COMPARISON BETWEEN ADOMIAN METHOD AND LAST SQUARE METHOD FOR SOLVING HIV/AIDS NON-LINEAR SYSTEM

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ABSTRACT
In this paper we present a simple methods to solve a differential equation system applying Adomian decomposition method and power series.

Key words: *Adomian decomposition method ,power series ,HIV/aids epidemic*.

1. INTRODUCTION

A system of first order differential equation can be considered as:

$$y' = f(x,y) \quad , \; x \in [0,T] \, , |y| < +\infty \, ,$$

$$y(0) = y_0 \, , \qquad (1)$$

Theoretical solution of the system is y(x). let y_n be an approximation $y(x_n)$ .here

$$y = [y_1, y_2, \dots, y_m]^T,$$

$$f = [f_1, f_2, \dots, f_m]^T,$$

And $$y_0 = [y_{01}, y_{02}, \dots, y_{0m}]^T.$$

Are elements of C^m . where each equation represent the first derivate of one of the unknown functions as a mapping depending on the independent variables x, and m unknown functions

$f_1, f_2, \dots, f_m$.

Since every ordinary differential equations of order n can be written as a system consisting of n ordinary differential equations of order one , we restrict our study to a system of differential equations of the first order.

The solutions of (1) can be assumed that

$$y = y_0 + ex \qquad (2)$$

where *e* is a vector function which is the same size as y_0 . Substitute (2) into (1) and neglect higher order term. We have the linear equation of *e* in the form

$$Ae = B \qquad (3)$$

where *A* and *B* is a constant matrixes. Solve this equation of (3), the coefficients of *x* in (2) can be determined. Repeating above procedure for higher order terms, we can get the arbitrary order series of the solutions for (1).
In this paper we consider the system of differential equation where

$$y' = f(t,y) \quad , \qquad t \in [0,T] \, , |y| < +\infty$$

$$y = [y_1, y_2]^T \quad , y_1 = S \, , \quad y_2 = I \, ,$$

$$f = [f_1, f_2]^T$$

And $$y_0 = [S_0, I_0]^T.$$

We can write the system:

$$\begin{cases} \dfrac{ds(t)}{dt} = \delta s(t) - \alpha_1 I(t)S(t) \\ \dfrac{dI(t)}{dt} = -\mu I(t) + \alpha_2 S(t)I(t) \end{cases}$$

Where

S(t) : the population of known susceptible at time 't'

I(t) : the population of known infectious at time 't'

$\propto_1$: invidious rate that provide from the known susceptible and become infectious.

$\propto_2$: rate of infectious invidious provide from unknown susceptibles.

μ: Death rate.

δ: immigration rate in the population of susceptible.

This system has two equilibrium point;

(S(t)=0;I(t)=0) et ($S(t) = \frac{\propto_2}{\mu}$;$I(t) = \frac{\delta}{\alpha_1}$)

This system is non linear and can't be solved by elementary functions .most of authors investigate the solution of this problem by power series method , Homotopy analysis method and Adomian decomposition method ,or power series ; with specific values of $\propto_1$,$\propto_2$,μ,δ.

In the HIV/AIDS epidemic always the parameters depends from time and in mathematique the problem appears when the parameters $\propto_1 (t)$,$\propto_2 (t)$,$\mu(t)$,$\delta(t)$ are analytic function and added to the model.
The new differential equations ,incorporating the above functions are as follows:

$$\begin{cases} \dfrac{ds(t)}{dt} = \delta(t)s(t) - \alpha_1(t)I(t)S(t) \\ \dfrac{dI(t)}{dt} = -\mu(t)I(t) + \alpha_2(t)S(t)I(t) \end{cases} \qquad (1)$$

With the initial conditions:

$S(0) = S_0$, $I(0) = I_0$.

In this paper we present a new methods for the solvability of the system (1) and compare it to the Modified Decomposition Method (ADM).

2. SOLVABILITY OF SYSTEM (1)

II-1 idea one

Dividing through the dI/dt equation by I gives the follow equation:

$$\frac{dlnI(t)}{dt} = \propto_2 S(t) - \mu(t)$$

Which can then be fit to a line y=$\propto_2 x - \mu$, where y= $\frac{dlnI(t)}{dt}$ and x=S, using the method of least squares. This will gives value for $\propto_2$ and μ and their standard deviation and variances.

$\propto_2$=cov(S, $\frac{dlnI(t)}{dt}$)/var(S)

Than this justified that the parameters depends for time.

II-2 idea two

Dividing through the dS/dt equation by S we obtain:

$$\frac{dlnS(t)}{dt} = -\propto_1 I(t) + \delta(t)$$

If we integer this equation we found

$$S(t) = exp[\int(-\propto_1 I(t) + \delta(t))dt] \qquad (2.2)$$

Than the system (1) can be transformed into another system in which the equations are decoupled and solvable separately. This separability can be obtained if we suppose from (2.2) that :

$$S(t) = \emptyset(t)I(t) \qquad (2.3)$$

Where $\emptyset(t)$ is unknown function with $\emptyset(0) = S_0/I_0$

The substitution (3) into (1) gives:

$$\begin{cases} \frac{dI(t)}{dt} = I(t)\left[\frac{\delta(t)\emptyset(t)-\emptyset'(t)}{\emptyset(t)}\right] -\propto_1(t)I^2(t) \\ \frac{dI(t)}{dt} = -\mu(t)I(t) +\propto_2(t)\emptyset(t)I^2(t) \end{cases} \qquad (2.4)$$

Where $\emptyset'(t) = \frac{d\emptyset}{dt}$.

Equating coefficients of system (2.4) we obtain:

$$\frac{\emptyset'(t)}{\emptyset(t)} = \delta(t) + \mu(t),$$

And

$$\emptyset(t) = -\frac{\propto_1(t)}{\propto_2(t)}$$

That is

$$\frac{\propto'_1(t)}{\propto_1(t)} - \frac{\propto'_2(t)}{\propto_2(t)} = \delta(t) + \mu(t) .$$

Than it may be shown that it is possible to solve the equations of system (2.4) separately.

Substitute $z = I^{-1}$,and multiply both sides by I^{-2} we obtain

$$z'(t) - \mu(t)z(t) = -\propto_2 (t)\emptyset(t)$$

Is a first linear differential equation can be solved easly and gives us

$$z(t)exp\left[-\int \mu(t)dt\right] = -\int \left[\propto_2 (t)\emptyset(t)exp\left(-\int \mu(t)dt\right)dt\right] + k$$

Solving this equation for z leads to an explicit solution. After z has been found ,the solution of (1) is given by $z = I^{-1}$, and by (2.3) we find s(t). than we have proved the following result.

Lemma 1

The system (1) may be transformed into another system in which the equations are decoupled and solvable separately by $S(t) = \emptyset(t)I(t)$ *if and only if the following conditions are satisfied:*

$$\frac{\propto'_1(t)}{\propto_1(t)} - \frac{\propto'_2(t)}{\propto_2(t)} = \delta(t) + \mu(t).$$

3. MODIFIED DECOMPOSITION

In this section we present a variation of the decomposition method using concepts of decomposition method: partial solutions, the A_n and transformations of series using A_n ,we define the operator L in system (1) by L=d/dt and we expect the decomposition of the solutions S(t),I(t) as a sum of components to be defined by

$$S(t) = \sum_{n=0}^{\infty} I_n t^n \ , \quad I(t) = \sum_{n=0}^{\infty} I_n t^n.$$

We let

$$\delta(t) = \sum_{n=0}^{\infty} \delta_n t^n \ ,$$

$$\propto_1 (t) = \sum_{n=0}^{\infty} \propto_{1n} t^n,$$

$$\propto_2 (t) = \sum_{n=0}^{\infty} \propto_{2n} t^n,$$

$$\mu(t) = \sum_{n=0}^{\infty} \mu_n t^n,$$

The substitution yields

$$\sum_{n=0}^{\infty} S_n t^n = S_0 + \int_0^t \sum_{n=0}^{\infty} t^n \sum_{k=0}^{n} \delta_k S_{n-k} dt - \int_0^t \sum_{n=0}^{\infty} t^n \sum_{k=0}^{n} \propto_1 A_{n-k} dt,$$

$$\sum_{n=0}^{\infty} I_n t^n = I_0 - \int_0^t \sum_{n=0}^{\infty} t^n \sum_{k=0}^{n} \mu_k I_{n-k} dt + \int_0^t \sum_{n=0}^{\infty} t^n \sum_{k=0}^{n} \propto_2 A_{n-k} dt,$$

Where the polynomials A_k ,k=0,1,….,are called Adomian polynomials:

$$A_0 = S_0 I_0,$$

$$A_1 = S_1 I_0 + S_0 I_1,$$

$$A_1 = S_2 I_0 + S_1 I_1 + S_0 I_2,$$

….

We now carry out the above integration we write

$$\sum_{n=0}^{\infty} S_n t^n = S_0 + \sum_{n=0}^{\infty} \frac{t^{n+1}}{n+1} \sum_{k=0}^{n} \delta_k S_{n-k} - \sum_{n=0}^{\infty} \frac{t^{n+1}}{n+1} \sum_{k=0}^{n} \propto_1 A_{n-k} \,;$$

$$\sum_{n=0}^{\infty} I_n t^n = I_0 + \sum_{n=0}^{\infty} \frac{t^{n+1}}{n+1} \sum_{k=0}^{n} \propto_2 I_{n-k} - \sum_{n=0}^{\infty} \frac{t^{n+1}}{n+1} \sum_{k=0}^{n} \mu_k A_{n-k} \,;$$

In the right terms if we replace n by n-1 we find

$$\sum_{n=0}^{\infty} S_n t^n = S_0 + \sum_{n=0}^{\infty} \frac{t^n}{n} \sum_{k=0}^{n-1} \delta_k S_{n-1-k} - \sum_{n=0}^{\infty} \frac{t^n}{n} \sum_{k=0}^{n-1} \propto_1 A_{n-1-k} \,;$$

$$\sum_{n=0}^{\infty} I_n t^n = I_0 + \sum_{n=0}^{\infty} \frac{t^n}{n} \sum_{k=0}^{n-1} \propto_2 I_{n-1-k} - \sum_{n=0}^{\infty} \frac{t^n}{n} \sum_{k=0}^{n-1} \mu_k A_{n-1-k} \,;$$

Finally we can equate coefficients of like powers of t on the left side and on the right side to obtain the recurrence relations for the coefficients. Thus

$$S(0) = S_0$$

$$I(0) = I_0$$

$$S_n = \frac{1}{n} \left[\sum_{k=0}^{n-1} \delta_k S_{n-1-k} - \sum_{k=0}^{n-1} \propto_1 A_{n-1-k} \right], \quad n \geq 1 \,,$$

$$I_n = \frac{1}{n} \left[\sum_{k=0}^{n-1} \propto_2 I_{n-1-k} - \sum_{k=0}^{n-1} \mu_k A_{n-1-k} \right], \quad n \geq 1 \,,$$

The final solution are given by

$$S(t) = \sum_{n=0}^{\infty} S_n t^n \quad \text{and} \quad I(t) = \sum_{n=0}^{\infty} I_n t^n.$$

We can illustrate with examples to show that the results obtained are the same for the different methods gives in this paper.

Application

Example 1

Consider the following differential equation system

$$\frac{dS(t)}{dt} = 2I^2(t),$$

$$\frac{dI(t)}{dt} = e^{-t}I(t)\,, \qquad (1)$$

With the initial conditions:

S(0)=1, I(0)=1.

The exact solutions are:

$S(t) = e^{2t}$, $I(t) = e^{t}$.

From boundary conditions , the solution can be supposed as

$$S(t) = S_0 + e_1 t \;\Rightarrow S(t) = 1 + e_1 t$$

$$I(t) = I_0 + e_1 t \;\Rightarrow I(t) = 1 + e_2 t \qquad (2)$$

Substitute (2) into (1) and neglect higher order terms , we have

$$e_1 - 2 + \Phi(\mathrm{t}) = 0$$

$$e_2 - 1 + \Phi(\mathrm{t}) = 0 \qquad (3)$$

Consider the system (1) which we can formulate the system as

$$Ae=B \qquad (4)$$

Where $A = \begin{bmatrix} 1 & O \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$

From (4) we have linear equations

$$\begin{bmatrix} 1 & O \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Solve this linear equation .we have

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

And

$$S(t) = 1 + 2t$$

$$I(t) = 1 + t \qquad (5)$$

From (5) the solution of (1) can be supposed as

$$S(t) = 1 + 2t + \ e_1 t^2$$

$$I(t) = 1 + t + \ e_2 t^2 \qquad (6)$$

In like manners .substitute (6) into (1) and neglect higher order terms, we have

$$(2\,e_1 - 4)t + \Phi(\mathrm{t}^2) = 0$$

$$(2\,e_2 - 1)t + \Phi(\mathrm{t}^2) = 0$$

Where $A = \begin{bmatrix} 2 & O \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$, $e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$

We can found

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 2 \\ \frac{1}{2} \end{bmatrix}$$

There for

$$S(t) = 1 + 2t + \ 2t^2$$

$$I(t) = 1 + t + \frac{1}{2}\,t^2$$

Repeating above procedure we have

$$S(t) \approx 1 + 2t + \ 2t^2 + \frac{4}{3}t^3 + \frac{2}{3}t^4 + \frac{4}{15}t^5 \ldots$$

$$I(t) \approx 1 + t + \ \frac{1}{2}t^2 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 \ldots$$

Which are the partial sum of the taylor series of the solution S(t) and I(t),respectively.

This solution are coinciding with the exact solution of (1).

Example 2
Consider the system (1) which

$\delta(t) = -t,\ \propto_1 (t) = -t\ ,\ \mu(t) = t\ \ ,\ \propto_2 (t) = t\ , S_0 = 2\ ,\ I_0 = 2.$

All the conditions of lemma I are satisfied and direct calculation gives the exact solutions

$$S(t) = \frac{2}{2-e^{\frac{t^2}{2}}}\ ,\ I(t) = \frac{2}{2-e^{\frac{t^2}{2}}}$$

Applying the above scheme ,we obtain

$S_0 = 2\,, \qquad I_0 = 2,$

$S_1 = 0, \qquad I_1 = 0,$

$S_2 = 1, \qquad I_2 = 1,$

$S_3 = 0, \qquad I_3 = 0,$

$S_4 = \frac{3}{4}, \qquad I_3 = \frac{3}{4},$

…

Then the terms S(t) and I(t) can be written as

$$S(t) = I(t) = 2 + t^2 + \frac{3}{4}t^4 + \frac{13}{24}t^6 + \cdots,$$

We can easily observed that this last solution are equivalent to the exact solutions.

4. CONCLUSION

In the above discussion it was shown that, different new results concerning the solvability of a system of two type model of non linear equations can be solved and expressed in terms of usual mathematical functions . A technique has also been considered to avoid the computation of consistent initial conditions. The numerical example has been presented to show that the approach is promising and the research is worth to continue in this direction.

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