USING TRANSFORMATION TECHNIQUE TO SOLVE MULTI-OBJECTIVE LINEAR FRACTIONAL PROGRAMMING PROBLEM

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ABSTRACT
In this paper we used a new transformation technique for solving multi-objective linear fractional programming problem (MOLFPP) to single-Objective linear fractional programming problem (SOLFPP), through a new method using mean and median, and then solve the problem by modified simplex method [5]. The obtain results are compared with that of modified methods in [9].

Keywords: *Solve MOLFPP by using average mean and average median, new average mean and new average median techniques.*

1. INTRODUCTION
Linear fraction maximum problems (i.e. ratio objective that have numerator and denominator) have attracted considerable research and interest, since they are useful in production planning, financial and corporative planning, health care and hospital planning. Several methods to solve such problems are proposed in(1962)[2]. Their method depends on transforming the linear fractional programming to an equivalent linear program. Sing (1981) in his paper did a useful study about the optimality condition in fractional programming[8]. A multi-objective linear programming problem (MOLPP) is solved by Chandra Sen. in (1983)[6]; Sulaiman and Othman (2007)[7] suggested an approach to construct the multi-objective function. Also Sulaiman and Sadiq in (2006) studied the multi-objective function by using mean and median value[4]. In (1993) Abdil-kadir and Sulaiman[1] studied the multi-objective fractional programming problem. In (2008) Hamad Amin studied multi-objective linear programming problem using Arithmetic Average[3].Also Sulaiman and Salih in (2010) studied the MOLFPP by using mean and median value [9]. In order to extend this work we have defined a MOLFPP and investigated the algorithm to solve fractional programming problem for multi-objective function, irrespective of the number of objectives with less computational burden and suggest a new technique by using average mean, average median, new average mean and new average median values of objective functions, to generate the best optimal solution. The computer application of our algorithm has also been discussed by solving a numerical example. Finally we have shown results and comparisons between different techniques.

2. DEFINITION AND MATHEMATICAL MODELS
The mathematical from of MOLPP is given as follows:

$$\left.\begin{array}{l} Max.Z_1 = c_1^t x + \gamma_1 \\ Max.Z_2 = c_2^t x + \gamma_2 \\ \quad\vdots \\ Max.Z_r = c_r^t x + \gamma_r \\ Min.Z_{r+1} = c_{r+1}^t x + \gamma_{r+1} \\ \quad\vdots \\ Min.Z_s = c_s^t x + \gamma_s \end{array}\right] (2.1) \quad \begin{array}{l} \text{Subject to:} \\ \quad Ax = b \\ \quad x > 0 \end{array}$$

Where r is the number of objective function that to be maximized, s is the number of objective functions that is to be maximized and minimized and $s - r$is the number of objerctive function that is to be minimized, other symbols have the same meaning as previously mentioned, for more details see[7].

3. MULTI-OBJECTIVE FRACTIONAL PROGRAMMING PROBLEM
Multi-Objective function that are the ratio of two linear objective functions are said to be MOLFPP [1,9] then can be defined:

$$\left.\begin{array}{l} Max.Z_1 = (c_1^t x + \gamma_1)/(d_1^t x + \beta_1) \\ Max.Z_2 = (c_2^t x + \gamma_2)/(d_2^t x + \beta_2) \\ \quad\vdots \\ Max.Z_r = (c_r^t x + \beta_r)/(d_r^t x + \beta_r) \\ Min.Z_{r+1} = (c_{r+1}^t x + \beta_{r+1})/(d_{r+1}^t x + \beta_{r+1}) \\ \quad\vdots \\ Min.Z_s = (c_s^t x + \gamma_s)/(d_s^t x + \beta_s) \end{array}\right] \quad (3.2)$$

Subject to:

$Ax = b$(3.3)

$x \geq 0$ (3.4)

Where be is m −dimensional vector of constants, x is n −dimensional vector of decision variables and A is a$m \times n$matrix of constants other symbols have the same meaning as before [7].

4. SOLVINGMOLFPPBY USING CHANDRA SEN. TECHNIQUE

The same approach which was taken by Sen. (1983)[6] is followed here to formulate the constraint objective function for the MOLFPP. Suppose we obtain a single value corresponding to each of the objective functions of the MOLFPP of equation (3.2).They are being optimized individually subject to the constraints (3.3)and(3.4) as follows:

$$\left.\begin{array}{l} Max.Z_1 = \varphi_1 \\ Max.Z_2 = \varphi_2 \\ \quad\vdots \\ Max.Z_r = \varphi_r \\ Min.Z_{r+1} = \varphi_{r+1} \\ \quad\vdots \\ Min.Z_s = \varphi_s \end{array}\right] \quad (4.5)$$

Where $\varphi_1, \varphi_2, \ldots, \varphi_s$are value of the objective functions, the level of the decision variable may not necessarily be the same for all optimal solutions in presence of conflicts among objectives. But we require the common set of decision variables to be the best compromising optimal solution that we can determine for the common set of the decision variables from the following combined objective function, which formulate the MOLFPP given in equation (4.5)

$$Max.Z = \sum_{i=1}^{r} \frac{Z_i}{|\varphi_i|} - \sum_{i=r+1}^{s} \frac{Z_i}{|\varphi_i|} \quad (4.6)$$

Where $\varphi_i \neq 0, i = 1,2,\ldots,s$. Subject to constraints (3.3) and (3.4),and the optimum value of the objective functions$\varphi_i, i = 1,2,\ldots,s$may be positive or negative.

5. NUMERICAL EXAMPLES

In this section, we present numerical examples.

Example 5.1: Solve the following MOLFPP by Chandra Sen. technique[6,9]

$$Max.Z_1 = (5x_1 + 3x_2)/(x_1 + x_2 + 1)$$
$$Max.Z_2 = (9x_1 + 5x_2)/(3x_1 + 3x_2 + 3)$$
$$Max.Z_3 = (3x_1 - 4x_2)/(x_1 + x_2 + 1)$$
$$Max.Z_4 = (3x_1 + 2x_2)/(2x_1 + 2x_1 + 2)$$

Subject to:

$$2x_1 + 4x_2 \geq 8$$
$$x_1 + x_2 \leq 3$$
$$x_1 + 2x_2 \leq 10$$
$$2x_1 + x_2 \leq 5$$
$$x_1 \leq 2$$
$$x_1, x_2 \geq 0$$

Solution: After finding the value of each of individual objective functions of example 5.1 by Modified Simplex method [5], the results obtained by using Chandra Sen.'s technique [6,9] are given and the numerical results as below in table 1:

Table 1: Results of example 5.1 by using modified simplex method

i	Z_i	x_i	φ_i	$AA_i = \lvert\varphi A_i\rvert$ $\forall\, i = 1,2,\dots,r$	$AL_i = \lvert\varphi A_i\rvert$ $\forall\, i = r+1,\dots,s$
1	$13/4$	(2,1)	$13/4$	$13/4$	
2	$23/12$	(2,1)	$23/12$	$23/12$	
3	$1/2$	(2,1)	$1/2$	$1/2$	
4	1	(2,1)	1	1	

$$TG = \sum_{i=1}^{r} \frac{Z_i}{AA_i} = \sum_{i=1}^{4} HG_i = \frac{(6341x_1 - 3114x_2)}{(598x_1 + 598x_2 + 598)}$$

$$TL = \sum_{i=r+1}^{s} \frac{Z_i}{AL_i} = \sum_{i=r+1}^{s} HL_i = 0$$

$$Max.Z = TG - TL = \frac{(6341x_1 - 3114x_2)}{(598x_1 + 598x_2 + 598)}$$

Subject to:

$$2x_1 + 4x_2 \geq 8$$
$$x_1 + x_2 \leq 3$$
$$x_1 + 2x_2 \leq 10$$
$$2x_1 + x_2 \leq 5$$
$$x_1 \leq 2$$
$$x_1, x_2 \geq 0$$

After solving it, we get $Max.Z = 4$ and $x_1 = 2,\ x_2 = 1$

Example 5.2: Solve the following MOLFPP by Chandra Sen. Technique [6,9]

$$Max.Z_1 = (3x_1 - 2x_2)/(x_1 + x_2 + 1)$$
$$Max.Z_2 = (9x_1 + 3x_2)/(x_1 + x_2 + 1)$$
$$Max.Z_3 = (3x_1 - 5x_2)/(2x_1 + 2x_2 + 2)$$
$$Min.Z_4 = (-6x_1 + 2x_2)/(2x_1 + 2x_2 + 2)$$
$$Min.Z_5 = (-3x_1 - x_2)/(x_1 + x_2 + 1)$$

Subject to:

$$x_1 + x_2 \leq 2$$
$$9x_1 + x_2 \leq 9$$
$$x_1, x_2 \geq 0$$

Solution: After finding the value of each of individual objective functions of example 5.2 by Modified Simplex method [5], the results obtained by using Chandra Sen.'s technique [6,9] are given and the numerical results as below in table 2:

Table 2: Results of example 5.2 by using modified simplex method

i	Z_i	x_i	φ_i	$AA_i = \|\varphi A_i\|$ $\forall\, i = 1,2,\dots,r$	$AL_i = \|\varphi A_i\|$ $\forall\, i = r+1,\dots,s$
1	$3/2$	(1,0)	$3/2$	$3/2$	
2	$9/2$	(1,0)	$9/2$	$9/2$	
3	$3/4$	(1,0)	$3/4$	$3/4$	
4	$-3/2$	(1,0)	$-3/2$		$3/2$
5	$-3/2$	(1,0)	$-3/2$		$3/2$

$$TG = \sum_{i=1}^{r} \frac{Z_i}{AA_i} = \sum_{i=1}^{3} HG_i = \frac{(18x_1 - 12x_2)}{(3x_1 + 3x_2 + 3)}$$

$$TL = \sum_{i=1}^{r} \frac{Z_i}{AL_i} = \sum_{i=4}^{5} HL_i = \frac{(-12x_1)}{(3x_1 + 3x_2 + 3)}$$

$$Max.Z = TG - TL = \frac{(10x_1 - 4x_2)}{(x_1 + x_2 + 1)}$$

Subject to:

$$x_1 + x_2 \leq 2$$
$$9x_1 + x_2 \leq 9$$
$$x_1, x_2 \geq 0$$

After solving it, we get $Max.Z = 5$ and $x_1 = 1,\ x_2 = 0$

The solution for example 5.1 when applying algorithm in [9] by using mean is the same optimal solution shown in table 1, then the combined objective linear fractional function is:

$$SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{4} Z_i = \frac{(75x_1 + 10x_2)}{(6x_1 + 6x_2 + 6)}, SN = \sum_{i=r+1}^{s} Z_i = 0$$

$$VM = \sum_{i=1}^{r} \frac{AA_i}{r} = \frac{5}{3}, VN = \sum_{i=r+1}^{s} \frac{AL_i}{s-r} = 0$$

$$S_1 = \frac{SM}{VM} = \frac{(15x_1 + 2x_2)}{(2x_1 + 2x_2 + 2)}, S_2 = \frac{SN}{VN} = 0$$

$$Max.Z = S_1 - S_2 = \frac{(15x_1 + 2x_2)}{(2x_1 + 2x_2 + 2)}$$

After solving it subject to the same constraints as before, we get

$Max.Z = 4$ and $x_1 = 2, x_2 = 1$

The solution for example 5.2 when applying algorithm in [9] by using mean is the same optimal solution shown in table 2, then the combined objective linear fractional function is:

$$SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{3} Z_i = \frac{(27x_1 - 3x_2)}{(2x_1 + 2x_2 + 2)}$$

$$SN = \sum_{i=r+1}^{s} Z_i = \sum_{i=4}^{5} Z_i = \frac{(-6x_1)}{(x_1 + x_2 + 1)}$$

$$VM = \sum_{i=1}^{r} \frac{AA_i}{r} = \frac{27}{12}, VN = \sum_{i=r+1}^{s} \frac{AL_i}{s-r} = \frac{3}{2}$$

$$S_1 = \frac{SM}{VM} = \frac{(18x_1 - 2x_2)}{(3x_1 + 3x_2 + 3)}, S_2 = \frac{SN}{VN} = \frac{(-4x_1)}{(x_1 + x_2 + 1)}$$

$$Max.Z = S_1 - S_2 = \frac{(30x_1 - 2x_2)}{(3x_1 + 3x_2 + 3)}$$

After solving it subject to the same constraints as before, we get
$Max\ .Z = 5$ and$x_1 = 1\ , x_2 = 0$
The solution for example 5.1 when applying algorithm in [9]by using median is the same optimal solution shown in table 1, then the combined objective linear fractional function is:

$$\text{To find median, first arrange} AA_i \text{ as: } \frac{1}{2}, 1, \frac{23}{12}, \frac{13}{4}$$

$$SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{4} \frac{(75x_1 + 10x_2)}{(6x_1 + 6x_2 + 6)}$$

$$SN = \sum_{i=r+1}^{s} Z_i = 0, WM = \frac{35}{24}, \qquad WN = 0$$

$$S_1 = \frac{SM}{WM} = \frac{\frac{(75x_1+10x_2)}{(6x_1+6x_2+6)}}{\frac{35}{24}} = \frac{(60x_1 + 8x_2)}{(7x_1 + 7x_2 + 7)}, S_2 = \frac{SN}{WN} = 0$$

$$Max.Z = S_1 - S_2 = \frac{(60x_1 + 8x_2)}{(7x_1 + 7x_2 + 7)}$$

After solving it subject to the same constraints as before, we get
$Max.Z = 4.57$ and $x_1 = 2, x_2 = 1$
The solution for example 5.2 when applying algorithm in [9] by using median is the same optimal solution shown in table 2, then the combined objective linear fractional function is:

$$\text{To find median, first arrange} AA_i \text{as : } \frac{3}{4}, \frac{3}{2}, \frac{9}{2} \text{ and } AL_i \text{as: } \frac{3}{2}, \frac{3}{2}$$

$$SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{3} Z_i = \frac{(27x_1 - 3x_2)}{(2x_1 + 2x_2 + 2)}$$

$$SN = \sum_{i=r+1}^{s} Z_i = \sum_{i=4}^{5} Z_i = \frac{(-6x_1)}{(x_1 + x_2 + 1)}, WM = \frac{3}{2}, WN = \frac{3}{2}$$

$$S_1 = \frac{SM}{WM} = \frac{\frac{(27x_1-3x_2)}{(2x_1+2x_2+2)}}{\frac{3}{2}} = \frac{(9x_1 - x_2)}{(x_1 + x_2 + 1)}$$

$$S_2 = \frac{SN}{WN} = \frac{\frac{(-6x_1)}{(x_1+x_2+1)}}{\frac{3}{2}} = \frac{(-4x_1)}{(x_1 + x_2 + 1)}$$

$$Max.Z = S_1 - S_2 = \frac{(13x_1 - x_2)}{(x_1 + x_2 + 1)}$$

After solving it subject to the same constraints as before, we get
$Max.Z = 6.5$ and $x_1 = 1, x_2 = 0$

6. SOLVING MOLFPP BY USING NEW TECHNIQUES

We formulate the combined objective function (4.6)as follows to determine the common set of decision variable.To solve MOLFPP by modified technique using average mean and average median, new average mean and new average median techniques consider below:

6.1 Solving MOLFPP by Using Average Mean and Average Median Techniques

$SM = \sum_{i=1}^{r} Z_i \,, SN = \sum_{i=r+1}^{s} z_i$

$Max.Z = \frac{SM-SN}{VM_2}$ (6.1.7)

$Max.Z = \frac{SM-SN}{WM_2}$ (6.1.8)

$VM_2 = \frac{VM+VN}{2}$ is average mean

$WM_2 = \frac{WM+WN}{2}$ is average median

Subject to the constraints(3.3) and(3.4)

VM: is the mean for all maximum value, $\forall\, i = 1,2,\dots,r$

VN: is the mean for all minimum value, $\forall\, i = r+1, r+2,\dots,s$

WM: is the median for all maximum value, $\forall\, i = 1,2,\dots,r$

WN: is the median for all minimum value, $\forall\, i = r+1,\dots,s$

6.1.1 Algorithm :(Using Average Mean and Average Median Techniques)

An algorithm for obtaining the optimal solution for the MOLFPP defined in equation (3.2) can be summarized as follows:

Step1: Assign arbitrary values to each of the individual objective functions which are to be maximized or minimized.

Step2: solve the first objective function by the modified simplex method, for linear fractional programming subject to constraints.

Step3: Check the feasibility of the solution obtained in step2, if it is feasible then go to step4, otherwise use dual simplex method to remove infeasibility.

Step4: Assign a name to the optimum value of the first objective function Z_1,sayφ_1.

Step5: Find the maximum mean, median of $\varphi_i, i = 1,2\dots,r$and find the minimum mean,median of $\varphi_i, i = r + 1,\dots,s$.

Step6: Construct the combined objective function which has formula(6.1.7) or (6.1.8).

Step7: Optimize the combined objective function under the same constraints (3.3)and(3.4)

The solution for example 5.1 when applying algorithm in the section 6.1.1 by using mean is the same optimal solution shown in table 1, then the combined objective linear fractional function is:

$$SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{4} Z_i = \frac{(75x_1 + 10x_2)}{(6x_1 + 6x_2 + 6)}, SN = \sum_{i=r+1}^{s} Z_i = 0$$

$$VM_2 = \frac{VM + VN}{2} = \frac{5}{6}$$

$$Max.Z = \frac{SM - SN}{VM_2} = \frac{\frac{(75x_1+10x_2)}{(6x_1+6x_2+6)}}{\frac{5}{6}} = \frac{(15x_1 + 2x_2)}{(x_1 + x_2 + 1)}$$

After solving it subject to the same constraints as before, we get

$Max.Z = 8$ and$x_1 = 2, x_2 = 1$

The solution for example 5.2 when applying algorithm in the section 6.1.1 by using mean is the same optimal solution shown in table 2, then the combined objective linear fractional function is:

$$SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{3} Z_i = \frac{(27x_1 - 3x_2)}{(2x_1 + 2x_2 + 2)}$$

$$SN = \sum_{i=r+1}^{s} Z_i = \sum_{i=4}^{5} Z_i = \frac{(-6x_1)}{(x_1 + x_2 + 1)}, VM_2 = \frac{VM + VN}{2} = \frac{15}{8}$$

$$Max.Z = \frac{SM - SN}{VM_2} = \frac{\frac{(39x_1 - 3x_2)}{(2x_1+2x_2+2)}}{\frac{15}{8}} = \frac{(52x_1 - 4x_2)}{(5x_1 + 5x_2 + 5)}$$

After solving it subject to the same constraints as before, we get

$Max\ .Z = 5.2$ and$x_1 = 1\ , x_2 = 0$

The solution for example 5.1 when applying algorithm in the section 6.1.1 by using median is the same optimal solution shown in table 1, then the combined objective linear fractional function is:

To find median, first arrangeAA_i as: $\frac{1}{2}, 1, \frac{23}{12}, \frac{13}{4}$

$$SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{4} Z_i = \frac{(75x_1 + 10x_2}{(6x_1 + 6x_2 + 6)}$$

$$SN = \sum_{i=r+1}^{s} Z_i = 0, WM_2 = \frac{WM + WN}{2} = \frac{35}{48}$$

$$Max.Z = \frac{SM - SN}{WM_2} = \frac{\frac{(75x_1+10x_2)}{(6x_1+6x_2+6)}}{\frac{35}{48}} = \frac{(120x_1 + 16x_2)}{(7x_1 + 7x_2 + 7)}$$

After solving it subject to the same constraints as before, we get

$Max.Z = 9.14$ and$x_1 = 2, x_2 = 1$

The solution for example 5.2 when applying algorithm in the section 6.1.1 by using median is the same optimal solution shown in table 2, then the combined objective linear fractional function is:

To find median, first arrangeAA_ias : $\frac{3}{4}, \frac{3}{2}, \frac{9}{2}$ and AL_ias: $\frac{3}{2}, \frac{3}{2}$

$$SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{3} Z_i = \frac{(27x_1 - 3x_2)}{(2x_1 + 2x_2 + 2)}$$

$$SN = \sum_{i=r+1}^{s} Z_i = \sum_{i=4}^{5} Z_i = \frac{(-6x_1)}{(x_1 + x_2 + 1)}, WM_2 = \frac{WM + WN}{2} = \frac{3}{2}$$

$$Max.Z = \frac{SM - SN}{WM_2} = \frac{\frac{(39x_1 - 3x_2)}{(2x_1+2x_2+2)}}{\frac{3}{2}} = \frac{(13x_1 - x_2)}{(x_1 + x_2 + 1)}$$

After solving it subject to the same constraints as before, we get

$Max.Z = 6.5$ and$x_1 = 1, x_2 = 0$

6.2Solving MOLFPP by Using New Average Mean and New Average Median Techniques

$$Max.Z = \frac{SM - SN}{M_s} \quad (6.2.9)$$

$$Max.Z = \frac{SM - SN}{WM_s} \quad (6.2.10\ )$$

$VM_s = \frac{VM + VN}{s}$ *is* new average mean and

$WM_s = \frac{WM + WN}{s}$ is new average median.

Subject to the some constraints (3.3) and (3.4) where VM_sdenoted the mean divided by s, WM_sdenoted the median divided by s, where s is the number of objective function.

6.2.1Algorithm :(Using New Average Mean and New Average Median Techniques)

An algorithm for obtaining the optimal solution for the MOLFPP defined in equation (3.2) can be summarized as follows:

Step1, step2, step3 and step4are the same as given in algorithm in the section 7 say $\varphi_i,\ i = 1,2,\dots,s$as before

Step5: Construct the combined objective function which has formula(6.2.9)or (6.2.10).

Step6: Optimize the combined objective function under the same constraints (3.3)and(3.4)

The solution for example 5.1 when applying algorithm in the section 9 by using mean is the same optimal solution shown in table 1, then the combined objective linear fractional function is:

$$SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{4} Z_i = \frac{(75x_1 + 10x_2)}{(6x_1 + 6x_2 + 6)}, SN = \sum_{i=r+1}^{s} Z_i = 0$$

$$VM_s = \frac{VM + VN}{S} = \frac{5}{12}$$

$$Max.Z = \frac{SM - SN}{VM_s} = \frac{\frac{(75x_1+10x_2)}{(6x_1+6x_2+6)}}{\frac{5}{12}} = \frac{(30x_1 + 4x_2)}{(x_1 + x_2 + 1)}$$

After solving it subject to the same constraints as before, we get

$Max.Z = 16$ and$x_1 = 2, x_2 = 1$

The solution for example 5.2 when applying algorithm in the section 6.2.1 by using mean is the same optimal solution shown in table 2, then the combined objective linear fractional function is:

$$SM = \sum_{i=1}^{r} Z_i = = \sum_{i=1}^{3} Z_i = \frac{(27x_1 - 3x_2)}{(2x_1 + 2x_2 + 2)}$$

$$SN = \sum_{i=r+1}^{s} Z_i = \sum_{i=4}^{5} Z_i = \frac{(-6x_1)}{(x_1 + x_2 + 1)}, VM_s = \frac{VM + VN}{S} = \frac{15}{20}$$

$$Max.Z = \frac{SM - SN}{VM_s} = \frac{\frac{(39x_1-3x_2)}{(2x_1+2x_2+2)}}{\frac{15}{20}} = \frac{(78x_1 - 6x_2)}{(3x_1 + 3x_2 + 3)}$$

After solving it subject to the same constraints as before, we get

$Max\ .Z = 13$ and$x_1 = 1\ , x_2 = 0$

The solution for example 5.1 when applying algorithm in the section 6.2.1 by using median is the same optimal solution shown in table 1, then the combined objective linear fractional function is:

To find median, first arrangeAA_i as: $\frac{1}{2}, 1, \frac{23}{12}, \frac{13}{14}$

$$SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{4} Z_i = \frac{(75x_1 + 10x_2)}{(6x_1 + 6x_2 + 6)}, \qquad SN = \sum_{i=r+1}^{s} Z_i = 0$$

$$WM_s = \frac{WM + WN}{S} = \frac{35}{96}$$

$$Max.Z = \frac{SM - SN}{WM_s} = \frac{\frac{(75x_1+10x_2)}{(6x_1+6x_2+6)}}{\frac{35}{96}} = \frac{(240x_1 + 32x_2)}{(7x_1 + 7x_2 + 7)}$$

After solving it subject to the same constraints as before, we get

$Max.Z = 18.28$ and$x_1 = 2, x_2 = 1$

The solution for example 5.2 when applying algorithm in the section 6.2.1 by using median is the same optimal solution shown in table 2, then the combined objective linear fractional function is:

To find median, first arrange AA_i as : $\frac{3}{4}, \frac{3}{2}, \frac{9}{2}$ and AL_i as: $\frac{3}{2}, \frac{3}{2}$

$$SM = \sum_{i=1}^{r} Z_i = = \sum_{i=1}^{3} Z_i = \frac{(27x_1 - 3x_2)}{(2x_1 + 2x_2 + 2)}$$

$$SN = \sum_{i=r+1}^{s} Z_i = \sum_{i=4}^{5} Z_i = \frac{(-6x_1)}{(x_1 + x_2 + 1)}, \qquad WM_s = \frac{WM + WN}{S} = \frac{3}{5}$$

$$Max.Z = \frac{SM - SN}{WM_s} = \frac{\frac{(39x_1 - 3x_2)}{(2x_1 + 2x_2 + 2)}}{\frac{3}{5}} = \frac{(65x_1 - 5x_2)}{(2x_1 + 2x_2 + 2)}$$

After solving it subject to the same constraints as before, we get

$$Max.Z = 16.25 \text{ and } x_1 = 1, x_2 = 0$$

7. COMPARISON OF THE NUMERICAL RESULTS

Now, we are going to comparison the numerical results which are obtained of the examples as below in table 3:

Table 3: Comparison between results of the numerical techniques

Techniques		Example 5.1:	Example 5.2:
Chandra Sen. Technique		$Max.Z = 4$	$Max.Z = 5$
Mean Technique		$Max.Z = 4$	$Max.Z = 5$
Median Technique		$Max.Z = 4.57$	$Max.Z = 6.5$
Average Techniques	Mean Technique	$Max.Z = 8$	$Max.Z = 5.2$
	Median Technique	$Max.Z = 9.14$	$Max.Z = 6.5$
New Average Techniques	Mean Technique	$Max.Z = 16$	$Max.Z = 13$
	Median Technique	$Max.Z = 18.28$	$Max.Z = 16.5$

In table 3; it is clear that the results obtained in examples 5.1, 5.2 when using new average technique are better than other results which are obtained by using new average mean and new average median, techniques.

8. DISCUSSION

In This paper, we have defined and discussed a number of techniques, which we have used in order to get the optimal solution of the MOLFPP. The comparisons of these techniques are based on the values of the objective functions; therefore we have tested two numerical examples. To show the best technique among these techniques we have obtained that new average mean and new average median was better than the techniques namely Chandra Sen., mean and median, and average mean and average median, and new average mean and new average median techniques, table 3was presented. We have used MATLAB program version [7.12.0.635 (R2011a)].

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