EXTENSIONAL $\left(\frac{G'}{G}\right)$-EXPANSION METHOD AND NEW EXACT SOLUTIONS FOR (N+1) –DIMENSIONAL C-I EQUATION

Xiu-xiu Wang, Ze-jian Cui & Shi-qiang Feng
College of Mathematics and Information, China West Normal University, Sichuan Nanchong 637009, China
E-mail: lingge112@163.com; cuizejian2007@163.com

ABSTRACT

By using the extensional $\left(\frac{G'}{G}\right)$-expansion method for studying the complex –valued (n+1)-dimensional Chafee-Infante equation, three types of new exact solutions of complex-valued Chafee-Infante equation have been found, and the exact solutions of constant state (n+1)-dimensional Chafee-Infante equation have been gotten.

Keywords: *Chafee-Infante equation; the* $\left(\frac{G'}{G}\right)$*-expansion method; the homogeneous balance principle; exact solutions*
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1. INTRODUCTION

In recent years, with the deep research of nonlinear equations, a series of effective approaches have been developed. In [4], Wang applied the homogeneous balance method to obtain the non-trivial wave front solution of the (n + 1)-dimensional Chaffee-Infante equation, and the constant state Chaffee-Infante equation was given exact solution; In [2],Zhang Guicheng, Li Zhibin used the hyperbolic function expansion method to get the solitary wave solution of the (1 + 1)-dimensional Chaffee-Infante equation . Reference [7], quoted the extensional $\left(\frac{G'}{G}\right)$-expansion method and got new explicit exact solution of the (1 + 1)-dimensional Chaffee-Infante equation . Literature [5], Xia Hongming managed to get Chaffee-Infante class of exact solutions of the equation by direct hypothesis method; In the document [1], Wang Xiankun, Shi Changbo discussed the exact solution of the (n + 1)-dimensional Wick stochastic Chaffee-Infante equation.

On the basis of Ref. [7], in this paper, we using the extensional $\left(\frac{G'}{G}\right)$-expansion method proposed by Wang et al. to consider (n + 1)-dimensional Chaffee-Infante equation:

$$u_t - \Delta u + \lambda(u^3 - u) = 0 \qquad (1.1)$$

Where $\lambda > 0$ is the diffusion coefficient., where $\Delta = \sum_{j=1}^{n} \frac{\partial^2}{\partial x_j^2}$ is the n-dimensional Laplace operator. This paper aims to look for the non-trivial and non-constant solutions of the equation(obviously, the equation has the trivial solution $u = 0$ and constant solution $u = \pm 1$) .Firstly, introducing the extended $\left(\frac{G'}{G}\right)$-expansion method; secondly, in part 3 using the expansive $\left(\frac{G'}{G}\right)$-expansion method to solve the (n + 1)-dimensional Chaffee-Infante equation, and finally, in the fourth part this method is to be used in constant state Chaffee-Infante equation

$$-\Delta u + \lambda(u^3 - u) = 0, \quad \lambda > 0 \qquad (1.2)$$

also received the new exact solution.

2. THE EXTENDED $\left(\frac{G'}{G}\right)$-EXPANSION METHOD

Suppose that a nonlinear partial differential equations

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \cdots) = 0, \qquad (2.1)$$

Say in two independent variables x and t, $u = u(x,t)$ is an unknown complex-valued function, P is a about $u = u(x,t)$ and the order of the partial derivative polynomial, contains the highest order derivative term and nonlinear term.

The main steps of the expansive $\left(\frac{G'}{G}\right)$ expansion method are the following:

Step 1 Take travelling wave transform, $\xi = \sum_{j=1}^{n} k_j x_j - ct + l$ which $k_j\ (j = 1,2,...,n)$ are wave numbers, c is frequency, are all undetermined constant, and l for arbitrary real constant. Then

$$u(x,t) = u(z), z = i\xi$$
$$u_t = -icu', u_{x_j} = ik_j u', u_{tt} = c^2 u'', u_{x_j t} = ck_j u'', u_{x_j x_j} = -k_j^2 u'' \qquad (2.2)$$

Thus (2.1) will become about $u = u(z)$ the ordinary differential equation (ODE)

$$P(u, -icu', ik_j u', c^2 u'', ck_j u'', -k_j^2 u'', \cdots) = 0, \qquad (2.3)$$

where $u' = \frac{du}{dz}$.

Step 2 Supposing that the solution of the differential equation can be expressed by a polynomial in $\left(\frac{G'}{G}\right)$ as the following:

$$\mathrm{u(z)} = \sum_{i=0}^{m} a_i \left(\frac{\mathrm{G}'}{\mathrm{G}}\right)^i \qquad (2.4)$$

Among them, $a_i\ (i = 0,1,2,\cdots,m)$ is undetermined constant, and m value is sure by considering the homogeneous balance principle between the highest order derivatives and the nonlinear term of the equation. $G = G(z)$ satisfy the following the second order ordinary differential equation:

$$G'' + \alpha G' + \beta G = 0, \qquad (2.5)$$

Among them, α and β are real constants to be determined later.

Step 3 By substituting (2.4) into eq. (2.3) and using the ODE (2.5) collecting all terms with the same order of $\left(\frac{G'}{G}\right)$ together. We can convert the eq. (2.3) into another polynomial in $\left(\frac{G'}{G}\right)$, equating each coefficient of this polynomial to zero, yields a set of nonlinear algebraic equations for $a_i(i = 0,1,2,\cdots,m)$, $k_j(j = 1,2,...,n), c, \alpha, \beta,\ l$.

Step 4 Assuming that the constant $a_i(i = 0,1,2,\cdots,m)$, can be obtained by solving the algebraic equations in step 3. Then substituting (2.5) and $a_i(i = 0,1,2,\cdots,m)$, into eq. (2.4), we will get more travelling wave solutions of (2.1).

Next we will introduce of the $\left(\frac{G'}{G}\right)$ expansion method to solve the exact solution of Chaffee-Infante equation (n + 1)-dimensional in details.

3. NEW EXACT SOLUTION OF THE (N+1)-DIMENSIONAL COMPLEX-VALUED CHAFFEE-INFANTE

We use the second part of the $\left(\frac{G'}{G}\right)$ expansion method for the equation (1.1) of the exact travelling wave solutions.

First of all, supposing (n + 1)-dimensional Chaffee-Infante equation has the following form of travelling wave solutions:

$$u(x,t)=u(z), z=i(\sum_{j=1}^{n}k_j x-ct+l) \qquad (3.1)$$

Where $k_j\,(j=1,2,...,n)$ and c are real constants to be determined later, and substituting (3.1) into eq. (1.1), then

$$-ciu'+\sum_{j=1}^{n}k_j^2u''+\lambda(u^3-u)=0 \qquad (3.2)$$

Where $u'=\frac{du(z)}{dz}$.Considering the homogeneous balance principle， that $m+2=3m$ we obtain $m=1$.

It can be set of the solution of the equation (3.2) for the following contains $\left(\frac{G'}{G}\right)$ the form of polynomial:

$$u(z)=a_1\left(\frac{G'}{G}\right)+a_0 \qquad (3.3)$$

Where $G'=\frac{dG}{dz}$.

And then

$$u'(z)=a_1\frac{G''G-(G')^2}{G^2}=a_1(-\alpha\frac{G'}{G}-\beta-(\frac{G'}{G})^2) \qquad (3.4)$$

$$u''(z)=a_1(-\alpha\frac{G''G-(G')^2}{G^2}-2\frac{G'}{G}\frac{G''G-(G')^2}{G^2})$$

$$=a_1[(\alpha^2+2\beta)\frac{G'}{G}+3\alpha(\frac{G'}{G})^2+2(\frac{G'}{G})^3+\alpha\beta] \qquad (3.5)$$

$$u^3-u=a_1^{\ 3}(\frac{G'}{G})^3+3a_1^{\ 2}a_0(\frac{G'}{G})^2+(3a_1a_0^{\ 2}-a_1)(\frac{G'}{G})+a_0^{\ 3}-a_0 \qquad (3.6)$$

and substituting (3.3)-(3.6) into eq. (3.2) then, the left hand side of equation is about $\left(\frac{G'}{G}\right)$ polynomial, make its each order coefficient is zero. Get the following about algebra equations for a_1, a_0, α, β :

$$(\frac{G'}{G})^3:\ \lambda a_1^{\ 3}+2a_1\sum_{j=1}^{n}k_j^2=0$$

$$(\frac{G'}{G})^2:\ 3\lambda a_1^{\ 2}a_0+3a_1\alpha\sum_{j=1}^{n}k_j^2+ca_1i=0$$

$$(\frac{G'}{G})^1: \; c\alpha a_1 i + a_1(\alpha^2+2\beta)\sum_{j=1}^{n}k_j^2 + \lambda(3a_1 a_0{}^2 - a_1) = 0$$

$$(\frac{G'}{G})^0: \; c\beta a_1 i + a_1\alpha\beta\sum_{j=1}^{n}k_j^2 + \lambda(a_0{}^3 - a_0) = 0$$

Solving the algebraic equations above, yields:

$$a_0 = \frac{\sqrt{3}}{3}\sqrt{\frac{\alpha^2}{\alpha^2-3\beta}} \;, \qquad a_1 = \sqrt{\frac{3}{\alpha^2-3\beta}}$$

$$c = -\frac{3}{2}\lambda\frac{\alpha}{\alpha^2-3\beta}\mathrm{i}, \sum_{j=1}^{n}k_j^2 = -\frac{3}{2}\lambda\frac{1}{\alpha^2-3\beta} \tag{3.7}$$

Substituting (3.7) into (3.3) then

$$u(z) = \sqrt{\frac{3}{\alpha^2-3\beta}}\left(\frac{G'}{G}\right) + \frac{\sqrt{3}}{3}\sqrt{\frac{\alpha^2}{\alpha^2-3\beta}} \tag{3.8}$$

Substituting (2.5) into (3.8), we can obtain three types travelling wave solutions of (3.2) :

1）when $\alpha^2-4\beta>0$， $G(z) = c_1 e^{\frac{-\alpha+\sqrt{\alpha^2-4\beta}}{2}z} + c_2 e^{\frac{-\alpha-\sqrt{\alpha^2-4\beta}}{2}z}$,

Then $G'(z) = c_1(\frac{-\alpha+\sqrt{\alpha^2-4\beta}}{2})e^{\frac{-\alpha+\sqrt{\alpha^2-4\beta}}{2}z} + c_2(\frac{-\alpha-\sqrt{\alpha^2-4\beta}}{2})e^{\frac{-\alpha-\sqrt{\alpha^2-4\beta}}{2}z}$

$$\left(\frac{G'}{G}\right) = -\frac{\alpha}{2} + \frac{\sqrt{\alpha^2-4\beta}}{2}\frac{(c_1-c_2)\cosh\frac{\sqrt{\alpha^2-4\beta}}{2}z + (c_1+c_2)\sinh\frac{\sqrt{\alpha^2-4\beta}}{2}z}{(c_1-c_2)\sinh\frac{\sqrt{\alpha^2-4\beta}}{2}z + (c_1+c_2)\cosh\frac{\sqrt{\alpha^2-4\beta}}{2}z}$$

thus $u_1(z) = \sqrt{\frac{3}{\alpha^2-3\beta}}(-\frac{\alpha}{2}+\frac{\sqrt{\alpha^2-4\beta}}{2})\frac{(c_1-c_2)+(c_1+c_2)\tanh\frac{\sqrt{\alpha^2-4\beta}}{2}z}{(c_1+c_2)+(c_1-c_2)\tanh\frac{\sqrt{\alpha^2-4\beta}}{2}z} + \frac{\sqrt{3}}{3}\alpha\sqrt{\frac{1}{\alpha^2-3\beta}}$ specially，take $c_1 = c_2$,

$$\tilde{u}_1(z) = \sqrt{\frac{3}{\alpha^2-3\beta}}(-\frac{\alpha}{2}+\sqrt{\frac{\alpha^2-4\beta)}{2}})\tanh\sqrt{\frac{\alpha^2-4\beta}{2}}z + \frac{\sqrt{3}}{3}\alpha\sqrt{\frac{1}{\alpha^2-3\beta}}$$

2）when $\alpha^2-4\beta=0$， $G(z) = (c_1+c_2 z)e^{-\frac{\alpha}{2}z}$ ， $\left(\frac{G'}{G}\right) = \frac{c_2}{c_1+c_2 z} - \frac{\alpha}{2}$

then $u_2(z) = \sqrt{\frac{3}{\beta}}(\frac{c_2}{c_1+c_2 z} - \frac{\alpha}{2}) + \frac{2\sqrt{3}}{3}$

3）when $\alpha^2-4\beta<0$， $G(z) = e^{-\frac{\alpha}{2}z}(c_1\cos\frac{\sqrt{\alpha^2-4\beta}}{2}z + c_2\sin\frac{\sqrt{\alpha^2-4\beta}}{2}z)$

$$\left(\frac{G'}{G}\right)=-\frac{\alpha}{2}+\frac{\sqrt{\alpha^2-4\beta}}{2}\frac{\mathrm{c}_2\cos\frac{\sqrt{\alpha^2-4\beta}}{2}z-\mathrm{c}_1\sin\frac{\sqrt{\alpha^2-4\beta}}{2}z}{\mathrm{c}_1\cos\frac{\sqrt{\alpha^2-4\beta}}{2}z+\mathrm{c}_2\sin\frac{\sqrt{\alpha^2-4\beta}}{2}z}$$

then $u_3(z)=\sqrt{\frac{3}{\alpha^2-3\beta}}\left(-\frac{\alpha}{2}+\frac{\sqrt{\alpha^2-4\beta}}{2}\frac{c_2-c_1\tan\frac{\sqrt{\alpha^2-4\beta}}{2}z}{c_1+c_2\tan\frac{\sqrt{\alpha^2-4\beta}}{2}z}\right)+\frac{\sqrt{3}}{3}\alpha\sqrt{\frac{1}{\alpha^2-3\beta}}$

where c_1, c_2 are arbitrary real constant.

Specially， when $c_2=0, c_1\neq 0$,

$$\tilde{u}_3(z)=\sqrt{\frac{3}{\alpha^2-3\beta}}\left(-\frac{\alpha}{2}+\frac{\sqrt{\alpha^2-4\beta}}{2}\right)\tan\frac{\sqrt{\alpha^2-4\beta}}{2}z+\frac{\sqrt{3}}{3}\sqrt{\frac{\alpha^2}{\alpha^2-3\beta}}$$

4. EXACT SOLUTIONS OF THE CONSTANT STATE CHAFFEE-INFANTE

Finally, we consider the constant state Chaffee-Infante equation：

$$-\Delta u+\lambda(u^3-u)=0$$

Applying the same way, then

$(\frac{G'}{G})^3$： $\lambda a_1^{\ 3}+2a_1\sum_{j=1}^{n}k_j^2=0$

$(\frac{G'}{G})^2$： $\lambda a_1^{\ 2}a_0+a_1\alpha\sum_{j=1}^{n}k_j^2=0$

$(\frac{G'}{G})^1$： $a_1(\alpha^2+2\beta)\sum_{j=1}^{n}k_j^2+\lambda(3a_1a_0^{\ 2}-a_1)=0$

$(\frac{G'}{G})^0$： $a_1\alpha\beta\sum_{j=1}^{n}k_j^2+\lambda(a_0^{\ 3}-a_0)=0$

get： $a_0=\sqrt{\frac{\alpha}{\alpha^2+4\beta}}$ ， $a_1=2\sqrt{\frac{1}{\alpha(\alpha^2+4\beta)}}$ ， $\sum_{j=1}^{n}k_j^2=-2\lambda\frac{1}{\alpha(\alpha^2+4\beta)}$.

Then the solution of equation is：

when $\alpha^2-4\beta>0$， then

$u_1(z)=2\sqrt{\frac{1}{\alpha(\alpha^2+4\beta)}}\left(-\frac{\alpha}{2}+\frac{\sqrt{\alpha^2-4\beta}}{2}\right)\frac{(c_1-c_2)+(c_1+c_2)\tanh\frac{\sqrt{\alpha^2-4\beta}}{2}z}{(c_1+c_2)+(c_1-c_2)\tanh\frac{\sqrt{\alpha^2-4\beta}}{2}z}+\sqrt{\frac{\alpha}{\alpha^2+4\beta}}$ when

$\alpha^2-4\beta=0$， then

$$u_2(z)=\sqrt{\frac{2}{\alpha^3}}\left(\frac{c_2}{c_1+c_2z}-\frac{\alpha}{2}\right)+\sqrt{\frac{1}{2\alpha}}$$

when $\alpha^2-4\beta<0$， then

$$u_3(z)=2\sqrt{\frac{1}{\alpha\left(\alpha^2+4\beta\right)}}\left(-\frac{\alpha}{2}+\frac{\sqrt{\alpha^2-4\beta}}{2}\,\frac{c_2-c_1\tan\frac{\sqrt{\alpha^2-4\beta}}{2}z}{c_1+c_2\tan\frac{\sqrt{\alpha^2-4\beta}}{2}z}\right)+\sqrt{\frac{\alpha}{\alpha^2+4\beta}}$$

where $z=\sum_{j=1}^{n}k_j x_j+l$.

5. CONCLUSION

6.On the basis of Ref. [7], the (n + 1)-dimensional complex-valued Chaffee-Infante equation is further discussed, at the same time, three forms of accurate solutions have been got, and several special solutions have been given. These solutions not only include the solutions of the [7], but also the two forms of the solution of the triangle function and rational function form which did not exist in other references. This paper riches the solutions of (n + 1)-dimensional Chaffee-Infante equation and gives exact solution of the constant state (n + 1)-dimensional Chaffee-Infante equation.

7. REFERENCES

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