THERMAL CONDUCTIVITY AND ITS EFFECTS ON COMPRESSIBLE BOUNDARY LAYER FLOW OVER A CIRCULAR CYLINDER

Oyem Onyekachukwu Anselm [1*] & Koriko O. K. [2]
[1]Department of Mathematics, Federal University Lokoja, Kogi State, Nigeria
[2]Department of Mathematical Science, Federal University of Technology Akure, Ondo State, Nigeria

ABSTRACT

In this paper, variable thermal conductivity on heat transfer over a circular cylinder is presented. The concept of assuming constant thermal conductivity on materials is however not efficient. Hence, the governing partial differential equation is reduced using non-dimensionless variables into a system of coupled non-linear ordinary differential equation, which is solved numerically. While the analysis on the stability and existence and uniqueness for different cases of variable thermal conductivity are shown, and as the temperature increases, the points of separation at surface temperature, decreases to an asymptotic value.

Keywords: *Thermal conductivity, compressible boundary layer, Boundary conditions, Circular Cylinder, stability and Numerical solution*

1. INTRODUCTION

Thermal conductivities of materials vary dramatically both in magnitude and temperature from one material to another due to differences in sample sizes. Though, literature has shown that successful studies had been carried out, but with a limitation of assuming constant thermal conductivity on the effect of heat transfer on forced convection boundary layer flow past a circular cylinder in a viscous compressible fluid, several authors have studied the effect of heat transfer on compressible boundary layer flow of various kinds of dynamic systems. Brown [1] studied the effect of heat transfer on the growth of the boundary layer in the impulsive motion of a cylinder in a viscous compressible fluid.

Hossain *et al.* [4] investigated the effect of heat transfer on compressible boundary layer flow over a circular cylinder, where they showed that heat transfer parameters has effect in moving the boundary layer upstream, Milena *et al*. [6] examined the thermal conductivity and specific heat capacity of several types of granular agricultural products, namely of spring oat and soybean and were measured in dependence on moisture content from the dry state to the water fully saturated state, bearing in mind that the obtained results will find use in the selection of suitable methods for processing of agricultural products, in a qualified assessment of optimal modes of technological processes, and the development of modern fully automatic agricultural equipments. Dominguez-Muñoz *et al.* [2] highlighted that increasing attention is being paid to the application of uncertainty and sensitivity analysis methods to model validation and building simulation by presenting polynomial fits for the average thermal conductivity and its standard deviation as functions of density for typical insulation materials. They explained further that insulation materials are extensively used to reduce the heat losses (or gains) from thermal systems like buildings, pipes and ducts.

Xinwei Wang *et al.* [10] studied thermal conductivity of Nanoparticle-fluid mixture, where the effective thermal conductivity of mixtures of fluids and nanometer-size particles were measured by steady state parallel-plate method, using two types of Nanoparticle (Al_2O_3 and CuO), dispersed in water, vacuum pump fluid, engine oil and ethylene glycol and the result showed that the thermal conductivities of Nanoparticle-fluid mixtures are higher than those of the base fluids. In this research, we develop sufficient base on the analysis varying different thermal conductivity and heat transfer through a laminar boundary layer in the flow of a viscous fluid over a body of arbitrary shape and arbitrary specified surface temperature constitutes a very important problem in the field of heat transfer. The difference in the temperature initiates the physical contact between the particles, creating kinetic energy and momentum.

2. PROBLEM FORMULATION

The equations describing the steady flow of compressible, laminar two-dimensional boundary layer flow under the assumption that the viscosity (μ) is proportional to the absolute temperature (T) and the Prandtl number (σ) is unity [4], is given as:

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)-u\frac{\partial p}{\partial x}=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

with

$$p=\rho RT \tag{4}$$

$$\mu=\mu_0\left(\frac{T}{T_o}\right) \tag{5}$$

subject to the following boundary conditions;

$$\begin{aligned} &u=v=0, \quad T=T_w \quad at \quad y=0\\ &u=U_1, \quad T=T_1 \quad at \quad y=\infty \end{aligned} \tag{6}$$

where T_w is the constant wall temperature, (x,y) are the Cartesian coordinates with $x-$ and $y-$ axes along and normal to the surface of the cylinder respectively, (u,v) are the velocity components along $x-$ and $y-$ axes, p is the pressure, ρ is the density, k is the thermal conductivity, C_p is the specific heat at constant pressure, R is the gas constant and the suffix 0, refers to some standard state, say, $x=0$. The main stream velocity U_1 is taken as the velocity in the irrotational motion of an incompressible fluid. Thus, if a is the radius of the cylinder, then,

$$U_1(x)=U_\infty \sin(x/a) \tag{7}$$

3. METHOD OF SOLUTION

In obtaining a solution describing the flow and heat transfer equations, equations (1), (2) and (3) are further reduced to almost an incompressible form by applying the Stewartson's transformations [9].

$$Y=\frac{a_1}{a_0\sqrt{v_0}}\int_0^y\frac{\rho}{\rho_0} \tag{8}$$

$$\rho u=\rho_0\sqrt{v_0}\,\frac{\partial\psi}{\partial y} \tag{9}$$

For an incompressible two-dimensional steady laminar flow with $\rho=$constant, we deduce from equation (1) by transformation (8) and (9), the values of

$$u=\left(\frac{a_1}{a_0}\right)\frac{\partial\psi}{\partial y} \tag{10}$$

$$v=-\frac{\rho_0}{\rho}\sqrt{v_0}\,\frac{\partial\psi}{\partial x} \tag{11}$$

$$\frac{\partial u}{\partial y}=\frac{a_1^{\,2}}{a_0^{\,2}\sqrt{v_0}}\frac{\partial^2\psi}{\partial Y^2} \tag{12}$$

$$\mu\frac{\partial u}{\partial y}=\frac{\mu_0 T}{T_0}\frac{a_1^{\,2}}{a_0^{\,2}\sqrt{\mu_0}}\frac{\partial^2\psi}{\partial Y^2} \tag{13}$$

Let $\mu_0\left(\frac{T}{T_0}\right)\equiv\mu_0\left(\frac{p}{p_0}\right)$, and equation (13) becomes

$$\mu\frac{\partial u}{\partial y}=\frac{\mu_0 p}{p_0}\frac{a_1{}^2}{a_0{}^2\sqrt{\mu_0}}\frac{\partial^2\psi}{\partial Y^2} \tag{14}$$

$$\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)=\left(\frac{\rho}{\rho_0}\right)\frac{a_1{}^3}{a_0{}^3}\frac{\partial^3\psi}{\partial Y^3} \tag{15}$$

furthermore, from equation (3),

$$\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)=\left(\frac{a_1{}^2}{a_0{}^2 v_0}\right)\frac{\partial k}{\partial Y}\frac{\partial T}{\partial Y}+k\left(\frac{a_1{}^2}{a_0{}^2 v_0}\right)\frac{\partial^2 T}{\partial Y^2}$$

$$\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)=\frac{a_1{}^2}{a_0{}^2 v_0}\left(\frac{\partial k}{\partial Y}\frac{\partial T}{\partial Y}+k\frac{\partial^2 T}{\partial Y^2}\right) \tag{16}$$

By the Eulerian equation of motion of an inviscid flow, assuming nobody force and steady flow

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{17}$$

If there be a constant flow along the x-direction, and the pressure gradient term is assumed to be known from Bernoulli's equation and applied to the outer inviscid flow, we have

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}\right)=U\frac{dU}{dx}+\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=U_1\frac{dU_1}{dx}+\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{18}$$

with $\left(u=U_1, T=T_1\right)$ from the boundary conditions in (6). Using the stream function, equation (18) becomes

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y}=\frac{p}{p_0}\left(\frac{a_1{}^3}{a_0{}^3}\right)\frac{\partial^3\psi}{\partial Y^3}+U_1\frac{dU_1}{dx} \tag{19}$$

and by the power law of Isentropic process,

$$\left(\frac{p}{p_0}\cdot\frac{a_1{}^3}{a_0{}^3}\right)^{-1}\left(\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y}\right)=\frac{\partial^3\psi}{\partial Y^3}+\left(\frac{p}{p_0}\cdot\frac{a_1{}^3}{a_0{}^3}\right)^{-1}U_1\frac{dU_1}{dx}$$

$$\left(\frac{T}{T_0}\cdot\frac{a_1{}^3}{a_0{}^3}\right)^{-1}\left(\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y}\right)=\frac{\partial^3\psi}{\partial Y^3}+\left(\frac{p}{p_0}\cdot\frac{a_1{}^3}{a_0{}^3}\right)^{-1}U_1\frac{dU_1}{dx}$$

$$\left(\left(\frac{a_1}{a_0}\right)^3\cdot\left(\frac{a_1}{a_0}\right)^{\gamma/\gamma-1}\right)^{-1}\left(\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y}\right)=\frac{\partial^3\psi}{\partial Y^3}+\left(\frac{p}{p_0}\cdot\frac{a_1{}^3}{a_0{}^3}\right)^{-1}U_1\frac{dU_1}{dx}$$

$$\left(\frac{a_1}{a_0}\right)^{3+\left(\gamma/\gamma-1\right)-1}\left(\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y}\right)=\frac{\partial^3\psi}{\partial Y^3}+\left(\left(\frac{a_1}{a_0}\right)^3\cdot\left(\frac{a_1}{a_0}\right)^{2\gamma/(\gamma-1)}\right)^{-1}U_1\frac{dU_1}{dx}$$

$$\left(\frac{a_1}{a_0}\right)^{3+\left(\gamma/\gamma-1\right)-1}\left(\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y}\right)=\frac{\partial^3\psi}{\partial Y^3}+\left(\left(\frac{a_1}{a_0}\right)^{(5\gamma-3)/(\gamma-1)}\right)^{-1}\times\left(\frac{T_0}{T}\right)^{-1}=\left(\frac{a_1}{a_0}\right)^{(5\gamma-3)/(\gamma-1)}\cdot\frac{T_0}{T}U_1\frac{dU_1}{dx}$$

$$\left(\frac{a_1}{a_0}\right)^{3\gamma-2/(\gamma-1)}\left(\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y}\right)=\frac{\partial^3\psi}{\partial Y^3}+\left(\frac{a_1}{a_0}\right)^{5\gamma-3/(\gamma-1)}\frac{T}{T_0}U_1\frac{dU_1}{dx} \tag{20}$$

Multiplying equation (2) by u, and adding it to equation (3), taking $C_p = C$, with the boundary condition (6) and taking the function S as relating to the absolute temperature T, with Mach number relation

$$\left(1+\frac{\gamma-1}{2}M_1{}^2\right)S = \frac{T}{T_1} - \frac{\gamma-1}{2}M_1{}^2\left(1-\frac{u^2}{U_1{}^2}\right)-1 \tag{21}$$

$$\frac{T}{T_1} = 1+\frac{\gamma-1}{2}M_1{}^2 \quad ; \quad M_1 = \frac{v}{a}$$

we have,

$$\rho\left(\left(\frac{a_1}{a_0}\right)\frac{\partial\psi}{\partial Y}\frac{\partial S}{\partial x} - \left(\frac{\rho_0}{\rho}\right)\sqrt{v_0}\,\frac{\partial\psi}{\partial x}\cdot\frac{\partial S}{\partial Y}\left(\frac{a_1}{a_0}\cdot\frac{\rho}{\rho_0\sqrt{v_0}}\right)\right) = \frac{a_1{}^2}{a_0{}^2 v_0}\left(\frac{\partial k}{\partial Y}\frac{\partial S}{\partial Y} + k\frac{\partial^2 S}{\partial Y^2}\right)$$

$$\rho\left(\left(\frac{a_1}{a_0}\right)\frac{\partial\psi}{\partial Y}\frac{\partial S}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial S}{\partial Y}\left(\frac{a_1}{a_0}\right)\right) = \rho\frac{a_1{}^2}{a_0{}^2}\left(\frac{\partial k}{\partial Y}\frac{\partial S}{\partial Y} + k\frac{\partial^2 S}{\partial Y^2}\right)$$

$$\left(\frac{a_1}{a_0}\right)\left(\frac{\partial\psi}{\partial Y}\frac{\partial S}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial S}{\partial Y}\right) = \frac{a_1{}^2}{a_0{}^2}\left(\frac{\partial k}{\partial Y}\frac{\partial S}{\partial Y} + k\frac{\partial^2 S}{\partial Y^2}\right) \tag{22}$$

We sufficiently consider a flow in which the Mach number <<1, replacing the factor a_0/a_1 by unity, hence, the equations describing the flow and heat transfer are

$$\frac{\partial\psi}{\partial Y}\frac{\partial^2\psi}{\partial Y\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial Y^2} = \frac{\partial^3\psi}{\partial Y^3} + U_1\frac{dU_1}{dx}(1+S) \tag{23}$$

$$\frac{\partial\psi}{\partial Y}\frac{\partial S}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial S}{\partial Y} = \frac{\partial k}{\partial Y}\frac{\partial S}{\partial Y} + k\frac{\partial^2 S}{\partial Y^2} \tag{24}$$

subject to the boundary conditions

$$\begin{aligned} &\psi = \frac{\partial\psi}{\partial Y} = 0 \qquad S = \frac{T_w}{T_1} - 1 = S_w \qquad at \qquad Y = 0 \\ &\frac{\partial S}{\partial Y} = U_1(x) \qquad S \to 0 \qquad as \qquad Y = \infty \end{aligned} \tag{25}$$

The computational constraints involved in solving differential equations, is usually cumbersome, hence, we introduce Merkin's [5] non-dimensionless variables

$$\xi = \frac{x}{a}, \quad \eta = Y\left(\frac{\sqrt{v_0}}{R}\right)\mathrm{Re}^{1/2} \tag{26}$$

$$\psi = \sqrt{v_0}\,\mathrm{Re}^{1/2}\,\xi f(\xi,\eta), \quad S(x,Y) = S(\xi,\eta) \tag{27}$$

We non-dimensionalize equation (23), (24) using (26) and (27), to get

$$\left[\begin{array}{l} \frac{v_0}{R}\mathrm{Re}\,\xi f'(\eta)\left(\frac{1}{a}\right)\cdot\frac{v_0}{R}\mathrm{Re}\left(f'(\xi,\eta)+\xi f''(\xi,\eta)\right)-\left(\frac{1}{a}\right)v_0^{\frac{1}{2}}\,\mathrm{Re}^{\frac{1}{2}}\left(f(\xi,\eta)+\xi f'(\xi,\eta)\right)\cdot\frac{v_0^{\frac{3}{2}}}{R^2}\mathrm{Re}^{\frac{3}{2}}\,\xi f''(\eta) \\ \qquad =\frac{v_0^{2}}{R^3}\mathrm{Re}^2\,\xi f'''(\eta)+U_1\frac{dU_1}{dx}\left(1+S\right) \end{array}\right]$$

$$\left[\begin{array}{l} \left(\frac{v_0}{R}\mathrm{Re}\,\xi f'(\eta)\cdot\left(\frac{1}{a}\right)\frac{v_0}{R}\mathrm{Re}\left(f'(\xi,\eta)+\xi f''(\xi,\eta)\right)\right)-\left(\frac{1}{a}\right)v_0^{\frac{1}{2}}\,\mathrm{Re}^{\frac{1}{2}}\left(f(\xi,\eta)+\xi f'(\xi,\eta)\right) \\ \qquad \cdot\frac{v_0^{\frac{3}{2}}}{R^2}\mathrm{Re}^{\frac{3}{2}}\,\xi f''(\eta)=\frac{v_0^{2}}{R^3}\mathrm{Re}^2\,\xi f'''(\eta)+U_\infty\sin\xi\cos\xi\left(1+S\right) \end{array}\right]$$

$$\left[\begin{array}{l} \frac{v_0^{2}}{R^3}\mathrm{Re}^2\,\xi f'''(\eta)+\frac{v_0^{2}}{aR^2}\mathrm{Re}^2\,\xi f(\xi)f''(\eta)+\frac{v_0^{2}}{aR^2}\mathrm{Re}^2\,\xi^2 f'(\xi)f''(\eta)-\frac{v_0^{2}}{aR^2}\mathrm{Re}^2\,\xi f'^2(\eta) \\ \qquad -\frac{v_0^{2}}{aR^2}\mathrm{Re}^2\,\xi^2 f'(\eta)f''(\xi)+U_\infty\sin\xi\cos\xi\left(1+S\right)=0 \end{array}\right]$$

$$\left[\begin{array}{l} \frac{v_0^{2}}{R^3}\mathrm{Re}^2\,f'''(\eta)+\frac{v_0^{2}}{aR^2}\mathrm{Re}^2\,f(\xi)f''(\eta)-\frac{v_0^{2}}{aR^2}\mathrm{Re}^2\,f'^2(\eta)+U_\infty\frac{\sin\xi\cos\xi}{\xi}\left(1+S\right) \\ \qquad =\frac{v_0^{2}}{aR^2}\mathrm{Re}^2\,\xi f'(\eta)f''(\xi)-\frac{v_0^{2}}{aR}\mathrm{Re}^2\,\xi f'(\xi)f''(\eta) \end{array}\right]$$

$$f'''(\eta)+f(\xi)f''(\eta)-f'^2(\eta)+\frac{\sin\xi\cos\xi}{\xi}\left(1+S\right)=\xi\left(f'(\eta)\frac{\partial f'}{\partial\xi}-f''(\eta)\frac{\partial f}{\partial\xi}\right) \tag{28}$$

From equation (24),

$$\left[\begin{array}{l} \frac{v_0}{R}\mathrm{Re}\,\xi f'(\eta)\cdot\left(\frac{1}{a}\right)\frac{\partial S}{\partial\xi}-\left(\frac{1}{a}\right)v_0^{\frac{1}{2}}\,\mathrm{Re}^{\frac{1}{2}}\left(f(\xi,\eta)+\xi f'(\xi,\eta)\right)\cdot\frac{v_0^{\frac{1}{2}}}{R}\mathrm{Re}^{\frac{1}{2}}\frac{\partial S}{\partial\eta}=\frac{v_0^{\frac{1}{2}}}{R}\mathrm{Re}^{\frac{1}{2}}\frac{\partial k}{\partial\eta}\cdot\frac{v_0^{\frac{1}{2}}}{R}\mathrm{Re}^{\frac{1}{2}}\frac{\partial S}{\partial\eta} \\ \qquad +k\frac{v_0}{R^2}\mathrm{Re}\frac{\partial^2 S}{\partial\eta^2} \end{array}\right]$$

$$\left[\begin{array}{l} \frac{v_0}{R}\mathrm{Re}\,\xi f'(\eta)\cdot\left(\frac{1}{a}\right)\frac{\partial S}{\partial\xi}-\left(\frac{1}{a}\right)v_0^{\frac{1}{2}}\,\mathrm{Re}^{\frac{1}{2}}\left(f(\xi,\eta)+\xi f'(\xi,\eta)\right)\cdot\frac{v_0^{\frac{1}{2}}}{R}\mathrm{Re}^{\frac{1}{2}}\frac{\partial S}{\partial\eta} \\ \qquad =\frac{v_0^{\frac{1}{2}}}{R}\mathrm{Re}^{\frac{1}{2}}\frac{\partial k}{\partial\eta}\cdot\frac{v_0^{\frac{1}{2}}}{R}\mathrm{Re}^{\frac{1}{2}}\frac{\partial S}{\partial\eta}+k\cdot\frac{v_0}{R^2}\mathrm{Re}\frac{\partial^2 S}{\partial\eta^2} \end{array}\right]$$

$$\frac{v_0}{aR}\mathrm{Re}\,\xi f'(\eta)\frac{\partial S}{\partial\xi}-\frac{v_0}{aR}\mathrm{Re}\,f(\xi)\frac{\partial S}{\partial\eta}-\frac{v_0}{aR}\mathrm{Re}\,\xi f'(\xi)\frac{\partial S}{\partial\eta}=\frac{v_0}{R^2}\mathrm{Re}\frac{\partial S}{\partial\eta}\frac{\partial k}{\partial\eta}+\frac{v_0}{R^2}\mathrm{Re}\,k\frac{\partial^2 S}{\partial\eta^2}$$

$$k\frac{\partial^2 S}{\partial\eta^2}+f(\xi)\frac{\partial S}{\partial\eta}+\frac{\partial S}{\partial\eta}\frac{\partial k}{\partial\eta}=\xi f'(\eta)\frac{\partial S}{\partial\xi}-\xi f'(\xi)\frac{\partial S}{\partial\eta}$$

$$kS''(\eta)+f(\xi)S'(\eta)+k'(\eta)S'(\eta)=\xi\left(f'(\eta)\frac{\partial S}{\partial\xi}-S'(\eta)\frac{\partial f}{\partial\xi}\right) \tag{29}$$

At the stagnation point $\xi=0$, equation (28) and (29) becomes

$$f'''+ff''-f'^2+S+1=0 \tag{30}$$

$$kS''+k'S'+fS'=0 \tag{31}$$

with the boundary conditions

$$f(0) = f'(0) = 0, \qquad S(0) = S_w$$
$$f'(\infty) = 1, \qquad S(\infty) = 0 \tag{32}$$

3.1 CASE I

If we consider thermal conductivity that is linear in S , i.e.

$$k = 1 + \alpha S \tag{33}$$

then the resulting coupled ordinary differential equation is

$$f''' + ff'' - f'^2 + S + 1 = 0 \tag{34}$$

$$(1 + \alpha S)S'' + (\alpha S' + f)S' = 0 \tag{35}$$

subject to the boundary conditions of (32)

3.2 CASE II

If the thermal conductivity is quadratic in S , i.e.

$$k = 1 + \alpha^2 S^2 \tag{36}$$

then, we have

$$f''' + ff'' - f'^2 + S + 1 = 0 \tag{37}$$

$$(1 + \alpha^2 S^2)S'' + (2\alpha^2 SS' + f)S' = 0 \tag{38}$$

subject to the boundary conditions of (32). Equations (34), (35) and (37), (38) along with the boundary conditions (32) are solved numerically using the Equilibrium (Boundary-Value) method [3] for the different values of surface temperature and surface curvature parameter at two different Mach angles $\alpha = 0.3 \, and \, \alpha = 0.075$.

4. RESULTS AND DISCUSSION

The result is presented as temperature, separation parameter of thermal conductivity over a circular cylinder in figures 1 to 4. And the results show that at $\alpha = 0.3$ and $\alpha = 0.075$ for both cases of non-linear ODE, as the temperature increases, the points of separation at surface temperature decreases to an asymptotic value. Generally, an increase in temperature S_w , makes its temperature coefficients increase also, thus, it is higher at the initial stagnation point $\xi = 0$ and later break down at the separation point ξ for values of S_w .

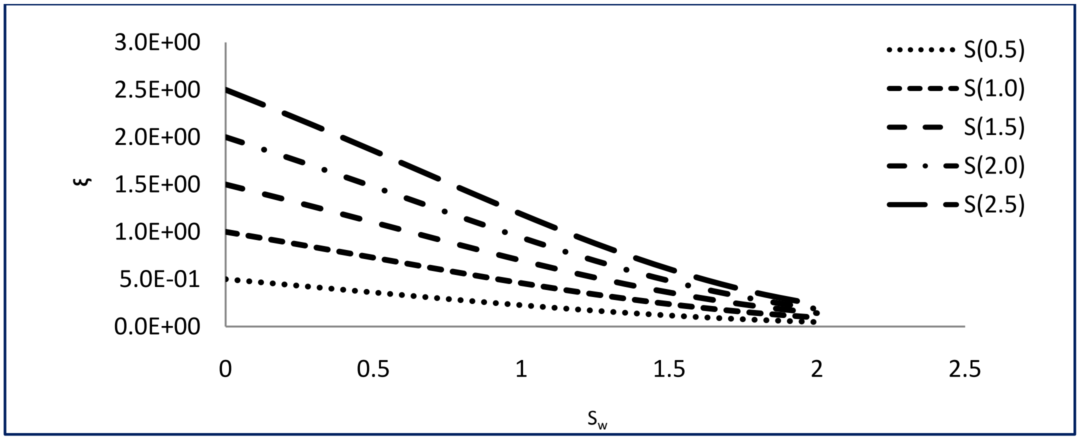

Figure 1: Variation of Surface temperature with separation Parameter at $\alpha = 0.3$ for $k = 1 + \alpha S$

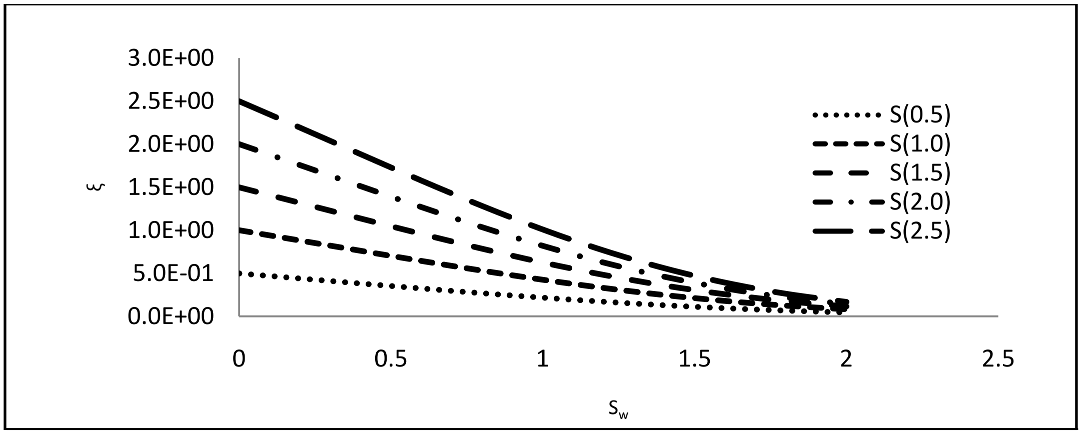

Figure 2: Variation of Surface temperature with separation Parameter at $\alpha = 0.075$ *for* $k = 1 + \alpha S$

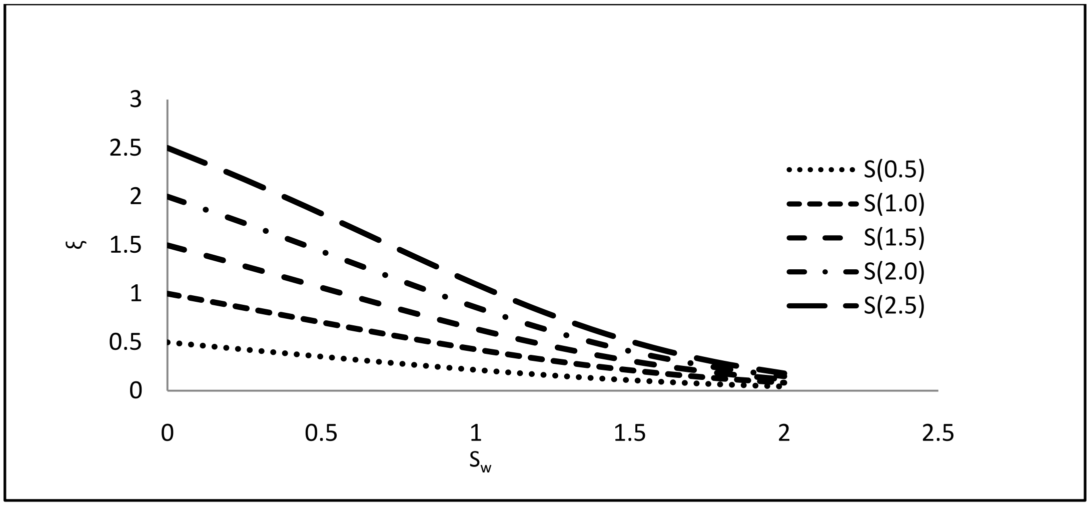

Figure 3: Surface temperature profile with separation Parameter at $\alpha = 0.3$ $k = 1 + \alpha^2 S^2$

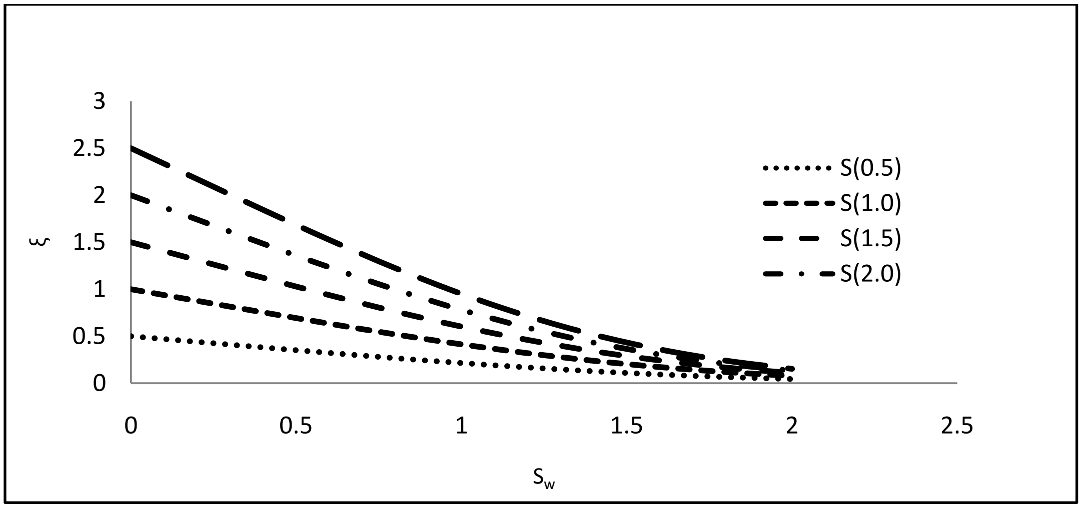

Figure 4: Surface temperature profile with separation Parameter at $\alpha = 0.075$ $k = 1 + \alpha^2 S^2$

5. CONCLUSION

The problem of heat transfer effect over a cylindrical cylinder at different values of thermal conductivity establishes that thermal conductivity cannot be assumed to be constant. The reason being that thermal conductivity varies dramatically both in magnitude and temperature from one material to another due to differences in sample sizes. Consequently, whenever the property of any material is considered, thermal conductivity should not be regarded as a constant.

6. REFERENCES

[1] S. N. Brown, The effect of Heat Transfer on Boundary Layer Growth, *Proc. Camb. Phil. Soc.29*, 789-802 (1963).

[2] F. Domínguez-Muñoz, B. Anderson, J. M. Cejudo-López, A. Carrillo-Andrés, Uncertainty in the Thermal Conductivity of Insulation Materials, *Eleventh International IBPSA Conference, Glasgow Scotland*, 1008-1013 (2009).

[3] J. D. Hoffman, Numerical Methods for Engineers and Scientist (2nd ed.), Marcel Dekker, Inc., New York (2001).

[4] M. A. Hossian, I. Pop, T. Y. Na, Effect of Heat Transfer on Compressible Boundary Layer Flow over a Circular Cylinder, *Acta Mechanica* 131, 267-272 (1998).

[5] J. H. Merkin, Mixed Convection from a Horizontal Circular Cylinder, *International Journal of Heat and Mass Transfer* 20, 73-77 (1977).

[6] J. Milena, P. Zbyšek, C. Robert, Thermal Properties of Biological Agricultural Materials, *Seminar Preceedings of Thermophysics*, 68-71 (2006).

[7] O. A. Oyem, Effects of Heat Transfer on Compressible Boundary Layer Flow over a Circular Cylinder with Variable Thermal Conductivity, M.Tech Thesis, Department of Mathematical Sciences, Federal University of Technology, Akure, Nigeria (*unpublished*) (2011).

[8] B. C. Pak, Y. I. Cho, Hydrodynamic and Heat Transfer Study of Dispersed Fluids with Submicron Metallic Oxide Particles, *Experimental Heat Transfer*, 11(2), 151–170 (1998).

[9] K. Stewartson, Correlation Incompressible and Compressible Boundary layers, *Proc. Roy Soc.* London Ser., A 200 84-100 (1949).

[10] W. Xinwei, X. Xianfan, S. C. Stephen, Thermal Conductivity of Nanoparticle-Fluid Mixture, *Journal of Thermophysics and Heat Transfer*, 13(4), 474-480 (1999).