STATISTICAL ANALYSIS OF REDUCED ROUND COMPRESSION FUNCTIONS OF SHA-3 FINALISTS

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ABSTRACT

National Institute of Standards and Technology announced a competition in 2008, of which the winner was acknowledged as the new hash standard SHA-3. There were 5 finalist algorithms which are selected among 51 first round candidates. In this paper, we apply statistical analysis to the finalists by using two different methods, and observe how conservative the algorithms are in terms of randomness. The first method evaluates 256-bit outputs, obtained from reduced round versions of the algorithms, through statistical randomness tests. On the other hand, the second method evaluates the randomness of the reduced round compression functions based on certain cryptographic properties. This analysis gives a rough idea on the security factor of the compression functions.

Keywords: *Statistical Randomness Testing, Cryptographic Randomness Testing, Hash Functions, SHA-3 Competition*

1. INTRODUCTION

Following the cryptanalysis of the widely used hash functions MD5 and SHA-1, National Institute of Standards and Technology (NIST) settled a public competition, of which the winner was acknowledged as the new hash standard SHA-3 [1]. 51 submissions out of 64 were qualified as first round candidates as of October 2008, and 14 second round and 5 finalist algorithms were announced in July 2009 and December 2010 respectively. The winner of the competition was announced on October 2012.

A basic property that block ciphers and hash functions are expected to satisfy is indistinguishability from a random mapping. Therefore, the statistical analysis of the algorithms is of great importance. The statistical analysis of the Advanced Encryption Standard (AES) candidate algorithms was done by Soto [2], using the NIST Test Suite [3]. However, this suite is designed to evaluate Pseudo Random Number Generators (PRNGs). Moreover, some of the tests in the suite require sequences of length at least 10^6 bits, due to the approximations and the asymptotic approaches used in the necessary computations. Soto overcame this problem by applying the sixteen tests in the NIST Test Suite to the concatenation of the outputs of candidate algorithms, which correspond to eight data types such as key avalanche, low density plaintext, high weight keys, and the like, to obtain 189 p-values. In 2010, Sulak et. al. proposed an alternative approach, where the tests are applied to sets of outputs, rather than their concatenations [4]. In that work, the authors choose seven tests suitable for 256-bit sequences, and they improve the evaluation method of the NIST Test Suite for short sequences.

Recently, a package of cryptographic randomness tests is introduced by Doğanaksoy et. al. [5]. This package consists of statistical tests designed based on certain cryptographic properties of block ciphers and hash functions such as diffusion, confusion and collision resistance. The authors apply the package to the AES finalist algorithms, and observe the number of rounds where the randomness is achieved more precisely than the previous methods.

We focus on black-box analysis of the algorithms from a randomness perspective. We define the reduced versions of the algorithms, and apply two methods to observe the randomness properties of the compression functions of the candidate algorithms. In the first method, we apply the proper tests for 256-bit sequences in the NIST Test Suite and use the evaluation method mentioned in [4]. For this purpose, we construct data sets having certain structures and test the corresponding output sets. In the second method, we apply the package of cryptographic randomness tests presented in [5] to the compression functions of the algorithms, and observe how conservative the designs are.

This paper is organized as follows. In Section 2, we explain the statistical randomness tests used in this work; the method to form data sets from the candidate algorithms, and the evaluation method. In Section 3, we give the brief descriptions of the cryptographic randomness tests. In section 4 we apply both methods to SHA-3 finalist algorithms and present the results. And in the last section we conclude the paper with our plans for the future work.

2. STATISTICAL RANDOMNESS TESTING

Statistical randomness tests are functions that take arbitrary length input and produce a real number between 0 and 1 called the *p*-value, by evaluating certain randomness properties of the given input. When testing generators for

randomness, generally more than one statistical randomness test is applied, and there are several test suites [3, 6-9] designed for this purpose, which include a variety of tests to observe randomness properties of sequences.
We apply the method of Sulak et. al. [4] to the SHA-3 candidates. We define 5 data sets similar to those described in [2], and apply proper statistical randomness tests of the NIST Test Suite to the algorithms.

2.1. NIST Test Suite

NIST Test Suite (published in 2001) originally consisted of 16 tests but the *Fast Fourier Transform Test* was later discarded due to a problem observed, by NIST in 2009. The proper tests for 256-bit sequences in the mentioned suite are Frequency Test, Frequency Test within a Block, Runs Test, Test for the Longest Run of Ones in a Block, Serial Test, Approximate Entropy Test and Cumulative Sums Tests. In this subsection, we give brief descriptions of the mentioned statistical randomness tests.
The subject of Frequency Test and Frequency Test within a Block is the weight of the sequence. In the Frequency Test, the p-value depends only on the weight of the sequence. Frequency Test within a Block separates the sequence into m bit blocks and the variables used for computing the *p*-value are the weights of each block. We take *m=32* for this test.
The subject of the Runs Test is the number of runs (an uninterrupted maximal subsequence of identical bits) in the sequence, and the p-value is determined by the weight of the sequence and the number of runs in the sequence. Whereas, Test for the Longest Run of Ones in a Block separates the sequence into *m* bit blocks and the subject of this test is the longest run of ones of these blocks. We take *m=8* for this test.
Serial Test and Approximate Entropy Test are related to the entropy and the subjects of these tests are overlapping *m* and *m+1* bit blocks of the sequence. We take *m = 2* for Serial Test and *m = 1* for Approximate Entropy Test. The *p*-values for both tests are determined by the frequencies of 1-bit and 2-bit blocks.
Cumulative Sums Test evaluates the sequence as a random walk and the subject of the test is the maximum distance of the random walk from the *x*-axis. This test is applied twice, forward and backward through the input sequence, and two *p*-values are obtained.

2.2. Data Sets

Any cryptographic module must have the property that the redundancies at the input should not leak to the output. Depending on this understanding, we construct data sets having certain structures. Then we try to observe whether this structure is inherited by the module. In all of the tests, it is assumed that the compression function is a mapping $F_2^m \times F_2^n \mapsto F_2^n$, where *m* is the size of the message block and *n* is the size of the chaining variable.

2.2.1. Low Density Message (LW)

The data set is formed by binary strings of low weight. The message length of candidate algorithms are 32, 256, 512, 1088 and 1536 bits. In the 32-bit case, the data set consists of 32-bit binary strings of weight not exceeding 5. In 256-bit case, this bound is 3 and in the remaining cases the strings of weights 1 and 2 are taken. The number of sequences for different message lengths is given in Table 1.

2.2.2. High Density Message (HW)

The data set is formed by choosing high density messages and they are formed similar to the low density message case. In other words, the inputs of the low density messages are complemented bitwise.

Table 1. The number of sequences for different message lengths

m	weight	# Sequences
32	≤ 5	242 824
256	≤ 3	2 796 416
512	≤ 2	131 328
1088	≤ 2	592 416
1536	≤ 2	1 242 676

2.2.3. 1-Bit Message Avalanche (Av1)

A random message *R* of length *m* is chosen. Then each time by flipping another bit of *R*, a set of *m* messages is formed and the corresponding hash values are obtained. The same procedure is applied to *k* different messages to get a set of *mk* sequences. The values of *k* for different input lengths are given in Table 2.

Table 2. Selection of k for 1-Bit Message Avalanche and Message Rotation

m	*k*	*mk*
32	32768	1 048 576
256	4096	1 048 576
512	2048	1 048 576
1088	963	1 047 744
1536	682	1 047 552

2.2.4. 8-Bit Message Avalanche (Av8)
A random message $R = b_0b_1...b_s$ of length $m=8s$ is chosen, where b_i is a 8-bit word for $i=0,1,...,s$. Then, by assigning all possible 256 values to each word, $255m/8$ different messages are formed and the corresponding hash values are obtained. The same procedure is applied to k different messages and the values of k for different input lengths are given in Table 3.

Table 3. Selection of k for 8-Bit Message Avalanche

m	*k*	*mk*
32	1028	1 048 560
256	128	1 044 480
512	64	1 044 480
1088	30	1 040 400
1536	21	1 028 160

2.2.5. Message Rotation (Rot)
A random message R of length m is chosen. A set of m messages is formed by consecutive 1-bit rotations of R and the corresponding hash values are obtained. The same procedure is applied to k different messages to get a set of mk sequences.

2.3. Evaluation Method
Each data set produces a large set of p-values. Our first task is to obtain a single p-value associated with the data set under consideration. We apply χ^2 Goodness of Fit Test by partitioning the interval [0,1] into 10 equal subintervals as follows. Let t be the number of sequences in a data set, and F_i be the number of p-values observed in subinterval i for $i=1,2,...,10$. Then the χ^2 value and the corresponding p-value are calculated as

$$\chi^2 = \sum_{i=1}^{10} \frac{(F_i - t \cdot p_i)^2}{t \cdot p_i} \text{ and } p-value = igamc(\frac{9}{2}, \frac{\chi^2}{2})$$

where *igamc* is the incomplete gamma function. The subinterval probabilities can be found in [4].

If we assume that the tests are independent, then the probability of Type I error (the data is random, but the null hypothesis is rejected) is computed as $1-(1-\alpha)^s$, where α is the level of level of significance, and s is the number of tests. We choose α =0,0001, in other words, if p-value≥0,0001 we conclude the data is random, and *0,0799%* random data will be concluded as non-random.
In the case of α =0,001, *0,797%* of random data will be concluded as non-random. Since we test several reduced versions of the algorithms, it is not much surprising to obtain a p-value <0,001. In that case we put a flag and repeat the test with a different data set (In the case of LW and HW, as it is not possible to change the data set, we change the weights of the strings to obtain new data sets.). If p-value <0,001 is obtained for the same test, we conclude that the hash output is non-random.

3. CRYPTOGRAPHIC RANDOMNESS TESTS
Apart from statistical randomness testing of the outputs of the algorithms using test packages which consist of statistical randomness tests designed to evaluate PRNGs, a different method can be applied to evaluate the randomness based on cryptographic properties. A recent package is designed by Doğanaksoy et. al [5] to evaluate block ciphers and hash functions, via cryptographic randomness tests. Since the authors obtain more precise results for AES finalist algorithms, we believe it is worthwhile to test the SHA-3 finalist algorithms using the mentioned package.

In this section, brief descriptions of the cryptographic randomness tests used in the work are given. The details of these tests with the computation of probabilities can be found in [5].

3.1. Strict Avalanche Criterion Test (SAC Test)

A function is said to satisfy the strict avalanche criterion, whenever a single input bit is complemented, each of the output bits changes with probability a half. The aim of the *SAC Test* is to measure if any flipped bit of the plaintext bit affects the ciphertext bits as if it is a random mapping. In this test, an $n \times m$ matrix, so called *SAC Matrix*, is formed and this matrix is evaluated for randomness.

3.2. Linear Span Test

Nonlinearity is one of the basic design criteria for hash functions. The *Linear Span Test*, a randomness test based on nonlinearity, examines the linear dependence of the outputs formed from a highly linearly dependent set of inputs. In this test, a matrix is formed by the outputs of the compression function corresponding to this set, and the rank of this matrix is compared to the rank of a random binary matrix.

3.3. Collision Test

Collision resistance is an important design criterion for hash functions, which means that it should be hard to find two messages with the same hash value. The *Collision Test* is designed to evaluate randomness based on collision resistance. However, as the output space is large, the collisions in a subset of output space are considered. In other words, the focus of the *Collision Test* is the number of collisions in specific bits of the output, which can be considered as near collision.

3.4. Coverage Test

Hash functions are one-way functions and required to behave like a random mapping, which implies that the expected coverage is about *63%* of the size of the input set. The *Coverage Test* examines the size of the output set (coverage) formed from a subset of the domain.

Similar to the statistical randomness tests, if p-value<0,0001, we consider the data as non-random for the tests mentioned in this section. If p-value<0,001, we apply the test once more to check whether such a case is coincidental or not. If p-value<0,001 is obtained again we consider the data as non-random.

4. APPLICATION

In this section, we apply statistical analysis to the SHA-3 finalist algorithms. For this purpose, first we define reduced versions of the finalists. Then, we apply statistical randomness tests and cryptographic randomness tests to observe the number of rounds that the algorithms achieve randomness.

When defining reduced versions of the algorithms, we only reduce the number of compression function rounds and ignore the finalization functions, if exist. Reduced rounds of finalization functions can also be taken into account, but this process together with the reduced rounds of compression functions increases the complexity of the tests by introducing plenty of versions for only one algorithm.

For the sake of simplicity and ease of computations, we apply statistical analysis only to the 256-bit versions of the algorithms. In the case of cryptographic randomness testing, we apply the package directly to the compression functions of the candidate algorithms. Therefore, initialization and padding are excluded. In the case of statistical randomness testing, all the algorithms are tested for 256-bit output values, since the internal state size of the algorithms vary from 256 bits to 1600 bits and one needs to find the exact χ^2 distribution bin values of all the statistical randomness tests for each state size, which is not feasible.

4.1. Statistical Randomness Test Results

When applying statistical randomness tests to the candidates, we reduce the algorithms by excluding finalization, setting IV and salt values to 0. Moreover, we discard the padding operation and the output is generated from a single message block which is iterated only once.

For statistical tests, we produce approximately 2^{24} 256-bit outputs for each algorithm from the data sets mentioned in Section 2.2. In Table 4, for each algorithm, the maximum number of rounds that the non-randomness is observed is given together with the data sets which produce the corresponding results.

Table 4. Statistical randomness test results for the 256-bit versions of the algorithms

Algorithm	#Rounds	#Rounds Random	Data Sets
Blake [10]	14	2	Av1
Grøstl [11]	10	2	LW, HW, Av1, Rot
JH [12]	42	7	LW, HW, Av1, Av8
Keccak [13]	24	2	LW, HW, Av1, Av8
Skein [14]	72	7	All

4.2. Cryptographic Randomness Test Results

Different from statistical testing of the outputs of the algorithms, cryptographic randomness tests are applied directly to the compression functions of the candidate algorithms. As mentioned in [5], the core operations of each test are advised to be repeated 2^{20} times. However, especially in Coverage Test and Collision Test, 2^{12} calls (or 2^{14} calls depending on the chosen parameter) to the compression functions of the algorithms is required. This poses some limitations towards the number of times the cores of the tests can be repeated, since the performances of the reference codes provided in the submissions of the finalist algorithms vary significantly. Hence, we repeat the core operations of SAC Test 2^{20} times, Linear Span Test 2^{16} times, Collision Test(16) 2^{16} times, Collision Test(20) 2^{12} times, Coverage Test(12) 2^{16} times, Coverage Test(14) 2^{12} times to have reasonable running times for all algorithms.

In Table 5, the maximum number of rounds that the non-randomness is observed is given together with the tests from which the best results are obtained for each algorithm.

Table 5. Cryptographic randomness test results for the compression functions of the 256-bit versions of the algorithms

Algorithm	#Rounds	#Rounds Random	Tests that give the best results
Blake	14	1	Col. Test, Cov. Test, SAC Test
Grøstl	10	2	Col. Test (20)
JH	42	10	Cov. Test (12), SAC Test
Keccak	24	3	SAC Test
Skein	72	8	SAC Test

5. CONCLUSION AND FUTURE WORK

In this work, we applied statistical randomness tests and cryptographic randomness tests on reduced round variants of the compression functions of the SHA-3 competition finalist algorithms. We observed up to how many rounds a compression function behaves non-random. Although the results do not imply cryptographic weaknesses about the hash functions directly, it may provide a starting point for the cryptanalysis. The main outcome of this analysis is the knowledge of how conservative the algorithms are.

As a future work, the analysis can be extended to other hash sizes. Moreover, finalization functions can be considered and more reduced versions can be constructed. Also new data sets can be defined for the statistical randomness testing.

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