# A FAMILY OF TRAVERSABLE WORMHOLE

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**ABSTRACT**
In this paper, we constructed wormhole solutions, governed by the equation of state where it violates the null energy condition, so that P + $\rho$ < 0 , important ingredient to sustain traversable wormhole. Considering generic shape function, several exact solutions are found. Some physical properties and characteristics of these solutions are investigated.

**Keywords**: *Null energy condition, generic shape function, wormhole solutions.*

## 1. INTRODUCTION

One of the fundamental problems in the general theory of relativity is finding exact solutions of the Einstein field equations [1,2]. Some solutions found fundamental applications in astrophysics, cosmology and, more recently, in the developments inspired by string theory [2]. Different mathematical formulations that allow to solve Einstein's field equations have been used to describe the behavior of objects submitted to strong gravitational fields known as neutron stars and ultracompact objects [3,4,5]. Similarly, classical wormhole, just as black holes, represent consistent solutions of Einstein´s theory of general relativity [6]. Morris y Thorne [7] defined wormhole as a tunnel in the spacetime linking different universes or widely separated regions of our own universe.

It is generally accepted that the Universe is undergoing an accelerated phase of expansion and this cosmic acceleration is one most current problems in cosmology [8]. Several candidates as a dark energy models, phantom model, Chaplygin gas model, modified gravity and braneworld models, amongst others, has been proposed [9,10,11]. Configurations resulting from these theories could be potential candidates to ocurr in a natural way and are of great astrophysical interest [12].

Lobo [9] proposed a exotic form of energy, denoted phantom energy that posseses peculiar properties, such as the violation of null energy condition (NEC), which is the fundamental ingredient to sustain traversable wormhole, defined by $p + \rho < 0$, where $\rho$ is the matter energy density and $p$ is the spatially homogeneous pressure. The notion of phantom energy is that of a homogeneously distributed fluid but it can extend to inhomogeneous spherically symmetric spacetime configurations, for what in the equation of state $\omega = p/\rho < -1$, $p$ is the radial pressure and the transverse pressure is the determined via the field equations.

In the recent paper, Lobo and Oliveira [13] discuss the possibility of the existence traversable wormhole geometries in the context of f(R) modified theories of gravity and Lemos et al. [12] find some exact solutions considering specific shape functions and several equations of state.

In this work, it proposes an equation of state what depends of an adjustable parameter and that one presents a dependence of the pressure with the density, there are solved the equations of gravitational field and we obtain new exact solutions for traversable wormhole. The physical properties of these solutions are studied and compared to some previous known models [12] .

This article is organized as follows: In Sec. II, we present Einstein´s field equations and a general solution of a traversable wormhole. In Sec. III, we construct specific traversable wormhole geometries from a shape funcion that it depends on a adjustable parameter n. In Sec. IV, we outline a general stability analysis procedure and we conclude in Sec. V.

## 2. TRAVERSABLE WORMHOLE SPACETIMES AND FIELD EQUATIONS

The spacetime metric representing a spherically symmetric and static wormhole is given by

$$ds^2 = -e^{-2\Phi(r)}dt^2 + \frac{dr^2}{1-b(r)/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{1}$$

where $\Phi(r)$ and b(r) are arbitrary functions of the radial coordinate, r, denoted as the redshift function, and the shape function, respectively [14,15,16]. A fundamental property of a wormhole is that in the throat $r = r_0$ the shape function is $b(r_0) = r = r_0$ and $b'(r_0) < 1$ [7]. Other conditions that it will have to fulfil it is $1 - \frac{b(r)}{r} > 0$ and the redshift function $\Phi(r)$ must be everywhere finite [13,14]. It is precisely these restrictions that impose the NEC violation.

Using the Einstein field equation,

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \tag{2}$$

for anisotropic distribution of matter , the energy –momentum tensor energy given by

$$T^{\mu\nu} = (\rho + P_t)U^{\mu}U^{\nu} - P_t g^{\mu\nu} + (P_r - P_t)\chi^{\mu}\chi^{\nu} \tag{3}$$

where $U^{\mu}$ is the four-velocity, $\chi^{\mu}$ is the unit spacelike vector in the radial direction, $\rho(r)$ is the energy density, $p_r$ is the radial pressure measured in the direction of $\chi^{\mu}$ and $p_t$ is the transverse pressure measured in the orthogonal direction to $\chi^{\mu}$. The stress-energy tensor components [9] are given by

$$\rho(r) = \frac{b'}{8\pi r^2} \tag{4}$$

$$p_r = \frac{1}{8\pi}\left[-\frac{b}{r^3} + 2\left(1 - \frac{b}{r}\right)\frac{\Phi'}{r}\right] \tag{5}$$

$$p_t = \frac{1}{8\pi}\left(1 - \frac{b}{r}\right)\left[\Phi'' + (\Phi')^2 - \frac{b'r - b}{2r(r-b)}\Phi' - \frac{b'r - b}{2r^2(r-b)} + \frac{\Phi'}{r}\right] \tag{6}$$

The conservation of the stress-energy tensor, $T^{\mu\nu}{}_{;\nu} = 0$ provides us with the following relationship

$$p_r' = \frac{2}{r}(p_t - p_r) - (\rho + p_r)\Phi' \tag{7}$$

where $\Phi'$ is given by

$$\Phi' = \frac{-8\pi\rho r^3 + b}{2r(r-b)} \tag{8}$$

The prime denotes a derivate with respect to the radial coordinate, r.

## 3. TRAVERSABLE WORMHOLE MODELS

Based on the wormhole model proposed for Lemos et al. where $b(r) = \sqrt{rr_0}$ [6,12] for what $\frac{p_r}{\rho} = -2$ [6], in this article, we shall construct exact solutions of traversable wormhole considering a phantom energy equation of state what depend on a n adjustable parameter where $p = -(n+1)\rho$, with violates the null energy condition $p + \rho < 0$ . Some known solutions may be recovered for a specific values of these parameter. To find phantom energy traversable wormhole spacetimes, we use equation of state $p_r = -(n+1)\rho$ with $p_r$ as radial pressure. The equations (4) and (5) allow to deduce

$$\Phi' = \frac{b - (n+1)rb'}{2r^2\left(1 - \frac{b}{r}\right)} \tag{9}$$

Considering $\Phi'(r) = 0$ and the conditions $b(r_0) = r = r_0$ in the wormhole throat, from Eq. (9) one obtains

$$b(r) = \sqrt[n+1]{r_o^{\,n} r} \tag{10}$$

That is the shape function dependent on an adjustable parameter n.
For substitution of (4) in (8), we have

$$\Phi' = \frac{b - rb'}{2r^2\left(1 - \frac{b}{r}\right)} \tag{11}$$

where the equation for $b'$ is

$$b'(r) = \frac{r_0^{\,n}}{(n+1)\left(r_0^{\,n} r\right)^{\frac{n}{n+1}}} \tag{12}$$

Now, replacing (10) and (12) in (11), we obtain

$$\Phi' = \frac{n\left(r_0^{\,n} r\right)^{\frac{1}{n+1}}}{2(n+1)r^2\left[1 - \frac{\left(r_0^{\,n} r\right)^{\frac{1}{n+1}}}{r}\right]} \tag{13}$$

Integrating this equation we have an expression for the redshift function given for

$$\Phi(r) = \frac{1}{2}\ln\left\{1 - \left[\left(\frac{r_0}{r}\right)^{1/n+1}\right]^n\right\} \tag{14}$$

For (10) we also verified that $b(r_o) = r_0$, so that at the throat the condition $b'(r_0) = 1/n+1 < 1$ is satisfied for values of n < -1 and n > 0. The metric inside $r_0 \le r \le a$ is

$$ds^2 = -\left\{1 - \left[\left(\frac{r_0}{a}\right)^{1/n+1}\right]^n\right\}dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{n/n+1}} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{15}$$

The stress-energy tensor components $\rho$, $P_r$ and $P_t$, are given by

$$\rho = \frac{1}{8\pi(n+1)r^2}\sqrt[n+1]{\left(\frac{r_0}{r}\right)^n} \tag{16}$$

$$p_r = -\frac{1}{8\pi r^2}\sqrt[n+1]{\left(\frac{r_0}{r}\right)^n} \tag{17}$$

$$p_t = \frac{n}{16\pi(n+1)r^3}\sqrt[n+1]{r_0^{\,n} r} \tag{18}$$

NEC is violated since

$$p_r + \rho = -\frac{n}{(n+1)8\pi r^2}\sqrt[n+1]{\left(\frac{r_0}{r}\right)^n} < 0 \tag{19}$$

for the values of n allowed.

The quotient $\frac{p_r}{\rho} = -(n+1)$ ; for n=1 and n=-2, we obtain

$$b(r) = \sqrt{r_o\ r} \text{ and } b(r) = r_0^2 / r \tag{20}$$

shape functions proposed by Lobo [9,11], Lemos et al. [12] and Lobo and Oliveira [13]. For the particular case in which n=1, it is equivalent to $\omega$ =-2 [9], we would have an phantom equation of state [6], but the metric properties are different to (10).

For n=-1.5, we have found a new solution of a wormhole where the redshift and shape functions given by

$$\Phi(r) = \frac{1}{2}\ln\left[1 - \left(\frac{r_0}{a}\right)^3\right] \tag{21}$$

$$b(r) = r_0^3 / r^2 \tag{22}$$

respectively.

It is considered isotropic pressure, $p_r = p_t = p$, Eq. (7) is given for

$$p_r' = -(\rho + p_r)\Phi' \tag{23}$$

Using (19) in (23) we have

$$\frac{3n+2}{r} = n\Phi' \tag{24}$$

integrating (24), we obtain redshift function for isotropic pressure is

$$\Phi(r) = \left(\frac{3n+2}{n}\right)\ln\left(\frac{r}{r_0}\right) \tag{25}$$

Expression similar to the obtained for Lobo for the redshift function with isotropic pressure [9].

## 4. STABILITY ANALYSIS

Using the Darmois-Israel formalism [17,18] , the surface stress-energy tensor, $S^i{}_j$ , at the junction interface $\Sigma$ is provided by the Lanczos equations

$$S^i{}_j = -\frac{1}{8\pi}\left(\kappa^i{}_j - \delta^i{}_j \kappa^k{}_k\right) \quad , \tag{26}$$

where $K_{ij}$ is the discontinuity of the extrinsic curvatures across the surface $\Sigma$. The extrinsic curvatures is defined as $K_{ij} = \eta_{\mu;\nu} e^\mu{}_{(i)} e^\nu{}_{(j)}$ , where $\eta^\mu$ is the unit normal 4-vector to $\Sigma$, and $e^\mu{}_{(i)}$ are the components of the holonomic basis vectors tangent to $\Sigma$ .

The Lanczos equation (26), provide us with the following expressions for the surface stresses

$$\sigma = -\frac{1}{4\pi a}\left(\sqrt{1 - \frac{2M}{a} + \dot{a}^2} - \sqrt{1 - \frac{b(a)}{a} + \dot{a}^2}\right) \tag{27}$$

$$\text{P} = \frac{1}{8\pi a}\left[\frac{1 - \frac{M}{a} + \dot{a}^2 + a\ddot{a}}{\sqrt{1 - \frac{2M}{a} + \dot{a}^2}} - \frac{(1 + a\Phi')\left(1 - \frac{b}{a} + \dot{a}^2\right) + a\ddot{a} - \frac{\dot{a}^2(b - b'a)}{2(a-b)}}{\sqrt{1 - \frac{b(a)}{a} + \dot{a}^2}}\right] \tag{28}$$

where σ and P are the surface energy density and the tangential surface pressure, respectively.

We follow closely the Lobo and Crawford's [18] approach to analyze stability of wormhole. They included the momentum and flux term in the conservation identity, deduced from the Lanczos equation [19] and the ADM constraint, into the Ishak and Lake analysis [20], to deduce the master equation which governs the stable equilibrium. We shall describe the approach briefly, details can be found in Lobo and Crawford [18] and reference there in.

The equation of motion of the thin shell is obtained from (27) and the resulting potential V(a) is linearized by a Taylor expansion around the equilibrium radio $a_0$ of the static solution, to the second order.

$$\dot{a}^2 + V(a) = 0 \tag{29}$$

With V(a) defined as:

$$V(a) = \frac{1}{2}\left[\left(1-\frac{2m}{a}\right)+\left(1-\frac{2M}{a}\right)\right]-\left(\frac{m_s}{2a}\right)^2-\left\{\left(\frac{a}{m_s}\right)\frac{1}{2}\left[\left(1-\frac{2m}{a}\right)-\left(1-\frac{2M}{a}\right)\right]^2\right\} \tag{30}$$

Next, one imposes $V(a_0)=0$ and $V'(a_0)=0$, therefore the solution will be stable if and only if V(a) has a local minimum at $a_0$ and $V''(a_0)>0$ is verified.

$$V''(a) = \frac{1}{2}\left[\left(1-\frac{2m}{a}\right)+\left(1-\frac{2M}{a}\right)\right]''-2\left(\frac{m_s}{2a}\right)\left(\frac{m_s}{2a}\right)''-\left\{\left[\left(\frac{a}{m_s}\right)\frac{1}{2}\left[\left(1-\frac{2m}{a}\right)-\left(1-\frac{2M}{a}\right)\right]\right]^2\right\}'' \tag{31}$$

The latter stability condition may be written as

$$\left(\frac{m_s}{2a}\right)\left(\frac{m_s}{2a}\right)'' < \Psi - \Gamma^2 \tag{32}$$

Where Γ is defined from the condition of $V'(a_0)=0$ as

$$\Gamma = \left(\frac{a_0}{m_s}\right)\left\{\frac{1}{2}\left[\left(1-\frac{2m}{a}\right)+\left(1-\frac{2M}{a}\right)\right]'-\left\{\left[\left(\frac{a}{m_s}\right)\frac{1}{2}\left[\left(1-\frac{2m}{a}\right)-\left(1-\frac{2M}{a}\right)\right]\right]^2\right\}'\right\}_{a_0} \tag{33}$$

And Ψ is given as

$$\Psi = \frac{1}{2}\left[\left(1-\frac{2m}{a}\right)+\left(1-\frac{2M}{a}\right)\right]''-\left(\frac{a}{m_s}\right)\frac{1}{2}\left[\left(1-\frac{2m}{a}\right)-\left(1-\frac{2M}{a}\right)\right]\left[\left(\frac{a}{m_s}\right)\frac{1}{2}\left[\left(1-\frac{2m}{a}\right)-\left(1-\frac{2M}{a}\right)\right]\right]'$$
$$-\left\{\left[\left(\frac{a}{m_s}\right)\frac{1}{2}\left[\left(1-\frac{2m}{a}\right)-\left(1-\frac{2M}{a}\right)\right]\right]'\right\}^2 \tag{34}$$

To obtain an expression for $(m_s/2a)''$ we shall make use of the conservation law [2,3] given by

$$S^i_{j|i} = \left[T_{\mu\nu}e^{\mu}_{(j)}\eta^{\nu}\right]^+_- \tag{35}$$

Eq.(32) provides us with

$$\sigma' = -\frac{2}{a}(\sigma + \mathrm{P}) + \Xi \tag{36}$$

where Ξ , defined for notational convenience, is given by

$$\Xi = -\frac{1}{4\pi a^2}\left[\frac{b'a-b}{2a(1-\frac{b}{a})}+a\Phi'\right]\sqrt{1-\frac{b}{a}+\dot{a}^2} \tag{37}$$

Using $m_s = 4\pi a^2\sigma$ and taking into account the radial derivate of σ´ , Eq. (33) can be rearranged to provide the following relationship

$$\left(\frac{m_s}{2a}\right)'' = \gamma - 4\pi\sigma'\eta \tag{38}$$

with the parameter η defined as η = P´/σ´ and γ given by

$$\gamma = \frac{4\pi}{a}(\sigma + \mathrm{P}) + 2\pi a\Xi' \tag{39}$$

Equations (33)-(39) in (32) are used to determine the stability regions of the respective solutions, where η is used as a parametrization of the stable equilibrium, so that there is no need to specify a surface equation of state [9].

Following Poisson and Visser [21] and Ishak and Lake [20], the parameter $\sqrt{\eta}$ is defined as the speed of sound [9,10]. Then, taking into consideration the requirement that the speed of sound should not exceed the speed of light, the values of η are constrained to $0 < \eta \leq 1$ . However, in the presence of exotic matter this condition can be violated. Therefore, in this work we consider values of η out of the previous interval.

For the analysis of stability, they were considered to be some specific cases for the redshift and shape funcions. The first case to considering is given for (21) and (22) , respectively. The radial derivate of the surface energy density $\sigma'$ [9], evaluated at the static solution, takes the following form

$$\sigma' = \frac{1}{4\pi a_0^2}\left( \frac{1-\frac{3M}{a_0}}{\sqrt{1-\frac{2M}{a_0}}} - \frac{1-\frac{5}{2}(\frac{r_0}{a_0})^3}{\sqrt{1-(\frac{r_0}{a_0})^3}} \right) \tag{40}$$

In the figures 1 and 2, we show the values of $\eta$ as a function of $\alpha = a_0/M$ for this solution. In agreement with Lobo [9], to determine the stability regions, we shall consider the following cases: $r_0/M = 1$ and $r_0/M = 0.5$, so that $2 < a_0/M < 4$ ; and for $r_0/M = 0.25$ and $r_0/M = 1.5$, we have $2 < a_0/M < 3.5$. Note that as the stability regions increase slightly for low values of $r_0/M$ but for $r_0/M = 1.5$ there is a valuable increase of these regions. For this particular case the surface energy density is positive.

For n=-2, we obtain $b(r) = r_0^2/r$ , shape function presented by Lobo and Oliveira [13] and Lemos et al. [16]. This case is represented in Figures 3 and 4, and the same trend appears that in the previous example, that is the stability regions only show a pronounced increase for $r_0/M = 1.5$. Again, also it has been considered that $\sigma > 0$.

In the Figures 5 and 6, for a fixed value n=2, we consider the particular cases of $r_0/M = 1.0$, so that $2 < a_0/M < 4$; $r_0/M = 0.5$, $2 < a_0/M < 8$; then, for $r_0/M = 0.25$ and $r_0/M = 1.5$, we have $2 < a_0/M < 16$. For the case $r_0/M = 1.5$, a vertical asymptote exists in $R/M \cong 5.754$. Note that the stability regions increase, for increasing values of $r_0/M$ and the values of η become less restricted. The radial derivate of the surface energy density is

$$\sigma' = \frac{1}{4\pi a_0^2}\left( \frac{1-\frac{3M}{a_0}}{\sqrt{1-\frac{2M}{a_0}}} - \frac{1-\frac{4}{3}\sqrt[3]{\left(\frac{r_0}{a_0}\right)^2}}{\sqrt{1-\sqrt[3]{\left(\frac{r_0}{a_0}\right)^2}}} \right) \tag{41}$$

The range of $a_0$ diminishes between $0.25 < r_0/M < 1.0$ and increases suddenly for $r_0/M$ =1.5.

For any n, it is possible then establish a general expression for $\sigma$ evaluated at a static solution $a_0$, in fact

$$\sigma = -\frac{1}{4\pi a}\left( \sqrt{1-\frac{2M}{a}} + -\sqrt{1-\sqrt[n+1]{\left(\frac{r_0}{a_0}\right)^n}} \right) \tag{42}$$

The equation for $\sigma'$ takes the form

$$\sigma' = \frac{1}{4\pi a_0^{\;2}}\left(\frac{1-\frac{3M}{a_0}}{\sqrt{1-\frac{2M}{a_0}}} - \frac{1-\left[\frac{3n+2}{2(n+1)}\right]\sqrt[n+1]{\left(\frac{r_0}{a_0}\right)^n}}{\sqrt{1-\sqrt[n+1]{\left(\frac{r_0}{a_0}\right)^n}}}\right) \tag{43}$$

For a given value of n and ${r_0}/{M}$ the vertical asymptotes exist if $\sigma' = 0$.

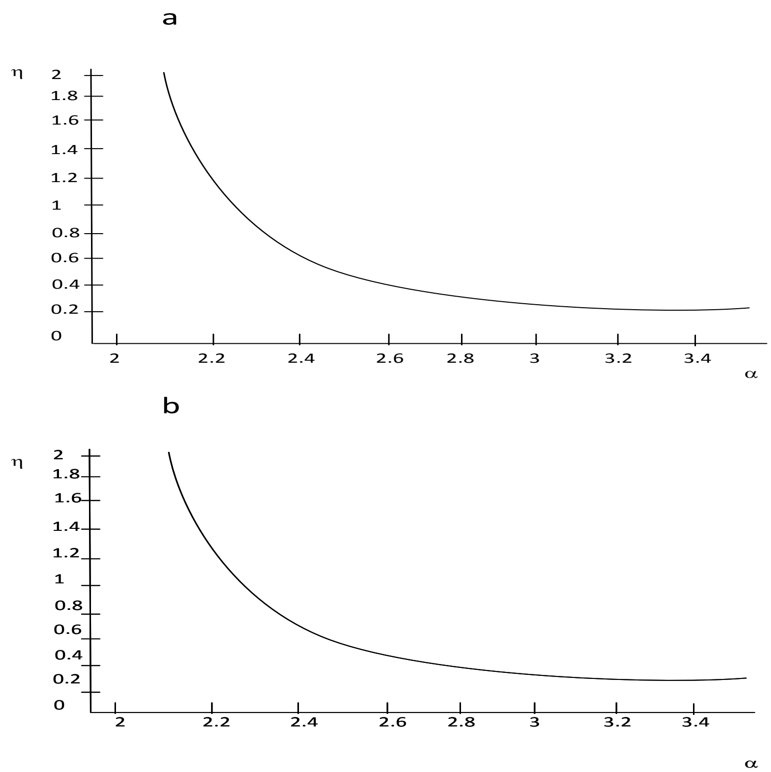



**Figure 1**. The solution for the model with n=-1.5. We have defined $\alpha = a_0 / M$ . The plots a and b correspond to $r_0 / M = 0.25$ and $r_0 / M = 0.5$, respectively . In both cases surface energy density is positive.

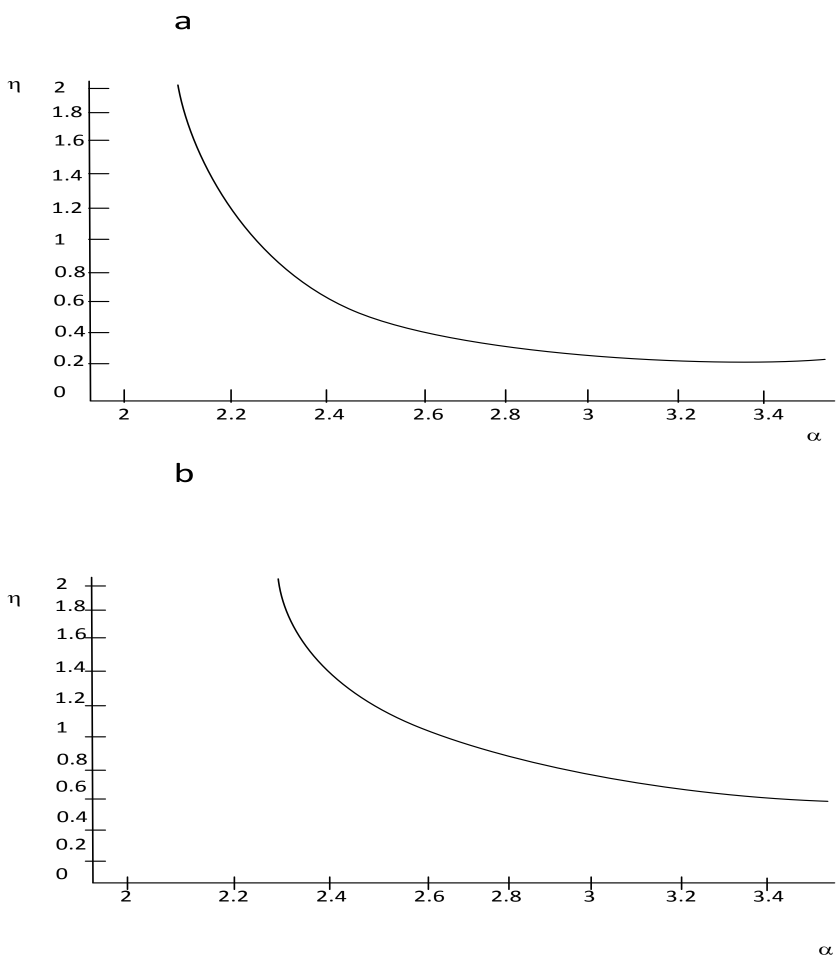



**Figure 2.** The solution for the model with n=-1.5 where $\alpha = a_0 / M$ and the plots a and b correspond to $r_0 / M = 1$ and $r_0 / M = 1.5$, respectively . In both cases surface energy density is positive.

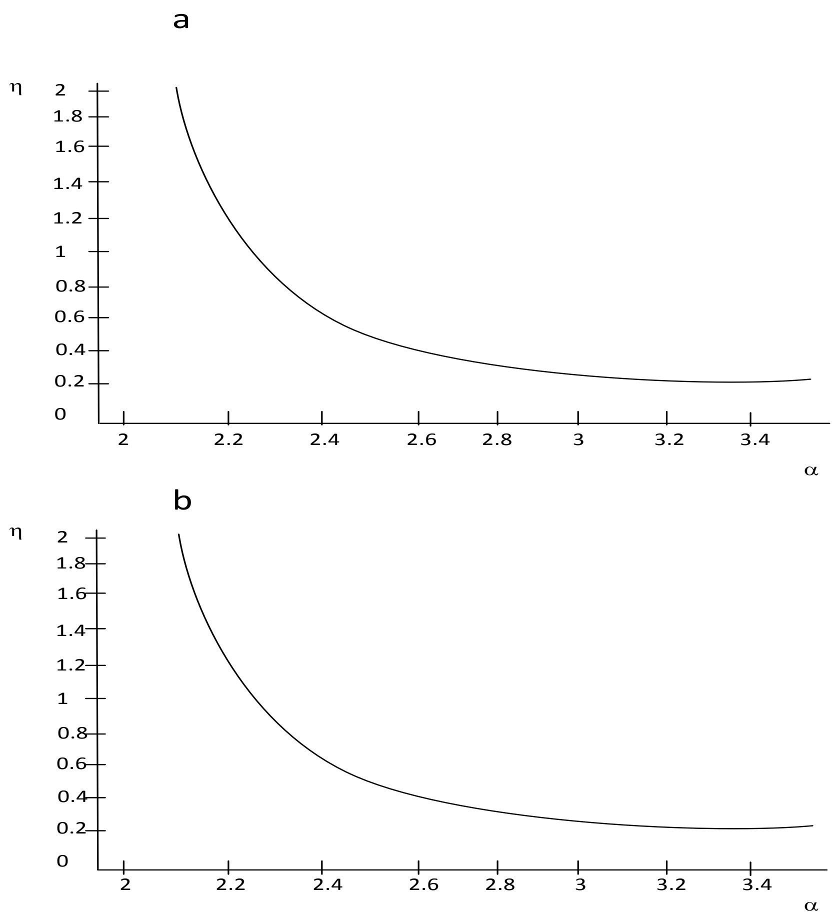
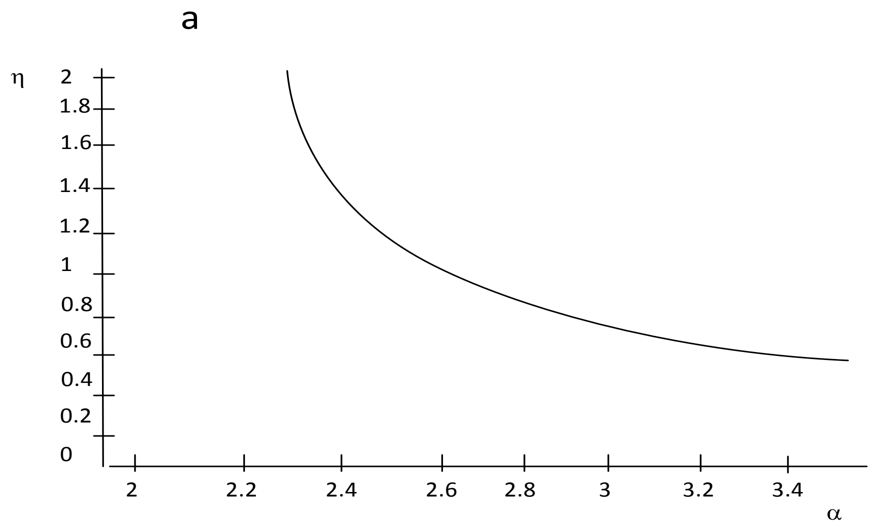



**Figure 3.** The solution for the model with n= -2.0. Again $\alpha = a_0 / M$ and the plots a and b correspond to $r_0 / M = 0.25$ and $r_0 / M = 0.5$, respectively. Also in both cases surface energy density is positive.



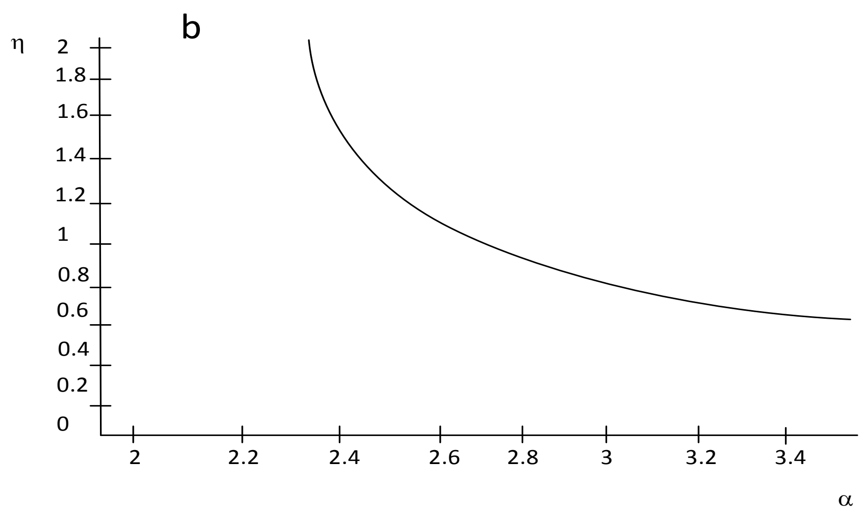
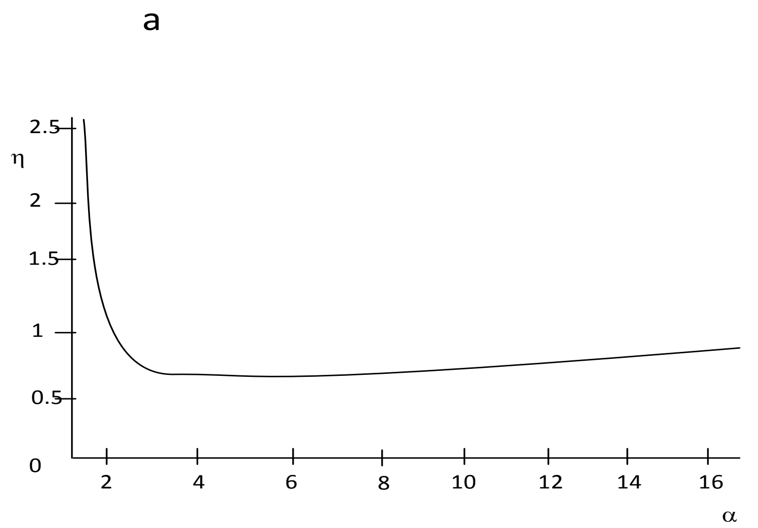




**Figure 4.** The solution for the model with n=- 2.0. Again $\alpha = a_0 / M$ and the plots a and b correspond to $r_0 / M = 1$ and $r_0 / M = 1.5$, respectively. Also in both cases surface energy density is positive.



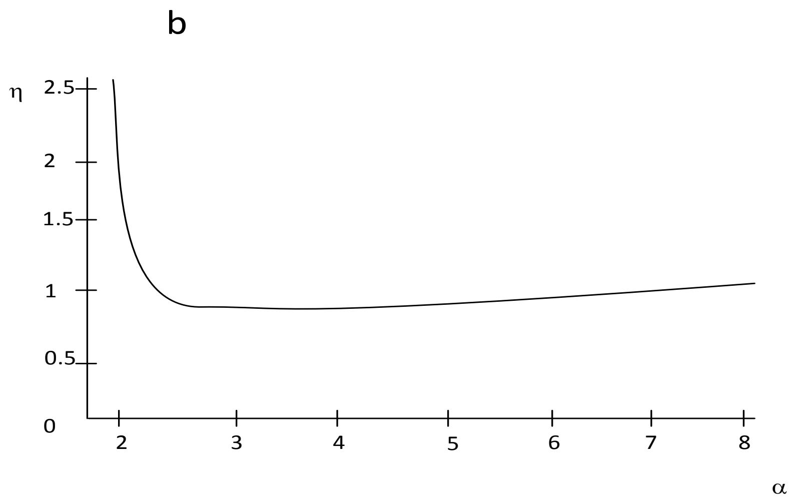




**Figure 5.** The solution for the model with n= 2.0. Again $\alpha = a_0 / M$ and the plots a and b correspond to $r_0 / M = 0.25$ and $r_0 / M = 0.5$, respectively. In the first plot , surface energy density is positive and the plot b , $\sigma < 0.$

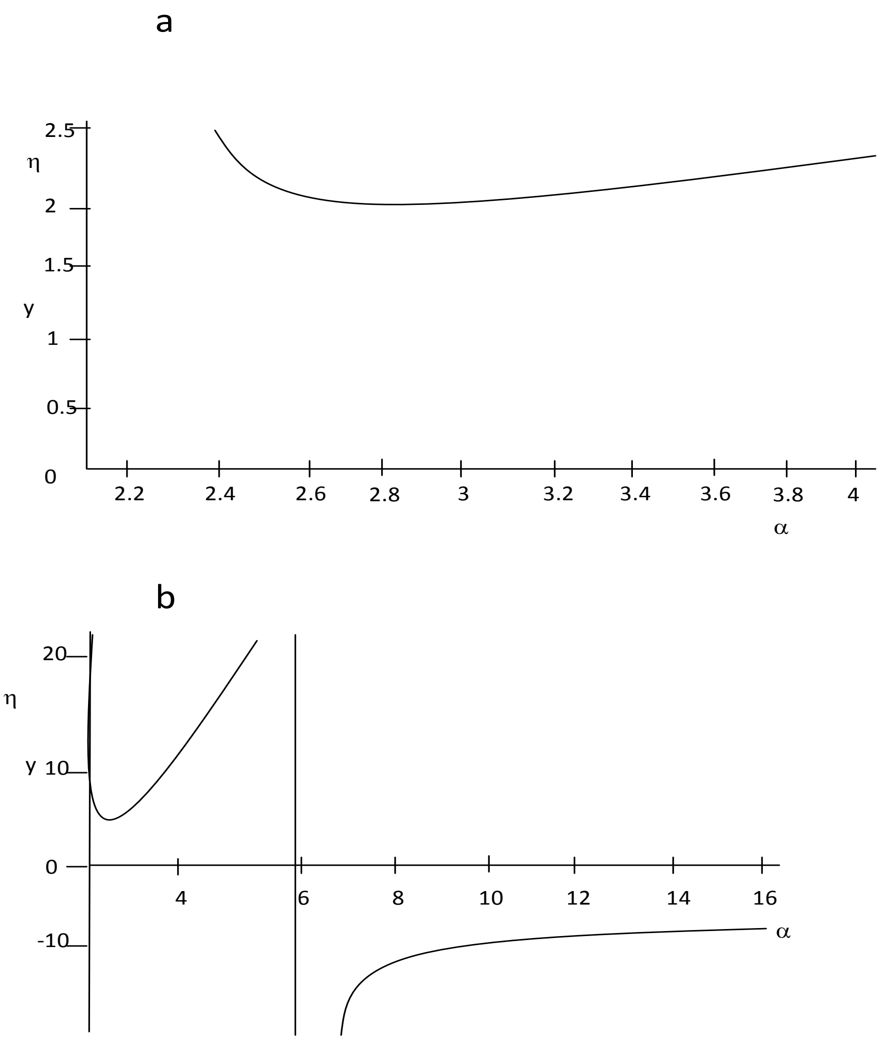



**Figure 6.** The solution for the model with n= 2.0. Again $\alpha = a_0 / M$ and the plots a and b correspond to $r_0 / M = 1$ and $r_0 / M = 1.5$, respectively. In the first plot $\sigma > 0$, and the plot b , $\sigma < 0$.

## 5. CONCLUSION

In this paper we have constructed a new family of exact wormhole solutions considering a generic shape function that depends on an adjustable parameter n and that satisfies all conditions that are required to represent a wormhole. The obtained solutions have revealed and interesting and important feature of the wormholes. Some know solutions may be recovered for specific values of this parameter. For the case in that n=2, we have a phantom equation of state, equivalent to $\omega = -3$. When n=1 and $r_0/M = 1.5$, an asymptote exists in R/M=4.24, which coincides with the result of Lobo [9] for $\omega = -2$. We have analyzed the stability of these wormholes and found that for the cases in which n <0, the surface energy density $\sigma$ is positive and the stability regions do not present valuable variations when it changes the wormhole throat, whereas for n> 0 , the stability regions increase greatly when $r_0/M$ increases, but the surface energy density will be positive or negative, depending on the values of $r_0/M$.

As a final remark, the wormholes can be used for interstellar travel, with what the long distances would be crossed in very short times and also they might be used to design time machines, violating causality associated with the time travel [9,19] .

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