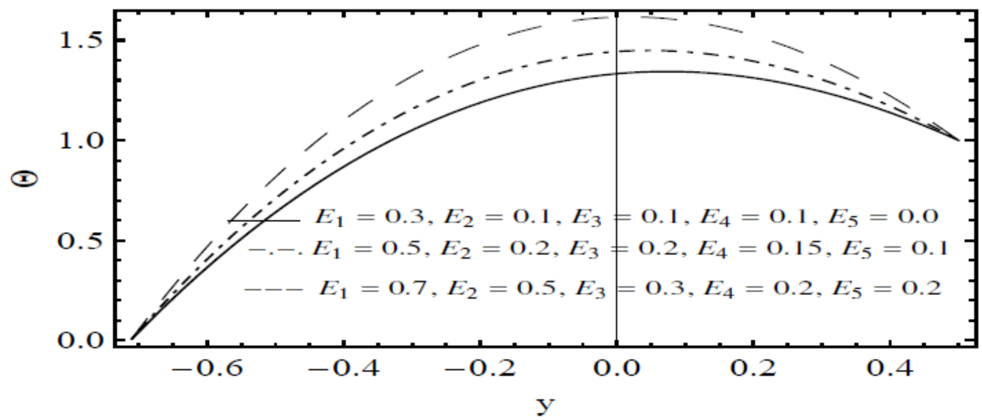

Figure 7. The temperature distribution Θ *is plotted against* y *for different values of wall parameters* E_1, E_2, E_3, E_4, E_5, when $n = 2$, $E_c = 4$, $a = 0.5$, $b = 0.3$, $d = 0.5$, $x = 1$, $t = 0.5$, $\vartheta = \frac{3\pi}{4}$.

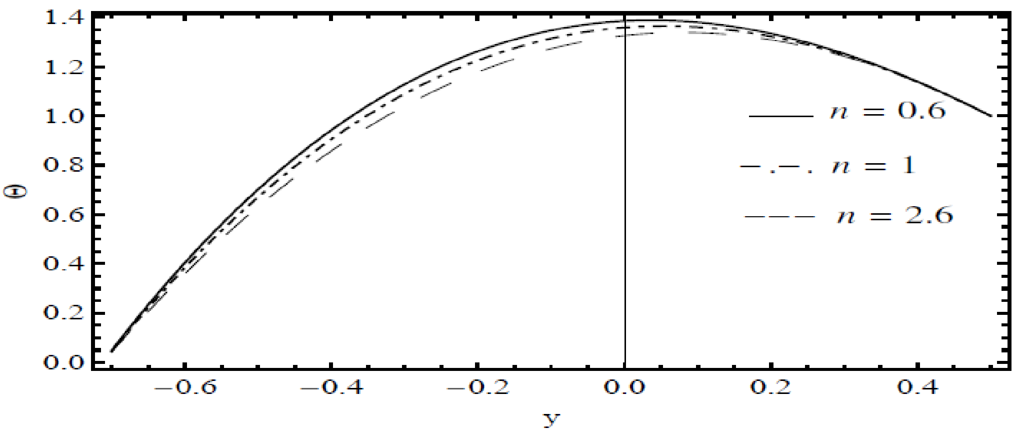

Figure 8. The temperature distribution Θ *is plotted against* y *for different values of the power-law index* n *when* $\vartheta = \frac{3\pi}{4}$, $E_1 = 0.3$, $E_2 = 0.1$, $E_3 = 0.1$, $E_4 = 0.1$, $E_5 = 0.1$, $E_c = 4$, $a = 0.5$, $b = 0.3$, $d = 0.5$, $x = 1$, $t = 0.5$.

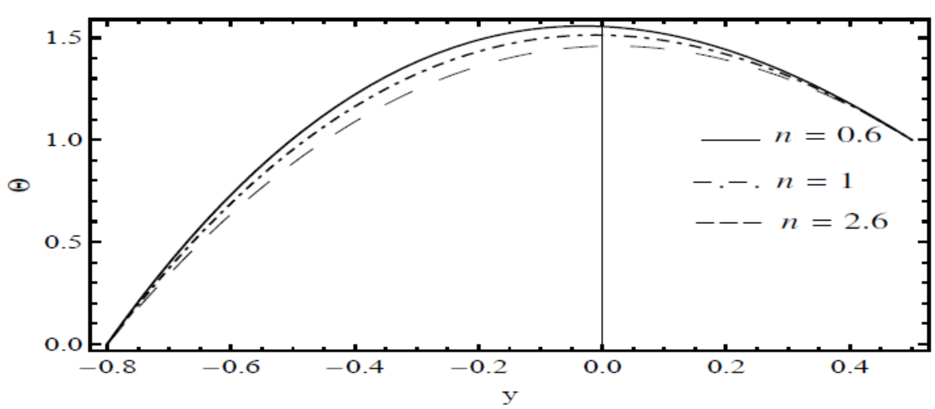

Figure 9. The temperature distribution Θ *is plotted against* y *for different values of the power-law index* n *when* $\vartheta = \pi$ $E_1 = 0.3$, $E_2 = 0.1$, $E_3 = 0.1$, $E_4 = 0.1$, $E_5 = 0.1$, $E_c = 4$, $a = 0.5$, $b = 0.3$, $d = 0.5$, $x = 1$, $t = 0.5$.

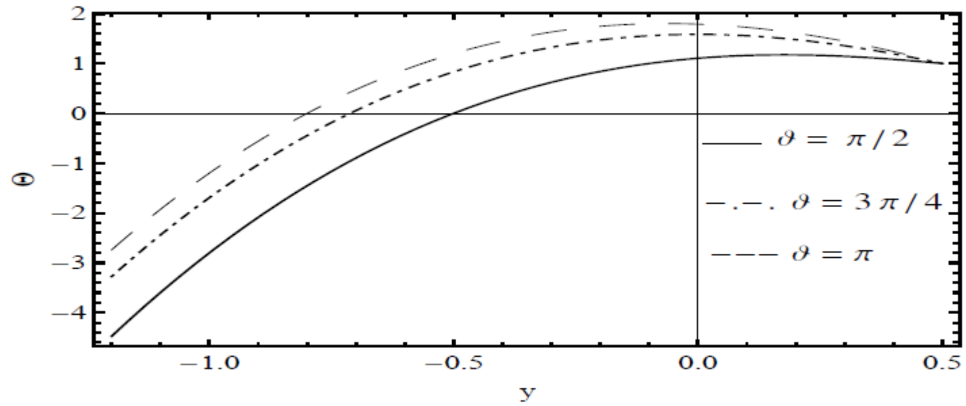

Figure 10. The temperature distribution Θ *is plotted against* y *for different values of the phase angle* ϑ *when* $n = 0.6$, $E_1 = 0.3$, $E_2 = 0.1$, $E_3 = 0.1$, $E_4 = 0.1$, $E_5 = 0.1$, $E_c = 5$, $a = 0.5$, $b = 0.3$, $d = 0.5$, $x = 1$, $t = 0.5$.

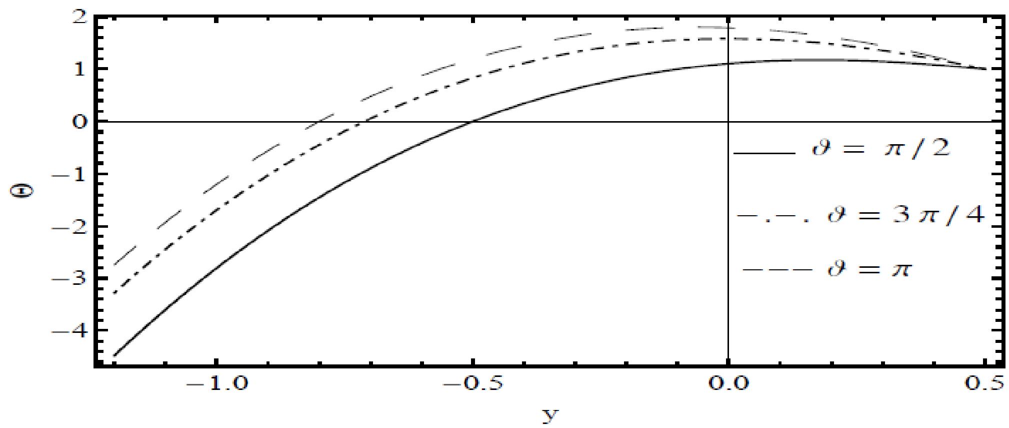

Figure 11. The temperature distribution Θ *is plotted against* y *for different values of the phase angle* ϑ *when* $n = 1$, $E_1 = 0.3$, $E_2 = 0.1$, $E_3 = 0.1$, $E_4 = 0.1$, $E_5 = 0.1$, $E_c = 5$, $a = 0.5$, $b = 0.3$, $d = 0.5$, $x = 1$, $t = 0.5$.

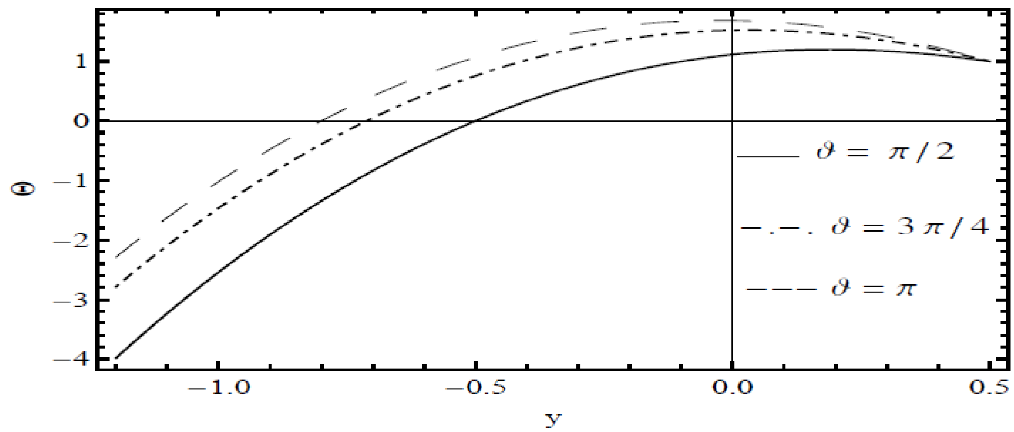

Figure 12. The temperature distribution Θ *is plotted against* y *for different values of the phase angle* ϑ *when* $n = 2$, $E_1 = 0.3$, $E_2 = 0.1$, $E_3 = 0.1$, $E_4 = 0.1$, $E_5 = 0.1$, $E_c = 5$, $a = 0.5$, $b = 0.3$, $d = 0.5$, $x = 1$, $t = 0.5$.

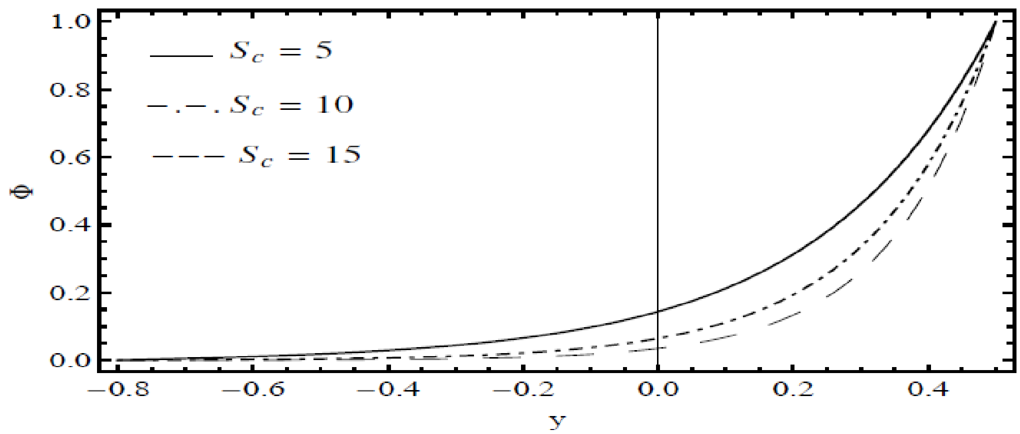

Figure 13. The concentration Φ *is plotted against* y *for different values of the Schmidt number* S_c *when* $\gamma = 3$, $a = 0.5$, $b = 0.3$, $d = 0.5$, $x = 1$, $t = 0.5$, $\vartheta = \pi$.

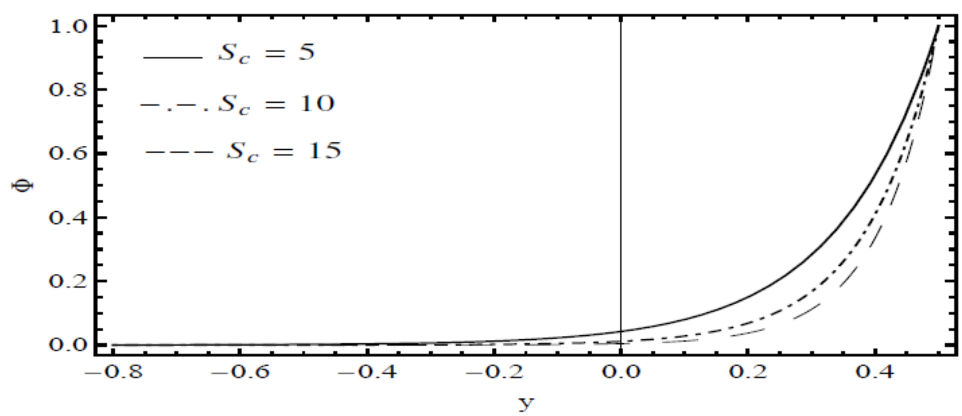

Figure 14. The concentration Φ *is plotted against* y *for different values of the Schmidt number* S_c *when* $\gamma = 8$, $a = 0.5$, $b = 0.3$, $d = 0.5$, $x = 1$, $t = 0.5$, $\vartheta = \pi$.

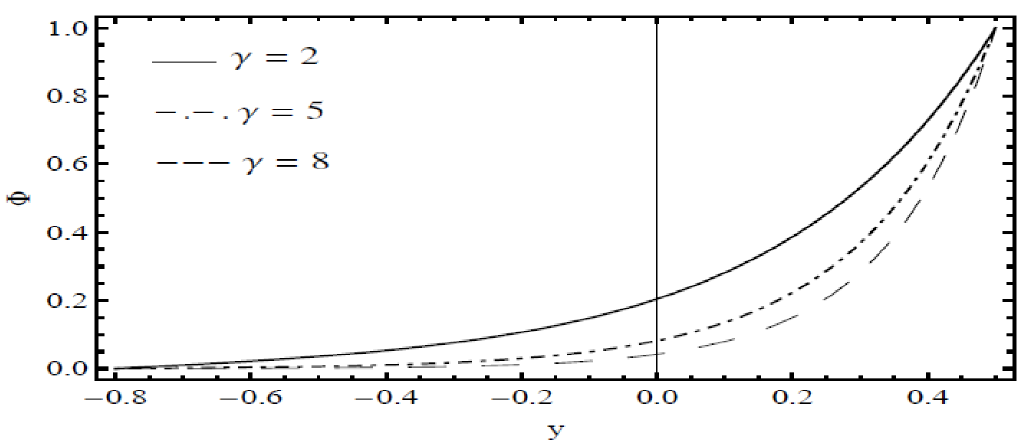

Figure 15. The concentration Φ *is plotted against* y *for different values of the dimensionless chemical reaction parameter* γ *when* $S_c = 5$, $a = 0.5$, $b = 0.3$, $d = 0.5$, $x = 1$, $t = 0.5$, $\vartheta = \pi$.

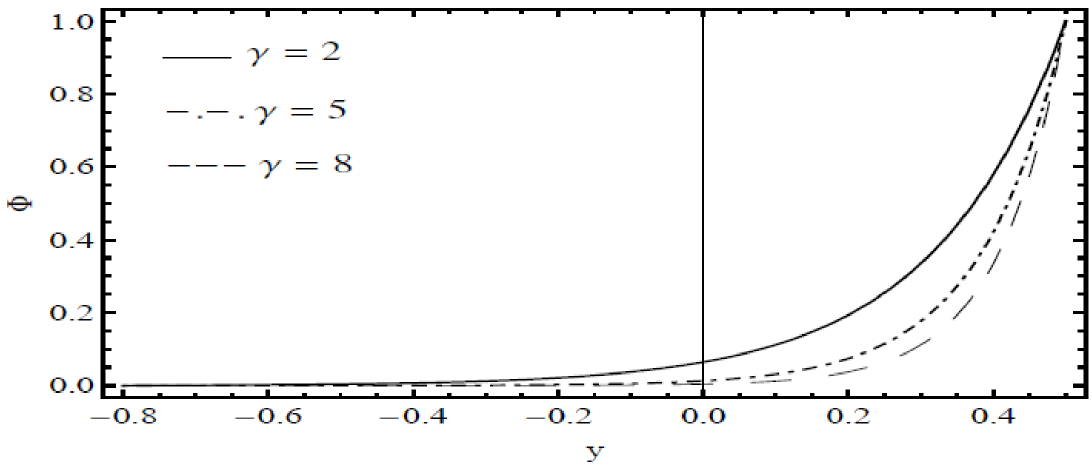

Figure 16. The concentration Φ *is plotted against* y *for different values of the dimensionless chemical reaction parameter* γ *when* $S_c = 15$, $a = 0.5$, $b = 0.3$, $d = 0.5$, $x = 1$, $t = 0.5$, $\vartheta = \pi$.

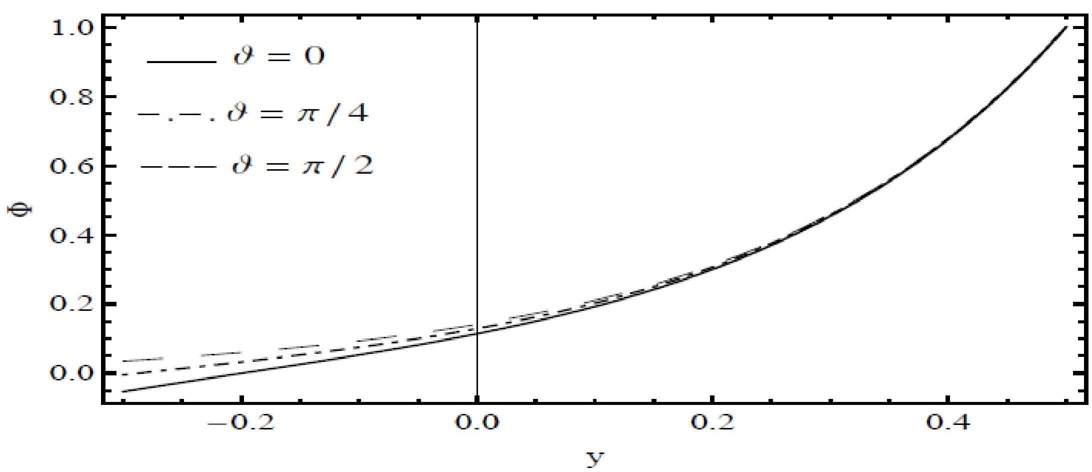

Figure 17. The concentration Φ *is plotted against* y *for different values of the phase angle* ϑ *when* $S_c = 5$, $\gamma = 3\pi$, $a = 0.5$, $b = 0.3$, $d = 0.5$, $x = 1$, $t = 0.5$.

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